Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.5-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [123]. This is test number [36].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (123)	% 0.00 (0)
Mathematica	% 100.00 (123)	% 0.00 (0)
Maple	% 98.37 (121)	% 1.63 (2)
Maxima	% 54.47 (67)	% 45.53 (56)
Fricas	% 90.24 (111)	% 9.76 (12)
Sympy	% 34.96 (43)	% 65.04 (80)
Giac	% 73.17 (90)	% 26.83 (33)
Mupad	% 43.09 (53)	% 56.91 (70)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

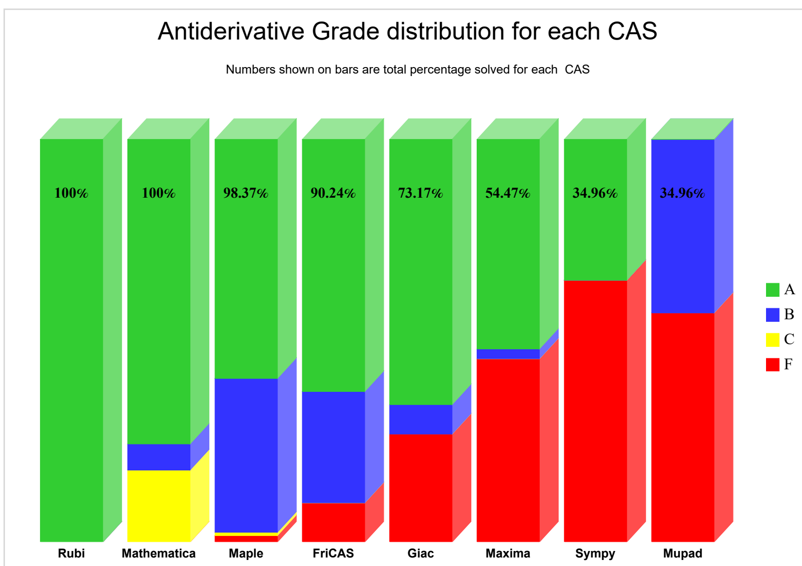
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

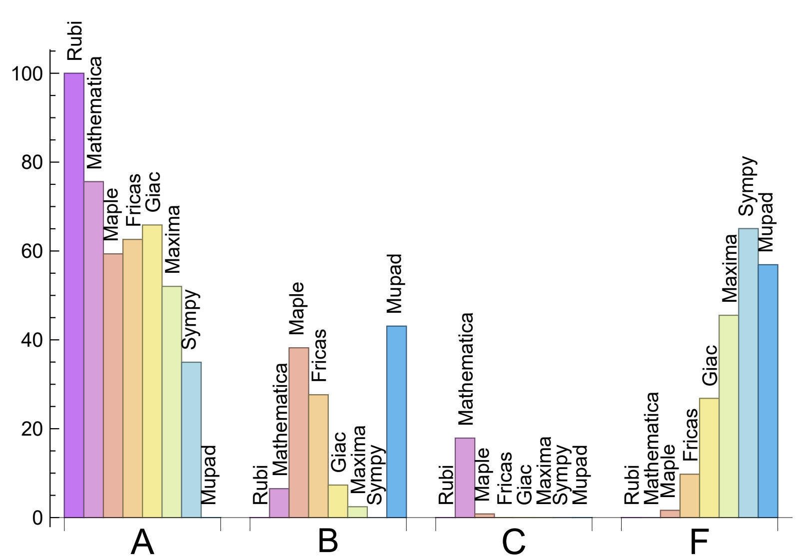
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.61	6.50	17.89	0.00
Maple	59.35	38.21	0.81	1.63
Maxima	52.03	2.44	0.00	45.53
Fricas	62.60	27.64	0.00	9.76
Sympy	34.96	0.00	0.00	65.04
Giac	65.85	7.32	0.00	26.83
Mupad	0.00	43.09	0.00	56.91

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	56	64.29 %	0.00 %	35.71 %
Fricas	12	41.67 %	58.33 %	0.00 %
Sympy	80	82.50 %	17.50 %	0.00 %
Giac	33	18.18 %	15.15 %	66.67 %
Mupad	70	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

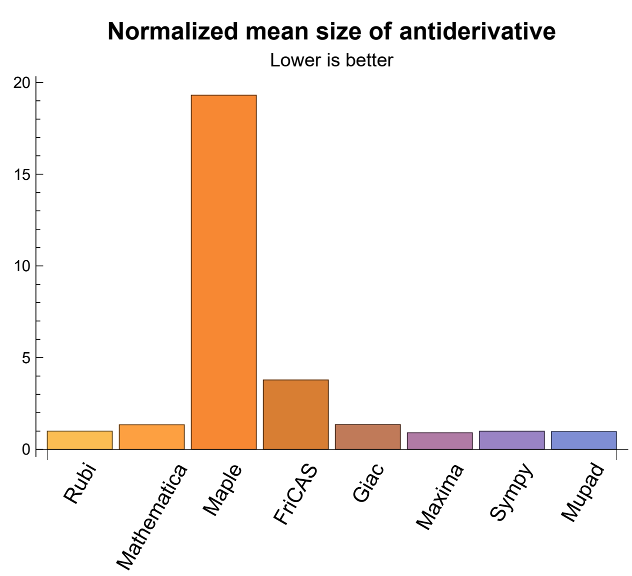
1.3 Performance

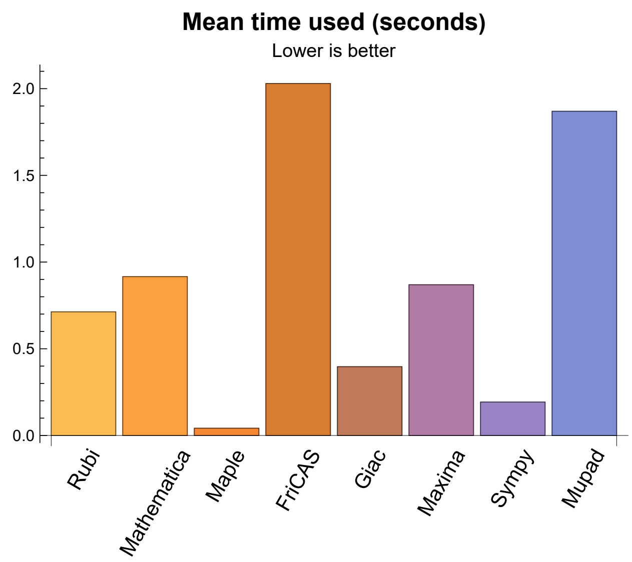
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.71	205.62	1.00	128.00	1.00
Mathematica	0.92	322.87	1.34	95.00	1.00
Maple	0.04	6251.50	19.31	136.00	0.82
Maxima	0.87	90.12	0.90	72.00	0.83
Fricas	2.03	827.68	3.79	112.00	1.24
Sympy	0.19	76.86	0.99	73.00	0.98
Giac	0.40	203.17	1.35	72.50	0.79
Mupad	1.87	105.40	0.96	64.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {6, 106, 107, 108, 113, 122, 123}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

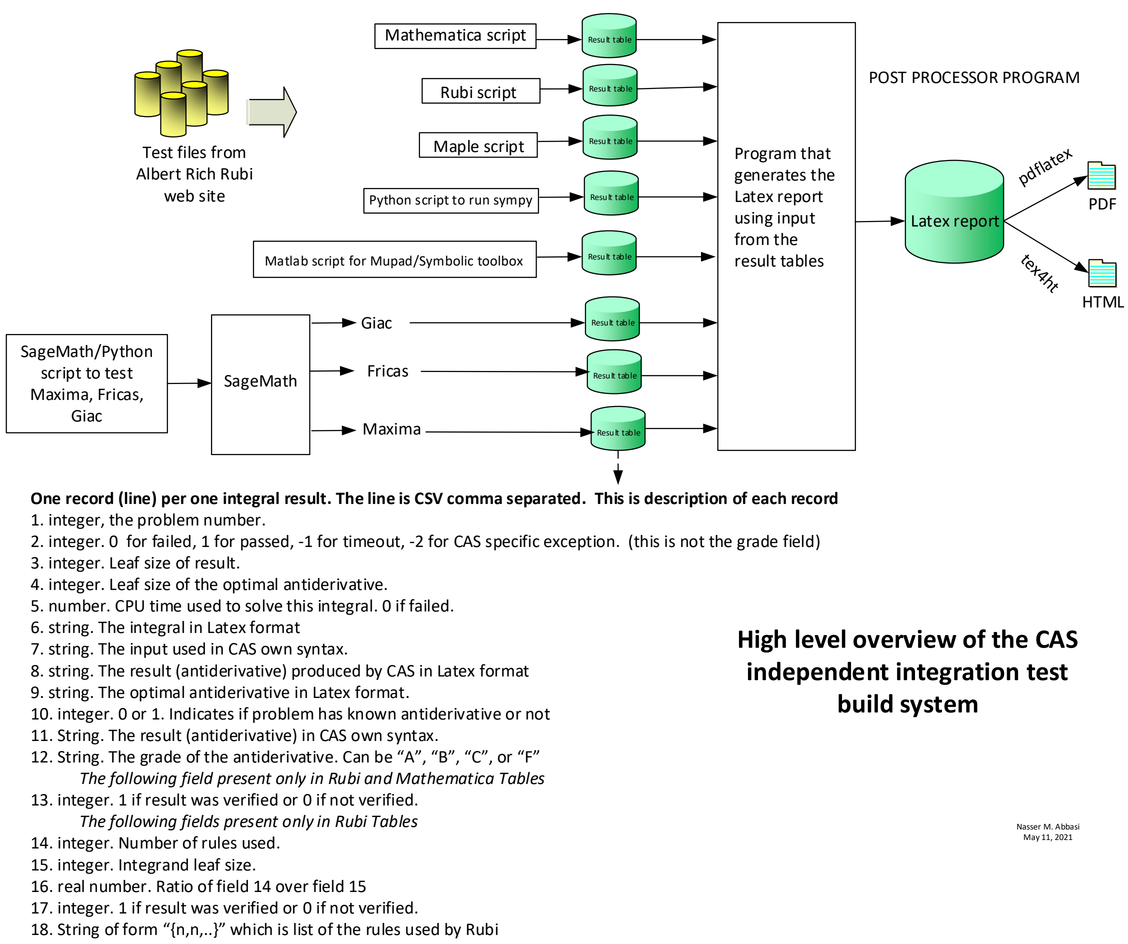
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 107, 108 }

C grade: { 8, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 121 }

F grade: { }

2.1.3 Maple

A grade: { 1, 8, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 96, 111, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 121 }

C grade: { 11 }

F grade: { 9, 10 }

2.1.4 Maxima

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 96 }

B grade: { 93, 94, 95 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

2.1.5 FriCAS

A grade: { 1, 8, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 93, 94, 96, 100, 101, 105, 109, 110, 111, 119 }

B grade: { 2, 3, 4, 5, 6, 7, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 95, 97, 98, 99, 104, 112, 114, 115, 116, 118, 120, 121 }

C grade: { }

F grade: { 9, 10, 13, 102, 103, 106, 107, 108, 113, 117, 122, 123 }

2.1.6 Sympy

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

2.1.7 Giac

A grade: { 1, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 118, 119, 120 }

B grade: { 2, 3, 4, 5, 8, 12, 14, 104, 121 }

C grade: { }

F grade: { 6, 7, 9, 10, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 108, 112, 113, 117, 122, 123 }

2.1.8 Mupad

A grade: { }

B grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 89, 96, 100, 101, 116, 120 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 121, 122, 123 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	87	136	0	205	0	84	-1
normalized size	1	1.00	0.85	1.33	0.00	2.01	0.00	0.82	-0.01
time (sec)	N/A	0.094	0.195	0.028	0.000	1.207	0.000	0.256	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	178	491	0	1079	0	847	-1
normalized size	1	1.00	2.17	5.99	0.00	13.16	0.00	10.33	-0.01
time (sec)	N/A	0.111	0.394	0.060	0.000	1.713	0.000	3.037	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	161	307	0	813	0	703	-1
normalized size	1	1.00	2.44	4.65	0.00	12.32	0.00	10.65	-0.02
time (sec)	N/A	0.079	0.228	0.029	0.000	1.428	0.000	1.898	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	296	829	0	1544	0	1166	-1
normalized size	1	1.00	2.29	6.43	0.00	11.97	0.00	9.04	-0.01
time (sec)	N/A	0.169	0.923	0.033	0.000	1.865	0.000	2.265	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	1746	1884	0	3818	0	2986	-1
normalized size	1	1.00	7.79	8.41	0.00	17.04	0.00	13.33	-0.00
time (sec)	N/A	0.427	6.382	0.032	0.000	9.944	0.000	4.870	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	3382	3695	0	8134	0	0	-1
normalized size	1	1.00	10.31	11.27	0.00	24.80	0.00	0.00	-0.00
time (sec)	N/A	0.970	6.607	0.038	0.000	40.657	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	490	1377	0	2005	0	0	-1
normalized size	1	1.00	3.02	8.50	0.00	12.38	0.00	0.00	-0.01
time (sec)	N/A	0.306	1.965	0.033	0.000	3.066	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	84	27	0	38	0	52	-1
normalized size	1	1.00	3.00	0.96	0.00	1.36	0.00	1.86	-0.04
time (sec)	N/A	0.017	0.074	0.022	0.000	0.844	0.000	0.219	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.143	0.253	0.000	0.908	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	172	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.243	0.302	0.000	0.905	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	27	16	0	22	0	49	-1
normalized size	1	1.00	0.56	0.33	0.00	0.46	0.00	1.02	-0.02
time (sec)	N/A	0.015	0.020	0.046	0.000	0.862	0.000	0.229	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	84	0	82	0	143	-1
normalized size	1	1.00	0.94	1.20	0.00	1.17	0.00	2.04	-0.01
time (sec)	N/A	0.051	0.088	0.021	0.000	0.912	0.000	0.260	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1077	1077	600	661	0	0	0	0	-1
normalized size	1	1.00	0.56	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.099	1.639	0.248	0.000	0.861	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	159	341	0	161	0	171	-1
normalized size	1	1.00	1.62	3.48	0.00	1.64	0.00	1.74	-0.01
time (sec)	N/A	0.210	0.395	0.030	0.000	0.955	0.000	0.215	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
normalized size	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.049	0.003	0.001	0.460	0.722	0.086	0.206	0.062

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
normalized size	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.038	0.002	0.001	0.447	0.720	0.081	0.181	0.034

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.029	0.001	0.001	0.434	0.691	0.075	0.209	0.025

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.016	0.001	0.001	0.432	0.692	0.062	0.182	0.023

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	49	33	35
normalized size	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.83
time (sec)	N/A	0.040	0.016	0.005	0.975	0.647	0.141	0.176	3.393

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	35
normalized size	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.81
time (sec)	N/A	0.027	0.015	0.005	0.969	0.734	0.157	0.209	0.038

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	63	46	55
normalized size	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.037	0.026	0.005	0.958	0.796	0.182	0.175	0.049

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	64	64	76	64	64
normalized size	1	1.00	1.00	0.81	0.80	0.80	0.95	0.80	0.80
time (sec)	N/A	0.060	0.003	0.002	0.452	0.772	0.102	0.201	0.085

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	55	54	54	63	54	54
normalized size	1	1.00	1.00	0.83	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.048	0.002	0.001	0.438	0.507	0.086	0.215	0.054

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	44	44
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.94	0.81	0.81
time (sec)	N/A	0.042	0.002	0.001	0.437	0.980	0.082	0.202	0.034

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	34	34	41	34	34
normalized size	1	1.00	1.00	0.76	0.74	0.74	0.89	0.74	0.74
time (sec)	N/A	0.030	0.001	0.001	0.435	1.108	0.072	0.192	0.025

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	44	43	43	63	43	45
normalized size	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.80
time (sec)	N/A	0.051	0.020	0.004	0.968	0.715	0.150	0.210	3.449

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	52	78	65	52	51
normalized size	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.81
time (sec)	N/A	0.061	0.032	0.006	0.961	0.871	0.198	0.172	0.050

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	63	46	55
normalized size	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.052	0.024	0.006	0.968	0.844	0.199	0.185	3.442

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	57	76	105	83	56	75
normalized size	1	1.00	0.74	0.67	0.89	1.24	0.98	0.66	0.88
time (sec)	N/A	0.063	0.044	0.007	0.962	0.841	0.229	0.183	3.468

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	75	74	74	92	74	74
normalized size	1	1.00	1.00	0.78	0.77	0.77	0.96	0.77	0.77
time (sec)	N/A	0.071	0.004	0.000	0.432	0.779	0.097	0.176	0.121

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	65	64	64	78	64	64
normalized size	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.056	0.002	0.001	0.426	0.803	0.090	0.205	0.081

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
normalized size	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.051	0.002	0.002	0.429	0.769	0.084	0.202	0.054

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
normalized size	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.032	0.001	0.001	0.434	0.826	0.077	0.201	0.033

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	76	53	55
normalized size	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.79
time (sec)	N/A	0.053	0.023	0.004	0.963	0.875	0.159	0.211	0.043

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	62	88	78	62	61
normalized size	1	1.00	1.00	0.79	0.81	1.14	1.01	0.81	0.79
time (sec)	N/A	0.072	0.027	0.008	0.961	0.845	0.188	0.206	3.431

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	63	72	118	85	62	71
normalized size	1	1.00	0.93	0.75	0.86	1.40	1.01	0.74	0.85
time (sec)	N/A	0.087	0.037	0.009	0.970	0.882	0.235	0.189	3.426

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	64	63	63	87	63	65
normalized size	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.77
time (sec)	N/A	0.057	0.028	0.006	0.976	0.802	0.167	0.211	3.441

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	73	53	55
normalized size	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.79
time (sec)	N/A	0.056	0.021	0.004	0.968	1.024	0.152	0.213	0.042

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	44	43	43	60	43	45
normalized size	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.80
time (sec)	N/A	0.050	0.017	0.004	0.958	0.852	0.155	0.182	3.445

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	46	33	35
normalized size	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.83
time (sec)	N/A	0.035	0.010	0.004	0.950	0.863	0.141	0.207	0.039

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	59	83	59	79
normalized size	1	1.00	1.00	0.82	0.81	0.81	1.14	0.81	1.08
time (sec)	N/A	0.053	0.032	0.006	0.963	0.807	0.244	0.191	0.187

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	78	117	102	78	95
normalized size	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.01
time (sec)	N/A	0.089	0.083	0.007	0.960	0.962	0.322	0.198	3.567

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	104	89	98	177	119	88	115
normalized size	1	1.00	0.90	0.77	0.85	1.54	1.03	0.77	1.00
time (sec)	N/A	0.124	0.155	0.008	0.959	1.019	0.363	0.180	0.179

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	71	72	98	90	72	72
normalized size	1	1.00	1.00	0.78	0.79	1.08	0.99	0.79	0.79
time (sec)	N/A	0.086	0.052	0.010	0.964	1.168	0.198	0.183	3.462

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	62	88	75	62	61
normalized size	1	1.00	1.00	0.79	0.81	1.14	0.97	0.81	0.79
time (sec)	N/A	0.073	0.028	0.008	0.960	0.732	0.191	0.195	3.424

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	52	78	61	52	52
normalized size	1	1.00	1.00	0.81	0.83	1.24	0.97	0.83	0.83
time (sec)	N/A	0.063	0.031	0.007	0.966	0.970	0.189	0.173	3.404

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	36
normalized size	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84
time (sec)	N/A	0.026	0.015	0.004	0.964	0.949	0.154	0.189	0.040

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	78	117	102	78	96
normalized size	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.02
time (sec)	N/A	0.088	0.061	0.007	0.963	0.926	0.319	0.192	3.578

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	106	94	96	167	122	96	115
normalized size	1	1.00	0.83	0.74	0.76	1.31	0.96	0.76	0.91
time (sec)	N/A	0.123	0.057	0.010	0.961	0.862	0.359	0.193	0.177

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	136	106	118	237	143	110	135
normalized size	1	1.00	0.92	0.72	0.80	1.60	0.97	0.74	0.91
time (sec)	N/A	0.161	0.076	0.011	0.968	1.070	0.400	0.202	3.586

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	73	82	128	95	72	81
normalized size	1	1.00	1.00	0.74	0.84	1.31	0.97	0.73	0.83
time (sec)	N/A	0.113	0.038	0.008	0.951	0.818	0.241	0.207	0.045

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	63	72	118	82	62	72
normalized size	1	1.00	1.00	0.75	0.86	1.40	0.98	0.74	0.86
time (sec)	N/A	0.087	0.037	0.007	0.958	0.820	0.229	0.183	0.049

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	56	75	63	46	56
normalized size	1	1.00	0.80	0.73	0.88	1.17	0.98	0.72	0.88
time (sec)	N/A	0.053	0.029	0.006	0.961	0.873	0.201	0.186	3.474

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	56	75	61	46	55
normalized size	1	1.00	0.80	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.033	0.028	0.007	0.960	0.917	0.183	0.205	0.044

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	99	89	98	177	122	88	116
normalized size	1	1.00	0.86	0.77	0.85	1.54	1.06	0.77	1.01
time (sec)	N/A	0.124	0.161	0.008	0.973	0.833	0.357	0.211	3.580

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	106	116	227	143	110	136
normalized size	1	1.00	0.85	0.66	0.72	1.42	0.89	0.69	0.85
time (sec)	N/A	0.160	0.124	0.011	0.974	0.868	0.427	0.217	3.597

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	151	118	138	297	163	116	155
normalized size	1	1.00	0.83	0.65	0.76	1.64	0.90	0.64	0.86
time (sec)	N/A	0.204	0.095	0.013	0.972	0.734	0.470	0.219	3.588

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	85	166	177	98	0	93	221
normalized size	1	1.00	0.41	0.80	0.85	0.47	0.00	0.45	1.06
time (sec)	N/A	0.309	0.296	0.031	1.006	0.660	0.000	0.249	5.028

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	75	132	143	88	0	83	187
normalized size	1	1.00	0.45	0.80	0.86	0.53	0.00	0.50	1.13
time (sec)	N/A	0.181	0.171	0.008	0.982	0.899	0.000	0.254	4.692

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	109	78	0	73	153
normalized size	1	1.00	0.52	0.79	0.88	0.63	0.00	0.59	1.23
time (sec)	N/A	0.099	0.105	0.007	0.976	0.847	0.000	0.217	4.185

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	55	64	75	68	0	63	119
normalized size	1	1.00	0.67	0.78	0.91	0.83	0.00	0.77	1.45
time (sec)	N/A	0.040	0.052	0.006	0.961	1.012	0.000	0.238	3.835

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	185	2065	0	2016	0	0	-1
normalized size	1	1.00	1.06	11.87	0.00	11.59	0.00	0.00	-0.01
time (sec)	N/A	0.441	0.457	0.149	0.000	2.105	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	214	16357	0	2102	0	0	-1
normalized size	1	1.00	1.14	87.01	0.00	11.18	0.00	0.00	-0.01
time (sec)	N/A	0.393	1.036	0.232	0.000	2.365	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	299	44343	0	2182	0	0	-1
normalized size	1	1.00	1.34	198.85	0.00	9.78	0.00	0.00	-0.00
time (sec)	N/A	0.459	2.075	0.372	0.000	2.816	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	95	185	206	108	0	103	-1
normalized size	1	1.00	0.41	0.80	0.89	0.47	0.00	0.45	-0.00
time (sec)	N/A	0.342	0.367	0.035	1.022	0.668	0.000	0.255	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	85	151	172	98	0	93	-1
normalized size	1	1.00	0.45	0.80	0.91	0.52	0.00	0.49	-0.01
time (sec)	N/A	0.190	0.231	0.007	0.991	0.988	0.000	0.256	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	138	88	0	83	-1
normalized size	1	1.00	0.51	0.80	0.94	0.60	0.00	0.56	-0.01
time (sec)	N/A	0.122	0.141	0.009	0.983	0.805	0.000	0.257	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	65	83	104	78	0	73	-1
normalized size	1	1.00	0.62	0.79	0.99	0.74	0.00	0.70	-0.01
time (sec)	N/A	0.050	0.080	0.006	0.963	0.739	0.000	0.230	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	310	3460	0	2027	0	0	-1
normalized size	1	1.00	1.57	17.56	0.00	10.29	0.00	0.00	-0.01
time (sec)	N/A	0.488	0.693	0.053	0.000	1.975	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	530	28185	0	2150	0	0	-1
normalized size	1	1.00	2.28	121.49	0.00	9.27	0.00	0.00	-0.00
time (sec)	N/A	0.575	2.562	0.158	0.000	2.643	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	1262	81552	0	2183	0	0	-1
normalized size	1	1.00	5.66	365.70	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.433	5.342	0.350	0.000	2.609	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	105	204	235	118	0	113	-1
normalized size	1	1.00	0.41	0.80	0.93	0.46	0.00	0.44	-0.00
time (sec)	N/A	0.373	0.452	0.040	1.021	0.811	0.000	0.536	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	95	170	201	108	0	103	-1
normalized size	1	1.00	0.45	0.80	0.95	0.51	0.00	0.49	-0.00
time (sec)	N/A	0.220	0.296	0.009	0.996	1.005	0.000	0.549	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	85	136	167	98	0	93	-1
normalized size	1	1.00	0.50	0.80	0.98	0.58	0.00	0.55	-0.01
time (sec)	N/A	0.130	0.183	0.007	1.028	0.795	0.000	0.516	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	75	102	133	88	0	83	-1
normalized size	1	1.00	0.59	0.80	1.04	0.69	0.00	0.65	-0.01
time (sec)	N/A	0.062	0.112	0.006	0.974	0.696	0.000	0.391	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	229	4860	0	2010	0	0	-1
normalized size	1	1.00	1.03	21.89	0.00	9.05	0.00	0.00	-0.00
time (sec)	N/A	0.539	1.047	0.053	0.000	2.250	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	685	40028	0	2161	0	0	-1
normalized size	1	1.00	2.69	156.97	0.00	8.47	0.00	0.00	-0.00
time (sec)	N/A	0.660	1.655	0.157	0.000	2.739	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	1009	119458	0	2240	0	0	-1
normalized size	1	1.00	3.59	425.12	0.00	7.97	0.00	0.00	-0.00
time (sec)	N/A	0.655	1.996	0.384	0.000	2.978	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	75	147	148	88	0	83	-1
normalized size	1	1.00	0.41	0.79	0.80	0.48	0.00	0.45	-0.01
time (sec)	N/A	0.312	0.241	0.019	0.999	0.769	0.000	0.503	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	65	113	114	78	0	73	-1
normalized size	1	1.00	0.45	0.79	0.80	0.55	0.00	0.51	-0.01
time (sec)	N/A	0.168	0.134	0.008	0.977	0.682	0.000	0.524	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	80	68	0	63	-1
normalized size	1	1.00	0.54	0.78	0.79	0.67	0.00	0.62	-0.01
time (sec)	N/A	0.088	0.075	0.008	0.965	0.731	0.000	0.487	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	46	58	0	53	-1
normalized size	1	1.00	0.76	0.76	0.78	0.98	0.00	0.90	-0.02
time (sec)	N/A	0.033	0.039	0.006	0.956	0.812	0.000	0.530	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	176	684	0	2002	0	0	-1
normalized size	1	1.00	1.19	4.62	0.00	13.53	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.300	0.004	0.000	2.250	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	287	5225	0	2102	0	0	-1
normalized size	1	1.00	1.53	27.79	0.00	11.18	0.00	0.00	-0.01
time (sec)	N/A	0.429	0.996	0.006	0.000	2.601	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	1277	13040	0	2183	0	0	-1
normalized size	1	1.00	5.73	58.48	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.469	6.230	0.007	0.000	2.372	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	95	166	148	112	0	82	-1
normalized size	1	1.00	0.57	1.00	0.89	0.67	0.00	0.49	-0.01
time (sec)	N/A	0.204	0.373	0.029	0.985	0.914	0.000	0.251	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	65	132	114	102	0	72	-1
normalized size	1	1.00	0.52	1.06	0.92	0.82	0.00	0.58	-0.01
time (sec)	N/A	0.127	0.232	0.008	0.973	0.945	0.000	0.281	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	80	92	0	62	-1
normalized size	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.071	0.143	0.008	0.959	0.884	0.000	0.237	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	64	46	82	0	53	87
normalized size	1	1.00	1.00	1.42	1.02	1.82	0.00	1.18	1.93
time (sec)	N/A	0.029	0.076	0.006	0.955	0.804	0.000	0.234	0.227

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	202	718	0	2083	0	0	-1
normalized size	1	1.00	1.15	4.08	0.00	11.84	0.00	0.00	-0.01
time (sec)	N/A	0.408	1.287	0.034	0.000	2.341	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	740	5942	0	2173	0	0	-1
normalized size	1	1.00	3.51	28.16	0.00	10.30	0.00	0.00	-0.00
time (sec)	N/A	0.473	1.525	0.099	0.000	2.526	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	231	18981	0	2263	0	0	-1
normalized size	1	1.00	0.94	77.16	0.00	9.20	0.00	0.00	-0.00
time (sec)	N/A	0.525	2.206	0.208	0.000	3.041	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	75	214	253	132	0	81	-1
normalized size	1	1.00	0.51	1.46	1.72	0.90	0.00	0.55	-0.01
time (sec)	N/A	0.167	0.513	0.031	1.002	0.647	0.000	0.252	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	65	180	219	122	0	72	-1
normalized size	1	1.00	0.62	1.71	2.09	1.16	0.00	0.69	-0.01
time (sec)	N/A	0.105	0.338	0.008	1.020	0.602	0.000	0.268	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	146	185	112	0	61	-1
normalized size	1	1.00	0.81	2.15	2.72	1.65	0.00	0.90	-0.01
time (sec)	N/A	0.061	0.236	0.007	0.978	0.684	0.000	0.267	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	51	0	29	29
normalized size	1	1.00	0.70	0.64	1.26	1.09	0.00	0.62	0.62
time (sec)	N/A	0.022	0.103	0.005	0.432	0.569	0.000	0.232	0.090

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	218	751	0	2133	0	0	-1
normalized size	1	1.00	1.10	3.77	0.00	10.72	0.00	0.00	-0.01
time (sec)	N/A	0.456	0.868	0.039	0.000	2.042	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	296	5975	0	2253	0	0	-1
normalized size	1	1.00	1.26	25.53	0.00	9.63	0.00	0.00	-0.00
time (sec)	N/A	0.543	1.153	0.104	0.000	1.992	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	242	19014	0	2343	0	0	-1
normalized size	1	1.00	0.90	70.68	0.00	8.71	0.00	0.00	-0.00
time (sec)	N/A	0.589	1.958	0.209	0.000	1.994	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	657	1429	0	1269	0	638	1299
normalized size	1	1.00	1.51	3.28	0.00	2.91	0.00	1.46	2.98
time (sec)	N/A	0.789	0.935	0.023	0.000	1.322	0.000	0.365	5.312

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	173	453	0	465	0	212	320
normalized size	1	1.00	0.99	2.59	0.00	2.66	0.00	1.21	1.83
time (sec)	N/A	0.164	0.273	0.007	0.000	0.770	0.000	0.299	3.906

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	417	6019	0	0	0	0	-1
normalized size	1	1.00	0.97	13.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.051	1.219	0.052	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	555	22287	0	0	0	0	-1
normalized size	1	1.00	1.14	45.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.929	5.093	0.037	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	829	2458	0	2179	0	1150	-1
normalized size	1	1.00	1.47	4.36	0.00	3.86	0.00	2.04	-0.00
time (sec)	N/A	0.935	1.762	0.025	0.000	3.790	0.000	0.598	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	392	862	0	839	0	417	-1
normalized size	1	1.00	1.66	3.65	0.00	3.56	0.00	1.77	-0.00
time (sec)	N/A	0.230	0.606	0.009	0.000	1.268	0.000	0.612	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	679	678	1232	22523	0	0	0	0	-1
normalized size	1	1.00	1.81	33.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.033	4.576	0.029	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	704	704	2843	72576	0	0	0	0	-1
normalized size	1	1.00	4.04	103.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.950	6.831	0.039	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	671	669	4727	178044	0	0	0	0	-1
normalized size	1	1.00	7.04	265.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.597	7.229	0.066	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	615	1930	0	1583	0	824	-1
normalized size	1	1.00	0.86	2.69	0.00	2.21	0.00	1.15	-0.00
time (sec)	N/A	2.710	1.346	0.028	0.000	3.195	0.000	0.528	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	251	706	0	637	0	304	-1
normalized size	1	1.00	0.79	2.23	0.00	2.02	0.00	0.96	-0.00
time (sec)	N/A	0.626	0.503	0.014	0.000	1.435	0.000	0.632	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	185	0	227	0	98	-1
normalized size	1	1.00	0.83	1.59	0.00	1.96	0.00	0.84	-0.01
time (sec)	N/A	0.111	0.146	0.008	0.000	1.455	0.000	0.499	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	376	761	0	11287	0	0	-1
normalized size	1	1.00	1.01	2.03	0.00	30.18	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.454	0.025	0.000	14.925	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	789	787	1377	3858	0	0	0	0	-1
normalized size	1	1.00	1.75	4.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.210	6.689	0.031	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	745	2827	0	3143	0	1099	-1
normalized size	1	1.00	1.15	4.36	0.00	4.84	0.00	1.69	-0.00
time (sec)	N/A	2.106	1.664	0.030	0.000	5.599	0.000	0.423	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	288	1011	0	1305	0	407	-1
normalized size	1	1.00	0.93	3.27	0.00	4.22	0.00	1.32	-0.00
time (sec)	N/A	0.447	0.733	0.015	0.000	3.263	0.000	0.352	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	249	0	429	0	122	143
normalized size	1	1.00	1.02	2.24	0.00	3.86	0.00	1.10	1.29
time (sec)	N/A	0.080	0.311	0.006	0.000	1.744	0.000	0.312	3.734

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	700	4099	0	0	0	0	-1
normalized size	1	1.00	1.05	6.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.829	5.343	0.028	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	891	891	872	4635	0	3995	0	1401	-1
normalized size	1	1.00	0.98	5.20	0.00	4.48	0.00	1.57	-0.00
time (sec)	N/A	1.768	2.191	0.031	0.000	12.868	0.000	0.455	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	387	1786	0	1581	0	587	-1
normalized size	1	1.00	0.87	4.02	0.00	3.56	0.00	1.32	-0.00
time (sec)	N/A	0.451	1.257	0.014	0.000	6.519	0.000	0.338	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	147	185	0	286	0	240	175
normalized size	1	1.00	1.12	1.41	0.00	2.18	0.00	1.83	1.34
time (sec)	N/A	0.085	0.378	0.007	0.000	4.021	0.000	0.322	3.708

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	144	0	154	0	205	-1
normalized size	1	1.00	1.59	2.82	0.00	3.02	0.00	4.02	-0.02
time (sec)	N/A	0.066	0.041	0.024	0.000	0.755	0.000	0.276	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1432	1432	670	928	0	0	0	0	-1
normalized size	1	1.00	0.47	0.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.219	2.341	0.272	0.000	1.039	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	390	418	0	0	0	0	-1
normalized size	1	1.00	0.60	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.677	0.611	0.468	0.000	0.930	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [14] had the largest ratio of [.4444]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	29	0.138
2	A	2	2	1.00	31	0.065
3	A	2	2	1.00	27	0.074
4	A	4	4	1.00	27	0.148
5	A	5	5	1.00	27	0.185
6	A	6	5	1.00	27	0.185
7	A	4	4	1.00	31	0.129
8	A	2	2	1.00	23	0.087
9	A	2	2	1.00	31	0.065
10	A	2	2	1.00	34	0.059
11	A	3	3	1.00	22	0.136
12	A	6	5	1.00	18	0.278
13	A	3	3	1.00	26	0.115
14	A	16	12	1.00	27	0.444
15	A	2	1	1.00	23	0.043
16	A	2	1	1.00	23	0.043
17	A	2	1	1.00	23	0.043
18	A	2	1	1.00	21	0.048
19	A	6	5	1.00	23	0.217
20	A	4	4	1.00	23	0.174
21	A	5	5	1.00	23	0.217
22	A	2	1	1.00	25	0.040
23	A	2	1	1.00	25	0.040
24	A	2	1	1.00	25	0.040
25	A	2	1	1.00	23	0.043
26	A	6	5	1.00	25	0.200
27	A	7	6	1.00	25	0.240
28	A	5	4	1.00	25	0.160
29	A	6	5	1.00	25	0.200
30	A	2	1	1.00	25	0.040
31	A	2	1	1.00	25	0.040
32	A	2	1	1.00	25	0.040
33	A	2	1	1.00	23	0.043
34	A	6	5	1.00	25	0.200
35	A	7	6	1.00	25	0.240

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	8	6	1.00	25	0.240
37	A	6	5	1.00	25	0.200
38	A	6	5	1.00	25	0.200
39	A	6	5	1.00	25	0.200
40	A	6	5	1.00	23	0.217
41	A	9	5	1.00	25	0.200
42	A	10	6	1.00	25	0.240
43	A	11	7	1.00	25	0.280
44	A	7	6	1.00	25	0.240
45	A	7	6	1.00	25	0.240
46	A	7	6	1.00	25	0.240
47	A	4	4	1.00	23	0.174
48	A	10	6	1.00	25	0.240
49	A	11	7	1.00	25	0.280
50	A	12	7	1.00	25	0.280
51	A	8	6	1.00	25	0.240
52	A	8	6	1.00	25	0.240
53	A	5	4	1.00	25	0.160
54	A	5	5	1.00	23	0.217
55	A	11	7	1.00	25	0.280
56	A	12	7	1.00	25	0.280
57	A	13	7	1.00	25	0.280
58	A	11	5	1.00	27	0.185
59	A	9	5	1.00	27	0.185
60	A	7	5	1.00	27	0.185
61	A	5	5	1.00	25	0.200
62	A	8	7	1.00	27	0.259
63	A	6	5	1.00	27	0.185
64	A	7	6	1.00	27	0.222
65	A	12	5	1.00	27	0.185
66	A	10	5	1.00	27	0.185
67	A	8	5	1.00	27	0.185
68	A	6	5	1.00	25	0.200
69	A	9	8	1.00	27	0.296
70	A	10	9	1.00	27	0.333
71	A	7	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	13	5	1.00	27	0.185
73	A	11	5	1.00	27	0.185
74	A	9	5	1.00	27	0.185
75	A	7	5	1.00	25	0.200
76	A	10	9	1.00	27	0.333
77	A	11	9	1.00	27	0.333
78	A	11	10	1.00	27	0.370
79	A	10	4	1.00	27	0.148
80	A	8	4	1.00	27	0.148
81	A	6	4	1.00	27	0.148
82	A	4	4	1.00	25	0.160
83	A	5	4	1.00	27	0.148
84	A	6	5	1.00	27	0.185
85	A	7	6	1.00	27	0.222
86	A	9	5	1.00	27	0.185
87	A	7	5	1.00	27	0.185
88	A	5	5	1.00	27	0.185
89	A	4	4	1.00	25	0.160
90	A	6	5	1.00	27	0.185
91	A	7	6	1.00	27	0.222
92	A	8	6	1.00	27	0.222
93	A	8	5	1.00	27	0.185
94	A	6	5	1.00	27	0.185
95	A	5	4	1.00	27	0.148
96	A	3	3	1.00	25	0.120
97	A	7	6	1.00	27	0.222
98	A	8	6	1.00	27	0.222
99	A	9	6	1.00	27	0.222
100	A	7	5	1.00	27	0.185
101	A	5	5	1.00	25	0.200
102	A	8	5	1.00	27	0.185
103	A	6	4	1.00	27	0.148
104	A	8	5	1.00	27	0.185
105	A	6	5	1.00	25	0.200
106	A	9	6	1.00	27	0.222
107	A	10	7	1.00	27	0.259

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	7	5	1.00	27	0.185
109	A	8	4	1.00	27	0.148
110	A	6	4	1.00	27	0.148
111	A	4	4	1.00	25	0.160
112	A	5	3	1.00	27	0.111
113	A	6	4	1.00	27	0.148
114	A	7	5	1.00	27	0.185
115	A	5	5	1.00	27	0.185
116	A	4	4	1.00	25	0.160
117	A	6	4	1.00	27	0.148
118	A	6	5	1.00	27	0.185
119	A	5	4	1.00	27	0.148
120	A	3	3	1.00	25	0.120
121	A	5	4	1.00	27	0.148
122	A	3	3	1.00	29	0.103
123	A	3	3	1.00	29	0.103

Chapter 3

Listing of integrals

3.1 $\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

```
[Out] 1/8*(8*a*f-b*(e+4*d*f/e))*arctanh(1/2*(2*f*x+e)/f^(1/2)/(f*x^2+e*x+d)^(1/2)
)/f^(3/2)+1/4*b*(f*x^2+e*x+d)^(1/2)/f+1/2*b*x*(f*x^2+e*x+d)^(1/2)/e
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 640, 621, 206}

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a
*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2
])])/(8*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx &= \frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\int \frac{\left(2a-\frac{bd}{e}\right)f+\frac{bfx}{2}}{\sqrt{d+ex+fx^2}}\,dx}{2f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\int \frac{1}{\sqrt{d+ex+fx^2}}\,dx}{8f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\text{Subst}\left(\int \frac{1}{4f-x^2}\,dx,x,\frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)}{4f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}-\frac{\left(be-8af+\frac{4bdf}{e}\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.85

$$\frac{2b\sqrt{f}(e+2fx)\sqrt{d+x(e+fx)}-\left(b\left(4df+e^2\right)-8aef\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+x(e+fx)}}\right)}{8ef^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (2*b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] - (-8*a*e*f + b*(e^2 + 4*d*f
))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + x*(e + f*x)])])/(8*e*f^(3/2))
```

fricas [A] time = 1.21, size = 205, normalized size = 2.01

$$\left[-\frac{\left(be^2+4(bd-2ae)f\right)\sqrt{f}\log\left(-8f^2x^2-8efx-e^2-4\sqrt{fx^2+ex+d}\left(2fx+e\right)\sqrt{f}-4df\right)-4\left(2bf^2x+bef\right.}{16ef^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(f)*log(-8*f^2*x^2 - 8*e*f*x - e^2
- 4*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(f) - 4*d*f) - 4*(2*b*f^2*x + b*e
*f)*sqrt(f*x^2 + e*x + d))/(e*f^2), 1/8*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(-
f)*arctan(1/2*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(-f)/(f^2*x^2 + e*f*x +
 d*f)) + 2*(2*b*f^2*x + b*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2)]
```

giac [A] time = 0.26, size = 84, normalized size = 0.82

$$\frac{1}{4}\sqrt{fx^2+xe+d}\left(2\,bxe^{(-1)}+\frac{b}{f}\right)+\frac{\left(4\,bdf-8\,afe+be^2\right)e^{(-1)}\log\left(\left|-2\left(\sqrt{f}x-\sqrt{fx^2+xe+d}\right)\sqrt{f}-e\right|\right)}{8\,f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(f*x^2 + x*e + d)*(2*b*x*e^(-1) + b/f) + 1/8*(4*b*d*f - 8*a*f*e + b
*e^2)*e^(-1)*log(abs(-2*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*sqrt(f) - e))/f
^(3/2)
```

maple [A] time = 0.03, size = 136, normalized size = 1.33

$$\frac{a\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{\sqrt{f}}-\frac{bd\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{2e\sqrt{f}}-\frac{be\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{8f^{\frac{3}{2}}}+\frac{\sqrt{f\,x^2+ex+}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 1/2*b*x*(f*x^2+e*x+d)^(1/2)/e+1/4*b*(f*x^2+e*x+d)^(1/2)/f-1/8*e*b/f^(3/2)*l
n((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))-1/2/e*b/f^(1/2)*d*ln((1/2*e+f*x)
/f^(1/2)+(f*x^2+e*x+d)^(1/2))+a*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))
/f^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x+\frac{b\,f\,x^2}{e}}{\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2),x)
```

```
[Out] int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ae}{\sqrt{d+ex+fx^2}}\,dx+\int \frac{bex}{\sqrt{d+ex+fx^2}}\,dx+\int \frac{bfx^2}{\sqrt{d+ex+fx^2}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] (Integral(a*e/sqrt(d + e*x + f*x**2), x) + Integral(b*e*x/sqrt(d + e*x + f*
x**2), x) + Integral(b*f*x**2/sqrt(d + e*x + f*x**2), x))/e
```

3.2 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

```
[Out] -2*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*x
+d)^(1/2))*e^(1/2)/(-a*e+b*d)^(1/2)/(-4*a*f+b*e)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {982, 208}

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f
]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx &= -\left((2e)\operatorname{Subst}\left(\int \frac{1}{e(be-4af)-(bd-ae)x^2}\,dx,x,\frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)\right)\\ &= -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}\end{aligned}$$

Mathematica [B] time = 0.39, size = 178, normalized size = 2.17

$$\frac{\sqrt{e}\left(\tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\left(e^2-4df\right)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (Sqrt[e]*(ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e
+ 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + ArcTanh[(Sqrt[b]*(
e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*
Sqrt[d + x*(e + f*x)])]))/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

fricas [B] time = 1.71, size = 1079, normalized size = 13.16

$$\left[\frac{1}{2}\sqrt{\frac{e}{b^2de - abe^2 - 4\left(abd - a^2e\right)f}}\log\left(\frac{8\,b^2d^2e^4 - 8\,abde^5 + a^2e^6 + 16\,a^2d^2e^2f^2 + \left(b^2e^4f^2 + 16\left(b^2d^2 - 8\,abde \cdots\right.\right.}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*log((8*b^2*d^2*e^4 -
 8*a*b*d*e^5 + a^2*e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 -
8*a*b*d*e + 8*a^2*e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*
e^5*f + 16*(b^2*d^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a
*b*e^4)*f^2)*x^3 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^
2*d^2*e^2 - 13*a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*
f)*x^2 - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^6 - 1
6*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 + 2*a^
2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16*(a*b^2*d
^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2 + 3*a^2*b*
e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2 - a^3*d*e^3
)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4*(b^3*d^2*e^
2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)*f)*x^2 - 4*(
3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a*b^2*e^6 + 32*
(a^2*b*d^2*e - a^3*d*e^2)*f^3 - 40*(a*b^2*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4
)*f^2 + 2*(4*b^3*d^2*e^3 - 11*a*b^2*d*e^4 + 7*a^2*b*e^5)*f)*x)*sqrt(f*x^2 +
 e*x + d)*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f)))/(b^2*f^2*x^4 +
 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b^2*e^2 + 2*a*b*e*f)*x^2)), -sqrt
(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*arctan(-1/2*(2*b*d*e^2 - a*e
^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 + (b*e^3 + 4*(b*d*e -
2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)*sqrt(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d
- a^2*e)*f))/(2*e*f^2*x^3 + 3*e^2*f*x^2 + d*e^2 + (e^3 + 2*d*e*f)*x))]
```

giac [B] time = 3.04, size = 847, normalized size = 10.33

$$\frac{\sqrt{-4\,abdfe + b^2de^2 + 4\,a^2fe^2 - abe^3}\,\log\left(\left|-4\left(\sqrt{f}\,x - \sqrt{fx^2 + xe + d}\right)^2 bdf^2 + 8\left(\sqrt{f}\,x - \sqrt{fx^2 + xe + d}\right)^2 af^2 \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*
x - sqrt(f*x^2 + x*e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d)
)^2*a*f^2*e - 4*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2
*f^2 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(
f*x^2 + x*e + d))*a*f^(3/2)*e^2 + 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f
*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2
 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*sqrt(f)*e^3 + 4*sqrt(-4*a*b*d*f*e
+ b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*
```

```
e + 2*a*f*e^3 + sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt
(f)*e^2))/(4*a*b*d*f - b^2*d*e - 4*a^2*f*e + a*b*e^2) - sqrt(-4*a*b*d*f*e +
 b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*x - sqrt(f*x^2 + x*
e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*a*f^2*e - 4*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2*f^2 - (sqrt(f)*x -
 sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*a
*f^(3/2)*e^2 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2 - (sqrt(f)*x - sqr
t(f*x^2 + x*e + d))*b*sqrt(f)*e^3 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2
*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*e + 2*a*f*e^3 - sqr
t(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt(f)*e^2))/(4*a*b*d*
f - b^2*d*e - 4*a^2*f*e + a*b*e^2)
```

maple [B] time = 0.06, size = 491, normalized size = 5.99

$$-\frac{e\ln\left(\frac{-\frac{2(ae-bd)}{b}+\frac{\sqrt{-(4af-be)be}\left(x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}\right)^2 f+\frac{\sqrt{-(4af-be)be}\left(x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}\right)}{b}-\frac{ae-bd}{b}}}{x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}}\right)}{\sqrt{-\left(4af-be\right)be}\sqrt{-\frac{ae-bd}{b}}}+\; e\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -e/(-b*e*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e
*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(
a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a
*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))
^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))+e/(-b*e*(4*a*f-b*e))^(
1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(
x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(
b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*f-b*e>0)', see `assume?` for mo
re details)Is 4*a*f-b*e    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f\,x^2+e\,x+d}\,\left(a+b\,x+\frac{b\,f\,x^2}{e}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)),x)
```

```
[Out] int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int \frac{1}{ae\sqrt{d+ex+fx^2}+bex\sqrt{d+ex+fx^2}+bfx^2\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) + b
*f*x**2*sqrt(d + e*x + f*x**2)), x)
```

3.3 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx$

Optimal. Leaf size=66

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

```
[Out] -2*arctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-
d)^(1/2)/(b^2-4*c*d)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {982, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx &= -\left((2b)\,\text{Subst}\left(\int \frac{1}{b\left(b^2-4cd\right)-(ab-bd)x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}\end{aligned}$$

Mathematica [B] time = 0.23, size = 161, normalized size = 2.44

$$\frac{\tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b\left(\sqrt{b^2-4cd}+b\right)}{4c\sqrt{a-d}\,\sqrt{a+x(b+cx)}}\right)+\tanh^{-1}\left(\frac{-2c\left(2a+x\sqrt{b^2-4cd}\right)-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\,\sqrt{a+x(b+cx)}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] (ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c
*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])] + ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] -
 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

fricas [B] time = 1.43, size = 813, normalized size = 12.32

$$\left[\log\left(\frac{8\,a^{2}b^{4}+\left(b^{4}c^{2}+24\,ab^{2}c^{3}+16\,a^{2}c^{4}+128\,c^{4}d^{2}-32\left(b^{2}c^{3}+4\,ac^{4}\right)d\right)x^{4}+2\left(b^{5}c+24\,ab^{3}c^{2}+16\,a^{2}bc^{3}+128\,bc^{3}d^{2}-32\left(b^{3}c^{2}+4\,abc^{3}\right)d\right)x^{3}+\left(b^{4}+24\,ab^{2}c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 -
32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 12
8*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*
c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^
2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*
c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3
 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 +
 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)
*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a
*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)
*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b^2 - 4*c*d^
2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2
 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (
b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2
 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3
*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(
b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x))/(a*b^2 + 4*c*d^
2 - (b^2 + 4*a*c)*d)]
```

giac [B] time = 1.90, size = 703, normalized size = 10.65

$$-\frac{\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^{2}+bx+a}\right)^{2}b^{2}c-4\left(\sqrt{c}\,x-\sqrt{cx^{2}+bx+a}\right)^{2}ac^{2}+8\left(\sqrt{c}\,x-\sqrt{cx^{2}+bx+a}\right)^{2}c^{2}d-\left(\sqrt{c}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d
- (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2
)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(a*b^
2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d +
 4*c*d^2) + log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
```

```
- sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d
- 4*a*c*d + 4*c*d^2)
```

maple [B] time = 0.03, size = 307, normalized size = 4.65

$$-\frac{\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\,\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\,\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2 c-d+\sqrt{b^2-4cd}\,\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\,\sqrt{a-d}}+\frac{\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\,\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-
4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-
4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d
)^(1/2)+b)/c))-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2
)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1
/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*d-b^2>0)', see `assume?` for mo
re details)Is 4*c*d-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(bx+cx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)
```

3.4 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

```
[Out] (b^2+4*c*(a-2*d))*arctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(a-d)^(3/2)/(b^2-4*c*d)^(3/2)-(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-
d)/(b^2-4*c*d)/(c*x^2+b*x+d)
```

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 12, 982, 208}

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x
^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
- 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\int -\frac{c^2\left(b^2+4c(a-2d)\right)(a-d)}{2\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx}{c^2(a-d)^2\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} - \frac{\left(b^2+4c(a-2d)\right)\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+c\cdots\right.}}{2(a-d)\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b\left(b^2+4c(a-2d)\right)\right)\text{Subst}\left(\int \frac{\cdots}{b\left(b^2-\cdots\right.}\right.}{(a-d)\left(b^2-\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b^2+4c(a-2d)\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b\cdots}{\sqrt{b^2-4cd}\,\sqrt{a\cdots}}\right.}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}$$

Mathematica [B] time = 0.92, size = 296, normalized size = 2.29

$$\frac{1}{2}\left(-\frac{8c(b+2cx)\sqrt{a+x(b+cx)}}{(a-d)\left(4cd-b^2\right)\left(\sqrt{b^2-4cd}-b-2cx\right)\left(\sqrt{b^2-4cd}+b+2cx\right)} - \frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-\cdots}{4c\sqrt{a-d}\,\sqrt{a+\cdots}}\right.}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] ((-8*c*(b + 2*c*x)*Sqrt[a + x*(b + c*x)])/((a - d)*(-b^2 + 4*c*d)*(-b + Sqr
t[b^2 - 4*c*d] - 2*c*x)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) - ((b^2 + 4*c*(a -
 2*d))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x
)/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3
/2)) - ((b^2 + 4*c*(a - 2*d))*ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a
 + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/((a - d)
^(3/2)*(b^2 - 4*c*d)^(3/2)))/2
```

fricas [B] time = 1.86, size = 1544, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 -
 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2*b
^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a
*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*
(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 +
 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c +
```

```
 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8
*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (
b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 +
 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 +
 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b
*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*
(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c +
4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c +
4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c
)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a
*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5
+ 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c
^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b^2 - 4*c*d^2 + (b^2
 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d -
 (b^3 + 4*a*b*c - 8*b*c*d)*x)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c
^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*
c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(
a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d
^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^
2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(a
*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*
a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*
a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)*
d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b
^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 +
16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x)]
```

giac [B] time = 2.27, size = 1166, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b
^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c)
 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d
+ 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^
2*c*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt
(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*
b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c -
 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*
c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))/(a*b^2
 - b^2*d - 4*a*c*d + 4*c*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*
sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2) - 8*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*c^(3/2)*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c - 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b*c*d + 3*a*b^2*sqrt(c) - 4*a^2*c^(3/2) - 2*b^2*sqrt(c)*d)/(
((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b*sqrt(c) + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2 - 2*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
```

```
c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*sqrt(c) + 4*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b*sqrt(c)*d - a*b^2 + a^2*c + b^2*d)*(a*b^2 - b^2*d -
4*a*c*d + 4*c*d^2))
```

maple [B] time = 0.03, size = 829, normalized size = 6.43

$$\frac{2c\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\,\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2 c-d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\left(b^2-4cd\right)^{\frac{3}{2}}\sqrt{a-d}}-\frac{2c\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2 \cdots}}{2c \cdots}\right.\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+
(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-
d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-
4*c*d)^(1/2))/c))+2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)
^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*
c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1
/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^2-4*c*d)/(a-d)/(x-1/2/c*(b^2-4*
c*d)^(1/2)+1/2/c*b)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(
1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2)+1/2/(b^2-4*c*d)^(1/2)/(a-d)^(3
/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^
(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^2-4*
c*d)/(a-d)/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2/c*b)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-1/2/(
b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4
*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2
-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)
^(1/2))/c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(bx+cx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)**2), x)
```

3.5 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=224

$$-\frac{\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}+\frac{3(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+b\cdots}}{4(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+\cdots\right.}$$

```
[Out] -1/4*(3*b^4+8*b^2*c*(a-4*d)+16*c^2*(3*a^2-8*a*d+8*d^2))*arctanh((2*c*x+b)*(
a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(5/2)/(b^2-4*c*d)^(
5/2)-1/2*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)^2+3/
4*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2/(b^2-4*c*d)^2/(c*
x^2+b*x+d)
```

Rubi [A] time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1060, 12, 982, 208}

$$-\frac{\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}+\frac{3(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+b\cdots}}{4(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(2*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^2) + (3*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*(a
 - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)) - ((3*b^4 + 8*b^2*c*(a - 4*d) +
16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(4*(a - d)^(5/2)*(b^2 - 4*c*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
```

```
0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{\int \frac{-\frac{1}{2}c^2(a-d)\left(3b^2+12ac-16cd\right)-4bc^3(a-\cdots}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\cdots\right.}}{2c^2(a-d)^2\left(b^2-4\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{\cdots}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{\cdots}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{\cdots}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{\cdots}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+\cdots\right.}$$

Mathematica [B] time = 6.38, size = 1746, normalized size = 7.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] (-2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b - Sqrt[b^2 - 4*c
*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*
(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (2
*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d]
 + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*(b^
2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (6*c^
2*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sq
rt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]*(
b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt
[a - d]*Sqrt[a + b*x + c*x^2])])/(2*(a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)*Sqrt[
a + x*(b + c*x)]) + (6*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sq
rt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*S
qrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[
b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(2*(a - d)^(3/2)*
(b^2 - 4*c*d)^(3/2)*Sqrt[a + x*(b + c*x)]) + (4*c^3*Sqrt[a + b*x + c*x^2]*(
((-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*Sqrt[a
 + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[
b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a - d]*(b*(-
2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2*(4*a*
c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d])*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTanh[(-4*
a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*d]))*x
)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^
2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + Sqrt[b^
2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(3/2)*(4*a*
c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a
 + x*(b + c*x)]) + (4*c^3*Sqrt[a + b*x + c*x^2]*(((-2*c^2*(b - 2*Sqrt[b^2 -
 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c^2
- 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(b + Sqrt[b^
2 - 4*c*d] + 2*c*x)) + (4*c*Sqrt[a - d]*(b*(2*c^2*(b - 2*Sqrt[b^2 - 4*c*d])
 + 2*c^2*(b + Sqrt[b^2 - 4*c*d])) - 2*(4*a*c^3 + c^2*(b - 2*Sqrt[b^2 - 4*c*
d])*(b + Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) -
(-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c
*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*
d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d]) + 4*c*(b + Sqrt[b^2 - 4*c*
d])^2))))/((b^2 - 4*c*d)^(3/2)*(4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c
*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)])
```

fricas [B] time = 9.94, size = 3818, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2 -
 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c + 8*
a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3
+ (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^2*c^
2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c^2 -
 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (3*b^
```

```
5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)
*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(
b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*
c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)
*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 -
2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2
+ 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4
*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c
*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d +
 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3
*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a
*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c
^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3
)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3
*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 +
 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*
b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c
^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*
a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*
a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 +
 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 1
44*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b
^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b
^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12
*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*
a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*
c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 -
32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 +
 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a
*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8
*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b
*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(
a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)
*x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d
^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c
+ 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*
x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^
2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c
^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (
3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*
c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c
)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)
*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 -
(b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)
*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2
)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4
 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 +
 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3
+ 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32
*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*
b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^
3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3
)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(
a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^
2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)
*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^
```

```
5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^
5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a
*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^
4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6
 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*
b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2
)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c
^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2
*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^
3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3
*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6
 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a
^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*
d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)*x)]
```

giac [B] time = 4.87, size = 2986, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d + 128*c^2*
d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^
2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^
(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 -
 b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2) - (3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d
+ 128*c^2*d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sq
rt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2))/(a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 1
6*a*b^2*c*d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4) -
 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(3/2) + 8*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(5/2) + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a^2*c^(7/2) - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(5/2)*d
 - 128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(7/2)*d + 128*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*c^(7/2)*d^2 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b^5*c + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^2 + 144*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^3 - 96*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*b^3*c^2*d - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^3*d
+ 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^3*d^2 + 9*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*b^6*sqrt(c) + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a*b^4*c^(3/2) + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(5/2)
- 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(7/2) - 78*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*b^4*c^(3/2)*d - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a*b^2*c^(5/2)*d + 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(7
/2)*d + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(5/2)*d^2 - 1152*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(7/2)*d^2 + 768*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*c^(7/2)*d^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^
```

```
7 - 10*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c - 288*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^3*b*c^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b
^5*c*d + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^2*d + 1344*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^3*d - 256*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^3*c^2*d^2 - 2304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*
c^3*d^2 + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^3 - 14*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*sqrt(c) - 71*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^2*b^4*c^(3/2) - 200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
3*b^2*c^(5/2) + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(7/2) + 23*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*sqrt(c)*d + 280*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*b^4*c^(3/2)*d + 1168*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^2*b^2*c^(5/2)*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^
(7/2)*d - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(3/2)*d^2 - 2048*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(5/2)*d^2 + 640*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a^2*c^(7/2)*d^2 + 1152*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*b^2*c^(5/2)*d^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7 -
47*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c - 56*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^3*b^3*c^2 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*
c^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*d + 136*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b^5*c*d + 496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
3*c^2*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^3*d - 80*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^5*c*d^2 - 896*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a*b^3*c^2*d^2 + 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^3*d^2
+ 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^3 - 11*a^2*b^6*sqrt(c)
- 11*a^3*b^4*c^(3/2) + 72*a^4*b^2*c^(5/2) - 48*a^5*c^(7/2) + 17*a*b^6*sqrt(
c)*d + 118*a^2*b^4*c^(3/2)*d - 256*a^3*b^2*c^(5/2)*d + 96*a^4*c^(7/2)*d - 6
*b^6*sqrt(c)*d^2 - 152*a*b^4*c^(3/2)*d^2 + 160*a^2*b^2*c^(5/2)*d^2 + 48*b^4
*c^(3/2)*d^3)/((a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 16*a*b^2*c*
d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4)*((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^4*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*s
qrt(c) + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2 - 2*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c*d - 2*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*b*sqrt(c)*d - a*b^2 + a^2*c + b^2*d)^2)
```

maple [B] time = 0.03, size = 1884, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/(b^2-4*c*d)^(3/2)/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x-1/
2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c))^(1/2)+3/4/(b^2-4*c*d)/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c
)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c))^(1/2)-3/8/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d+(b
^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*b^2+3/2/(b^2-4*c*d)^(3/2)/(
a-d)^(5/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2
*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*c*
d-1/(b^2-4*c*d)^(3/2)*c/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2
*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c))+1/2/(b^2-4*c*d)^(3/2)/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c*
d)^(1/2)/c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x
+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)+3/4/(b^2-4*c*d)/(a-d)^2/(x+1/2*b/c+1/2
*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^
```

```
(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)+3/8/(b^2-4*c*d)^(3/2)/(a-d)^(5
/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(
b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*b^2-3/2/(b^2-
4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d
)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c
*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c))*c*d+1/(b^2-4*c*d)^(3/2)*c/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)
*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c))+6*c^2/(b^2-4*c*d)^(5/2)/(a-d)^(1/2)*ln((2*a-2*
d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2
*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))-6*c^2/(b^2-4*c*d)^(5/2)/(a-
d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(
a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+3/(b
^2-4*c*d)^2*c/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x-1/2*(-b+(b^2-
4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^
(1/2)+3/(b^2-4*c*d)^2*c/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2
*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(cx^2 + bx + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2 + b\,x + a}\,\left(c\,x^2 + b\,x + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=328

$$-\frac{(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{24(a-d)^3\left(b^2-4cd\right)^3\left(bx+cx^2+d\right)}+\frac{\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2\right.\right.}{}$$

```
[Out] 1/8*(b^2+4*c*(a-2*d))*(5*b^4-8*b^2*c*(a+4*d)+16*c^2*(5*a^2-8*a*d+8*d^2))*ar
ctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(7
/2)/(b^2-4*c*d)^(7/2)-1/3*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(
c*x^2+b*x+d)^3+5/12*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2
/(b^2-4*c*d)^2/(c*x^2+b*x+d)^2-1/24*(15*b^4+8*b^2*c*(7*a-22*d)+16*c^2*(15*a
^2-44*a*d+44*d^2))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^3/(b^2-4*c*d)^3/(c*x
^2+b*x+d)
```

Rubi [A] time = 0.97, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {974, 1060, 12, 982, 208}

$$-\frac{(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{24(a-d)^3\left(b^2-4cd\right)^3\left(bx+cx^2+d\right)}+\frac{\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(3*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^3) + (5*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(12*(
a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)^2) - ((15*b^4 + 8*b^2*c*(7*a - 2
2*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(24*(a - d)^3*(b^2 - 4*c*d)^3*(d + b*x + c*x^2)) + ((b^2 + 4*c*(a - 2*d))
*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt
[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(8*(a - d)
^(7/2)*(b^2 - 4*c*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
```

```
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)\left(5b^2+20ac-24cd\right)-8bc^3(a-}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2}}{3c^2(a-d)^2\left(b^2-4}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+\right.}$$

Mathematica [B] time = 6.61, size = 3382, normalized size = 10.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] (-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d
] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^(5/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)])
- (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - Sqrt[b^2 - 4*c*d
] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b
^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) - (8
*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(5/2)*(b + Sqrt[b^2 - 4*c*d]
 + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^3*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (20*
c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]
*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*c^2*Sqrt[a + b*x + c*x^2]*
ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*
Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(5/2)*Sqr
t[a + x*(b + c*x)]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b +
 Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*
c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/
2)*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^
2]*(((-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*Sq
rt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + S
qrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a - d]*(
b*(-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2*(
```

```
4*a*c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d])*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTanh[
(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*d]
))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 + 2*b*c*(-b + Sqr
t[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + Sqr
t[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(5/2)*(
4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*Sq
rt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*(-1/2*((4*c^2*(-b + Sq
rt[b^2 - 4*c*d]) + 2*c^2*(2*b + 3*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2]
)/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2
)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)^2) - (((10*c^3*Sqrt[b^2 - 4*c*d]*(-b + S
qrt[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d]
))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-
b + Sqrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a -
 d]*(b*(10*c^3*Sqrt[b^2 - 4*c*d]*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^3*(10*b^2 -
 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*c*d]
 + c^3*(-b + Sqrt[b^2 - 4*c*d])*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 -
4*c*d])))*ArcTanh[(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b +
 Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2
 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2
 + 8*b*c*(-b + Sqrt[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2)))/(2*(4
*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2))))/
(3*(b^2 - 4*c*d)^2*(4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt
[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*((
(-2*c^2*(b - 2*Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a +
 b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2
- 4*c*d])^2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) + (4*c*Sqrt[a - d]*(b*(2*c^2*
(b - 2*Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d])) - 2*(4*a*c^3 + c
^2*(b - 2*Sqrt[b^2 - 4*c*d])*(b + Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b*(
b + Sqrt[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqr
t[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d])
 + c*(b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d]) +
 4*c*(b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(5/2)*(4*a*c^2 - 2*b*c*(b
 + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)])
 - (16*c^4*Sqrt[a + b*x + c*x^2]*(-1/2*((2*c^2*(2*b - 3*Sqrt[b^2 - 4*c*d])
- 4*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(
b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(b + Sqrt[b^2 - 4*c*d
] + 2*c*x)^2) - (((-10*c^3*Sqrt[b^2 - 4*c*d]*(b + Sqrt[b^2 - 4*c*d]) - 2*c^
3*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2]
)/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*
(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) + (4*c*Sqrt[a - d]*(b*(-10*c^3*Sqrt[b^2 -
4*c*d]*(b + Sqrt[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt
[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*c*d] + c^3*(b + Sqrt[b^2 - 4*c*
d])*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b
*(b + Sqrt[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*S
qrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d
]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d])
 + 4*c*(b + Sqrt[b^2 - 4*c*d])^2)))/(2*(4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c
*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2))))/(3*(b^2 - 4*c*d)^2*(4*a*c^2 - 2*b*c*
(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)
])
```

fricas [B] time = 40.66, size = 8134, normalized size = 24.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(1024*c^3*d^6 - (5*b^6*c^3 + 12*a*b^4*c^4 + 48*a^2*b^2*c^5 + 320*a
^3*c^6 - 1024*c^6*d^3 + 384*(b^2*c^5 + 4*a*c^6)*d^2 - 24*(3*b^4*c^4 + 8*a*b
```

```
^2*c^5 + 48*a^2*c^6)*d)*x^6 - 384*(b^2*c^2 + 4*a*c^3)*d^5 - 3*(5*b^7*c^2 +
12*a*b^5*c^3 + 48*a^2*b^3*c^4 + 320*a^3*b*c^5 - 1024*b*c^5*d^3 + 384*(b^3*c
^4 + 4*a*b*c^5)*d^2 - 24*(3*b^5*c^3 + 8*a*b^3*c^4 + 48*a^2*b*c^5)*d)*x^5 +
24*(3*b^4*c + 8*a*b^2*c^2 + 48*a^2*c^3)*d^4 - 3*(5*b^8*c + 12*a*b^6*c^2 + 4
8*a^2*b^4*c^3 + 320*a^3*b^2*c^4 - 1024*c^5*d^4 - 128*(5*b^2*c^4 - 12*a*c^5)
*d^3 + 24*(13*b^4*c^3 + 56*a*b^2*c^4 - 48*a^2*c^5)*d^2 - (67*b^6*c^2 + 180*
a*b^4*c^3 + 1104*a^2*b^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c +
48*a^2*b^2*c^2 + 320*a^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 +
320*a^3*b^3*c^3 - 6144*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b
^5*c^2 - 8*a*b^3*c^3 + 144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144
*a^2*b^3*c^3 - 320*a^3*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 1
2*a*c^4)*d^4 - 24*(13*b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c
+ 180*a*b^4*c^2 + 1104*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c
 + 48*a^2*b^4*c^2 + 320*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*
c^2 + 4*a*b*c^3)*d^4 + 24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b
^7 + 12*a*b^5*c + 48*a^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(a*b^2 + 4*c*
d^2 - (b^2 + 4*a*c)*d)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^
4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 +
 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24
*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2
*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*
(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b
*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*
d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*
b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d
^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d
*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(8*a^3*b^7 + 4608*b*c^3*d^6 - 2592*(b^3*
c^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3*b^2*c^5 +
 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 584*a*b^2*c^
5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2*c^5 + 960
*a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3)*d^4 + 5*
(15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4 - 1408*(b
^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2*b*c^5)*d^
2 - (15*b^7*c^2 + 292*a*b^5*c^3 + 1168*a^2*b^3*c^4 + 960*a^3*b*c^5)*d)*x^4
- (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3 + 4*(15*a*
b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*b^2*c^4 -
452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d^3 + 8*(27
*b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (15*b^8*c +
 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*a*b^7 + 58
4*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 120*a^3*b^5
*c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*(61*b^5*c^
2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^5*c^2 - 18
88*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 112*a^2*b^5*
c^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2*(5*a^2*b^
8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 + 4*(329*b^4
*c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^4*c^2 + 12
96*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304*a^2*b^4*c
^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b^4*c^2)*d)
*x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2*c^3 + 4*a
*c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*(b^6*c + 2
4*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*c + 576*a^
2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a^2*b^6*c +
 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^8 - 256*(b
^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7)*d^6 - 16
*(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^8*c^3 + 64
*a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^4 - 4*(a*b
^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 + 2*(3*a^2*b
^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 + 4*a^4*b^6*
c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 + 3*(a^4*b^
```

```
9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5*c^4 + 32*
a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a^2*b^3*c^5
 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4 + 1024*a^3
*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 96*a^3*b^5*
c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 48*a^4*b^5*
c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 + 4*a^4*b^6*
c)*d^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4*c^4 - 48*
a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - (15*b^8*c^2
 + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c + 60*a*b^8*
c^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45*a^2*b^8*c
^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30*a^3*b^8*c
^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4 + (a^4*b^
11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^5*c^3 + 80
*a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c^4 + 24*a^
3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 - 768*a^4*b*
c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c^4)*d^4 -
4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 + 20*a^3*b^9
*c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1024*a*c^5*
d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6*c^2 + 40*a
*b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2*b^4*c^3 -
256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3*b^4*c^3)*
d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c^3)*d^4 +
2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10 + 15*a^4*b
^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 + 4*a*b*c^4
)*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^7*c + 24*a
*b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c + 576*a^2
*b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24*a^2*b^7*c
+ 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^7*c + 48*a
^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x), -1/48*(3*(1024*c^3*d^6
 - (5*b^6*c^3 + 12*a*b^4*c^4 + 48*a^2*b^2*c^5 + 320*a^3*c^6 - 1024*c^6*d^3
+ 384*(b^2*c^5 + 4*a*c^6)*d^2 - 24*(3*b^4*c^4 + 8*a*b^2*c^5 + 48*a^2*c^6)*d
)*x^6 - 384*(b^2*c^2 + 4*a*c^3)*d^5 - 3*(5*b^7*c^2 + 12*a*b^5*c^3 + 48*a^2*
b^3*c^4 + 320*a^3*b*c^5 - 1024*b*c^5*d^3 + 384*(b^3*c^4 + 4*a*b*c^5)*d^2 -
24*(3*b^5*c^3 + 8*a*b^3*c^4 + 48*a^2*b*c^5)*d)*x^5 + 24*(3*b^4*c + 8*a*b^2*
c^2 + 48*a^2*c^3)*d^4 - 3*(5*b^8*c + 12*a*b^6*c^2 + 48*a^2*b^4*c^3 + 320*a^
3*b^2*c^4 - 1024*c^5*d^4 - 128*(5*b^2*c^4 - 12*a*c^5)*d^3 + 24*(13*b^4*c^3
+ 56*a*b^2*c^4 - 48*a^2*c^5)*d^2 - (67*b^6*c^2 + 180*a*b^4*c^3 + 1104*a^2*b
^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a
^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 + 320*a^3*b^3*c^3 - 6144
*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b^5*c^2 - 8*a*b^3*c^3 +
 144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144*a^2*b^3*c^3 - 320*a^3
*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 12*a*c^4)*d^4 - 24*(13*
b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c + 180*a*b^4*c^2 + 1104
*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c + 48*a^2*b^4*c^2 + 32
0*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*c^2 + 4*a*b*c^3)*d^4 +
 24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b^7 + 12*a*b^5*c + 48*a
^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d
)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d
+ (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*sqr
t(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2
*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)*d)*
x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)*d^
2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(8*a^3*b^7 + 4608*b*c^3*d^6 -
2592*(b^3*c^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3
*b^2*c^5 + 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 58
4*a*b^2*c^5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2
*c^5 + 960*a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3
)*d^4 + 5*(15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4
 - 1408*(b^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2
```

```
*b*c^5)*d^2 - (15*b^7*c^2 + 292*a*b^5*c^3 + 1168*a^2*b^3*c^4 + 960*a^3*b*c^
5)*d)*x^4 - (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3
+ 4*(15*a*b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*
b^2*c^4 - 452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d
^3 + 8*(27*b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (
15*b^8*c + 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*
a*b^7 + 584*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 1
20*a^3*b^5*c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*
(61*b^5*c^2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^
5*c^2 - 1888*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 11
2*a^2*b^5*c^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2
*(5*a^2*b^8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 +
4*(329*b^4*c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^
4*c^2 + 1296*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304
*a^2*b^4*c^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b
^4*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2
*c^3 + 4*a*c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*
(b^6*c + 24*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*
c + 576*a^2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a
^2*b^6*c + 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^
8 - 256*(b^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7
)*d^6 - 16*(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^
8*c^3 + 64*a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^
4 - 4*(a*b^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 +
2*(3*a^2*b^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 +
4*a^4*b^6*c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 +
 3*(a^4*b^9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5
*c^4 + 32*a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a
^2*b^3*c^5 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4
+ 1024*a^3*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 9
6*a^3*b^5*c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 4
8*a^4*b^5*c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 +
4*a^4*b^6*c)*d^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4
*c^4 - 48*a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - (
15*b^8*c^2 + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c +
 60*a*b^8*c^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45
*a^2*b^8*c^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30
*a^3*b^8*c^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4
 + (a^4*b^11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^
5*c^3 + 80*a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c
^4 + 24*a^3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 -
768*a^4*b*c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c
^4)*d^4 - 4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 +
20*a^3*b^9*c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1
024*a*c^5*d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6*
c^2 + 40*a*b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2*
b^4*c^3 - 256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3
*b^4*c^3)*d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c
^3)*d^4 + 2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10
+ 15*a^4*b^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 +
 4*a*b*c^4)*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^
7*c + 24*a*b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c
 + 576*a^2*b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24*
a^2*b^7*c + 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^
7*c + 48*a^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 3695, normalized size = 11.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+
(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/
c))^(1/2)*c*d+5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)
*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^
2-4*c*d)^(1/2))/c))^(1/2)*c*d-3/4/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*c*ln((2*a-2
*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)
^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/3/(b^2-4*c*d)^2/(a-d
)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)^3*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^
2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-3/(b^2-4*c*d
)^(5/2)*c^2/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)
^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*
d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2
))/c))-1/3/(b^2-4*c*d)^2/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^3*(a+(x-
1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d
)^(1/2))/c))^(1/2)+3/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c
*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2
-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+20*c^3/(b^2-4*c*d)^(7/2)/(a-d)^(1
/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^
(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-20*c^3/(b
^2-4*c*d)^(7/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-
4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^
(1/2))/c))-5/12/(b^2-4*c*d)^(3/2)/(a-d)^2/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/
c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(
b^2-4*c*d)^(1/2))/c))^(1/2)-5/16/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d-
(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(
b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2)
)/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*b^2+3/4/(b^2-4*c*d)^(3/2)/(a-
d)^(5/2)*c*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*
(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))+5/12/(
b^2-4*c*d)^(3/2)/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c))^(1/2)+5/16/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/
2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)
^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))
/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*b^2-5/8/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c
+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c
*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)*b^2-10/(b^2-4*c*d)^3*c^2/(
a-d)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)
^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)+2/(b^2-4*c*
d)^(5/2)*c/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x-1/2*(-b+(b^2-4
*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(
1/2)+3/2/(b^2-4*c*d)^(5/2)*c/(a-d)^(5/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))
/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2
```

```
*(-b+(b^2-4*c*d)^(1/2))/c))*b^2-6/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(5/2)*ln((2*a
-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*d-2/(b^2-4*c*d)^(5/2
)*c/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^(1
/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-7/3/
(b^2-4*c*d)^2*c/(a-d)^2/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b
^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)
)^(1/2)-3/2/(b^2-4*c*d)^(5/2)*c/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(
x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2)
)/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2
*(b+(b^2-4*c*d)^(1/2))/c))*b^2+6/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(5/2)*ln((2*a-
2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(
1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*d-10/(b^2-4*c*d)^3*c^2/(a
-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)
^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2)-5/8/(b^2-4
*c*d)^2/(a-d)^3/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2
)*b^2-7/3/(b^2-4*c*d)^2*c/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c))^(1/2)+5/4/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d-(b^2-4*c*
d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/
2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*c*d-5/4/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*
ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2
)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(cx^2 + bx + d\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2 + b\,x + a}\,\left(c\,x^2 + b\,x + d\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
[Out] Timed out
```

3.7 $\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

```
[Out] -(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*f
+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/(-4*a*f+b*e)^(3/2
)-b*(2*f*x+e)*(f*x^2+e*x+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+
a*e)
```

Rubi [A] time = 0.31, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {974, 12, 982, 208}

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e +
 b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]
*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/2)*
(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,\left(ae+bex+bfx^2\right)^2}\,dx = -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{\int \frac{b(bd-ae)f^2\left(8aef-b\left(e^2+4d\ldots\right.\right.}{2\sqrt{d+ex+fx^2}\,\left(ae+bex+bf\ldots\right)}}{be(bd-ae)^2f^2(be-4\ldots}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{\left(8aef-b\left(e^2+4df\right)\right)\ldots}{2e(bd-\ldots}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\left(8aef-b\left(e^2+4df\right)\right)\ldots}{\ldots}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\left(8aef-b\left(e^2+4df\right)\right)\ldots}{e^{3/2}(bd-a\ldots}$$

Mathematica [B] time = 1.96, size = 490, normalized size = 3.02

$$\frac{2f\left(-\frac{\left(b\left(4df+e^2\right)-8aef\right)\tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{e\tanh^{-1}\left(\frac{\sqrt{b}\left(e^2-4df\right)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}\sqrt{be-4af}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}(e+2fx)\sqrt{be-4af}-\ldots}{4f\sqrt{bd-ae}\sqrt{d+x(e\ldots}}\right)}{\sqrt{bd-ae}(be-4af)^{3/2}}\ldots\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] (2*f*(-((Sqrt[b]*Sqrt[e]*Sqrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*
(-(Sqrt[e]*Sqrt[b*e - 4*a*f]) + Sqrt[b]*(e + 2*f*x)))) - (Sqrt[b]*Sqrt[e]*S
qrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*(Sqrt[e]*Sqrt[b*e - 4*a*f]
 + Sqrt[b]*(e + 2*f*x))) - ((-8*a*e*f + b*(e^2 + 4*d*f))*ArcTanh[(-(Sqrt[b]
*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]
*f*Sqrt[d + x*(e + f*x)])])/(4*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) - (
e*ArcTanh[(Sqrt[b]*(e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(
4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])])/(4*(b*d - a*e)^(3/2)*f*Sqrt[b*
e - 4*a*f]) + ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) + Sqrt[e]*Sqrt[b*e - 4*a*f]
*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])]/(Sqrt[b*d - a*e]
*(b*e - 4*a*f)^(3/2))))/e^(3/2)
```

fricas [B] time = 3.07, size = 2005, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*
e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3
 + 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*log((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*e^6
 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*e^2)
*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d^2*e
- 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3 + (b^
2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*a*b*d*e
^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*sqrt(b^2*d*
e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*d*e^2*f +
 2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e - 2*a*e^2)
*f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)*sqrt(f*x^2
 + e*x + d) - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^
6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 +
 2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b
^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*b^2*d*e^2 - a^2
*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 - a*b^2*e^3)*f)*x
)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6 + a^3*b^2*e^7 + 1
6*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(a^2*b^3*d^2*e^2 -
2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3
*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^3*e^6)*f)*x^2 - 8*(a^2
*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*d^2*e^5 - 2*a*b^4*d*e^
6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*f^2 -
8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*x), 1/2*(sqrt(-b^2*d*e
^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*e^2*f + 4*(b^2*d -
2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3 + 4*(b^2*d*e - 2*a
*b*e^2)*f)*x)*arctan(-1/2*sqrt(-b^2*d*e^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)
*f)*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 +
(b*e^3 + 4*(b*d*e - 2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)/(b^2*d^2*e^3 - a*b
*d*e^4 - 2*(4*(a*b*d*e - a^2*e^2)*f^3 - (b^2*d*e^2 - a*b*e^3)*f^2)*x^3 - 3*
(4*(a*b*d*e^2 - a^2*e^3)*f^2 - (b^2*d*e^3 - a*b*e^4)*f)*x^2 - 4*(a*b*d^2*e^
2 - a^2*d*e^3)*f + (b^2*d*e^4 - a*b*e^5 - 8*(a*b*d^2*e - a^2*d*e^2)*f^2 + 2
*(b^2*d^2*e^2 - 3*a*b*d*e^3 + 2*a^2*e^4)*f)*x)) - 2*(b^3*d*e^3 - a*b^2*e^4
- 4*(a*b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d
*e^2 - a*b^2*e^3)*f)*x)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d
*e^6 + a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (
16*(a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 -
 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^
3*e^6)*f)*x^2 - 8*(a^2*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*
d^2*e^5 - 2*a*b^4*d*e^6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e
^4 + a^4*b*e^5)*f^2 - 8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*
x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[2]%%%},[8,2,0,0,0]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[1]%%%},[6,2,2,0,0]%%%}+%%%{%%%{-4,[2
]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[2]%%%},[6,1,1,1,0]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{[%%%{12,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,1,2,1,0]%%%}+%%%{1,[4,2,4,0,0]%%%}+%%%{%%%{-14,[1]%%%},[4,2,2,0,1]%%%}
+%%%{%%%{6,[2]%%%},[4,2,0,0,2]%%%}+%%%{%%%{26,[1]%%%},[4,1,3,1,0]%%%}+%%%{%
%%{-16,[2]%%%},[4,1,1,1,1]%%%}+%%%{%%%{16,[2]%%%},[4,0,2,2,0]%%%}+%%%{%%{[8
,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{[%%%{-12,[1]%%%},0]:[1,0
```

```
,%%%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,1,4,1,0]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,2,1,
1]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,3,2,0]%%%}+%
%%{-2,[2,2,4,0,1]%%%}+%%%{%%%{10,[1]%%%},[2,2,2,0,2]%%%}+%%%{%%%{-4,[2]%%%}
,[2,2,0,0,3]%%%}+%%%{2,[2,1,5,1,0]%%%}+%%%{%%%{-28,[1]%%%},[2,1,3,1,1]%%%}+
%%%{%%%{8,[2]%%%},[2,1,1,1,2]%%%}+%%%{%%%{24,[1]%%%},[2,0,4,2,0]%%%}+%%%{%%
{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{[%%%{4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,1,4,1,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,2,1
,2]%%%}+%%%{%%{[-8,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,5,2,0]%%%}+%%%{1,[0,2,4,
0,2]%%%}+%%%{%%%{-2,[1]%%%},[0,2,2,0,3]%%%}+%%%{%%%{1,[2]%%%},[0,2,0,0,4]%%
%}+%%%{-2,[0,1,5,1,1]%%%}+%%%{%%%{2,[1]%%%},[0,1,3,1,2]%%%}+%%%{1,[0,0,6,2,
0]%%%} / %%%{%%%{1,[3]%%%},[8,2,0,0,0]%%%}+%%%{%%{poly1[%%%{-4,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[2]%%%},[6,2,2,0,0]%%%}+%%
%{%%%{-4,[3]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[3]%%%},[6,1,1,1,0]%%%}+%%%{%%{p
oly1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{poly1
[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{poly1[%%%
{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,2,1,0]%%%}+%%%{%%%{1,[1]%%%},[
4,2,4,0,0]%%%}+%%%{%%%{-14,[2]%%%},[4,2,2,0,1]%%%}+%%%{%%%{6,[3]%%%},[4,2,0
,0,2]%%%}+%%%{%%%{26,[2]%%%},[4,1,3,1,0]%%%}+%%%{%%%{-16,[3]%%%},[4,1,1,1,1
]%%%}+%%%{%%%{16,[3]%%%},[4,0,2,2,0]%%%}+%%%{%%{poly1[%%%{8,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{poly1[%%%{-12,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[3,1,4,1,0]%%%}+%%%{%%{poly1[%%%{32,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[3,1,2,1,1]%%%}+%%%{%%{poly1[%%%{-32,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,0,3,2,0]%%%}+%%%{%%%{-2,[1]%%%},[2,2,4,0,1]%%%}+%%%{%%%{10,[2]%%%
},[2,2,2,0,2]%%%}+%%%{%%%{-4,[3]%%%},[2,2,0,0,3]%%%}+%%%{%%%{2,[1]%%%},[2,1
,5,1,0]%%%}+%%%{%%%{-28,[2]%%%},[2,1,3,1,1]%%%}+%%%{%%%{8,[3]%%%},[2,1,1,1,
2]%%%}+%%%{%%%{24,[2]%%%},[2,0,4,2,0]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{poly1[%%%{4,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{poly1[%%%{12,[1]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,1,4,1,1]%%%}+%%%{%%{poly1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,1,2,1,2]%%%}+%%%{%%{poly1[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[1,0,5,2,0]%%%}+%%%{%%%{1,[1]%%%},[0,2,4,0,2]%%%}+%%%{%%%{-2,[2]%%%},[
0,2,2,0,3]%%%}+%%%{%%%{1,[3]%%%},[0,2,0,0,4]%%%}+%%%{%%%{-2,[1]%%%},[0,1,5,
1,1]%%%}+%%%{%%%{2,[2]%%%},[0,1,3,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,6,2,0]%%%
} Error: Bad Argument Value
```

maple [B] time = 0.03, size = 1377, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -2/(4*a*f-b*e)/e*f/(-(4*a*f-b*e)*b*e)^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*
e-b*d)/b+(-(4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/
f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*
f+(-(4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a
*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f))-1/(4*a*f-b*e
)/e/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-(4*a*f-b*e)*b*e)^(1/2))*((x-1/2*(-b*e+(-
(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f+(-(4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4
*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2)+1/2/(4*a*f-b*e)/b/e*(-(4*a*
f-b*e)*b*e)^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-(4*a*
f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*
d)/b)^(1/2)*((x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f+(-(4*a*f-b*e)*
b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2)
)/(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f))-1/(4*a*f-b*e)/e/(a*e-b*d)/(x
+1/2/f*e+1/2/b/f*(-(4*a*f-b*e)*b*e)^(1/2))*((x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^
(1/2))/b/f)^2*f-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^(1/
```

```
2))/b/f)/b-(a*e-b*d)/b)^(1/2)-1/2/(4*a*f-b*e)/b/e*(-(4*a*f-b*e)*b*e)^(1/2)/
(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-(4*a*f-b*e)*b*e)^(1/2)*
(x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2
*(b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b
*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-(4*a*
f-b*e)*b*e)^(1/2))/b/f))+2/(4*a*f-b*e)/e*f/(-(4*a*f-b*e)*b*e)^(1/2)/(-(a*e-
b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-(4*
a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-(4*a*f-b*
e)*b*e)^(1/2))/b/f)^2*f-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-(4*a*f-b*e)*
b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^(1/2)
)/b/f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bfx^2+bex+ae\right)^2\sqrt{fx^2+ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,f\,x^2+b\,e\,x+a\,e\right)}^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.8 $\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {982, 204}

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, x, \frac{2+2x}{\sqrt{5+2x+x^2}}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 84, normalized size = 3.00

$$-\frac{i\left(\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ((-1/2*I)*(ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - Arc
Tanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]]))/Sqrt[3]
```

fricas [A] time = 0.84, size = 38, normalized size = 1.36

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}(x+1)-\frac{1}{3}\sqrt{3}\left(x^2+2x+4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(x
^2 + 2*x + 4))
```

giac [B] time = 0.22, size = 52, normalized size = 1.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt(
3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))
```

maple [A] time = 0.02, size = 27, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+2x+5}\left(x^2+2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

```
[Out] int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.9 $\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx$

Optimal. Leaf size=136

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}\left(2a+2cx^2+ex\right)^{p+q+1}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{(p+q+1)\sqrt{e^2-16ac}}$$

```
[Out] -2^(1+q)*(2*c*x^2+e*x+2*a)^(1+p+q)*hypergeom([-p-q, 1+p+q],[2+p+q],1/2*(e+4
*c*x+(-16*a*c+e^2)^(1/2))/(-16*a*c+e^2)^(1/2))*((-e-4*c*x+(-16*a*c+e^2)^(1/
2))/(-16*a*c+e^2)^(1/2))^(-1-p-q)/(1+p+q)/(-16*a*c+e^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {967, 624}

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}\left(2a+2cx^2+ex\right)^{p+q+1}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{(p+q+1)\sqrt{e^2-16ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

```
[Out] -((2^(1 + q)*(-((e - Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-1
 - p - q)*(2*a + e*x + 2*c*x^2)^(1 + p + q)*Hypergeometric2F1[-p - q, 1 + p
 + q, 2 + p + q, (e + Sqrt[-16*a*c + e^2] + 4*c*x)/(2*Sqrt[-16*a*c + e^2])]
)/(Sqrt[-16*a*c + e^2]*(1 + p + q)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 967

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_.)*(x_
)^2)^(q_.), x_Symbol] :> Dist[(c/f)^p, Int[(d + e*x + f*x^2)^(p + q), x], x
] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a
*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !IntegerQ[q] || LeafCount[d + e
*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

Rubi steps

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = 2^{-p}\int \left(2a+ex+2cx^2\right)^{p+q} \, dx$$

$$= -\frac{2^{1+q}\left(-\frac{e-\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-1-p-q}\left(2a+ex+2cx^2\right)^{1+p+q}\,{}_2F_1\left(-p-q,\right.}{\sqrt{-16ac+e^2}\,(1+p+q)}$$

Mathematica [A] time = 0.14, size = 142, normalized size = 1.04

$$\frac{2^{q-2}\left(-\sqrt{e^2-16ac}+4cx+e\right)\left(\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q}(2a+x(2cx+e))^{p+q}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{-e-4cx}{2\sqrt{}}\right.}{c(p+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

```
[Out] (2^(-2 + q)*(e - Sqrt[-16*a*c + e^2] + 4*c*x)*((e + Sqrt[-16*a*c + e^2] + 4
*c*x)/Sqrt[-16*a*c + e^2])^(-p - q)*(2*a + x*(e + 2*c*x))^(p + q)*Hypergeom
etric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16*a*c + e^2] - 4*c*x)/(
2*Sqrt[-16*a*c + e^2])])/(c*(1 + p + q))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,cx^2+ex+2\,a\right)^q\left(cx^2+\frac{1}{2}\,ex+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="fricas")
```

```
[Out] integral((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2\,cx^2+ex+2\,a\right)^q\left(cx^2+\frac{1}{2}\,ex+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+\frac{1}{2}ex+a\right)^p\left(2c\,x^2+ex+2a\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

```
[Out] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2\,cx^2+ex+2\,a\right)^q\left(cx^2+\frac{1}{2}\,ex+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+\frac{e\,x}{2}+a\right)^p\,\left(2\,c\,x^2+e\,x+2\,a\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x)
```

```
[Out] int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x**2)**p*(2*c*x**2+e*x+2*a)**q,x)
[Out] Timed out
```

3.10 $\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx$

Optimal. Leaf size=200

$$-\frac{\sqrt{c}\,2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{q+1}\left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1}{}_2F_1\left(-p-q,p+q+1;p+q+2;-\right.}{(p+q+1)\sqrt{ce^2-4af^2}}$$

```
[Out] -2^(1+p+q)*(a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^(1+q)*hypergeom([-p-q, 1+p
+q],[2+p+q],1/2*c^(1/2)*(e+2*f*x+(-4*a*f^2+c*e^2)^(1/2)/c^(1/2))/(-4*a*f^2+
c*e^2)^(1/2))*c^(1/2)*(-c^(1/2)*(e+2*f*x-(-4*a*f^2+c*e^2)^(1/2)/c^(1/2))/(-
4*a*f^2+c*e^2)^(1/2))^(-1-p-q)/(1+p+q)/(-4*a*f^2+c*e^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {968, 624}

$$-\frac{\sqrt{c}\,2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{q+1}\left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1}{}_2F_1\left(-p-q,p+q+1;p+q+2;-\right.}{(p+q+1)\sqrt{ce^2-4af^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] -((2^(1 + p + q)*Sqrt[c]*(-((Sqrt[c]*(e - Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2
*f*x))/Sqrt[c*e^2 - 4*a*f^2]))^(-1 - p - q)*(a + (c*e*x)/f + c*x^2)^p*((a*f
)/c + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q,
(Sqrt[c]*(e + Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f
^2])])/(Sqrt[c*e^2 - 4*a*f^2]*(1 + p + q)))
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rule 968

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x + c*x^2)^FracPart[p])/(
d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[(d + e*x + f*x^2)^(p + q),
 x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b
*d - a*e, 0] && !IntegerQ[p] && !IntegerQ[q] && !GtQ[c/f, 0]
```

Rubi steps

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx = \left(\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{-p}\right) \int \left(\frac{af}{c}+ex+fx^2\right)^{p+q} dx$$

$$= -\frac{2^{1+p+q}\sqrt{c}\left(-\frac{\sqrt{c}\left(e-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-1-p-q} \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex\right.}{\sqrt{ce^2-4af}}$$

Mathematica [A] time = 0.24, size = 172, normalized size = 0.86

$$\frac{2^{p+q-1}\left(\sqrt{c}(e+2fx)-\sqrt{ce^2-4af^2}\right)\left(a+\frac{cx(e+fx)}{f}\right)^p \left(\frac{af}{c}+x(e+fx)\right)^q \left(\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2-4af^2}}+1\right)^{-p-q} {}_2F_1\left(-p-q,p+q+\right.}{\sqrt{c}f(p+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] (2^(-1 + p + q)*((a*f)/c + x*(e + f*x))^q*(a + (c*x*(e + f*x))/f)^p*(-Sqrt[
c*e^2 - 4*a*f^2] + Sqrt[c]*(e + 2*f*x))*(1 + (Sqrt[c]*(e + 2*f*x))/Sqrt[c*e
^2 - 4*a*f^2])^(-p - q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q, 1/2
 - (Sqrt[c]*(e + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f^2])])/(Sqrt[c]*f*(1 + p + q)
)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{cfx^2+cex+af}{c}\right)^q \left(\frac{cfx^2+cex+af}{f}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((c*f*x^2 + c*e*x + a*f)/c)^q*((c*f*x^2 + c*e*x + a*f)/f)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+\frac{cex}{f}+a\right)^p \left(f\,x^2+ex+\frac{af}{c}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

```
[Out] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2 + \frac{cex}{f} + a\right)^p \left(fx^2 + ex + \frac{af}{c}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="maxima")
[Out] integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x + f\,x^2 + \frac{a\,f}{c}\right)^q \left(a + c\,x^2 + \frac{c\,e\,x}{f}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p,x)
[Out] int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)
[Out] Timed out
```

3.11 $\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{x^2+1}\,\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\,\sinh^{-1}(x)}{x+1}$$

```
[Out] arcsinh(x)*((1+x)^2)^(1/2)/(1+x)+(x^2+1)^(1/2)*((1+x)^2)^(1/2)/(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {970, 641, 215}

$$\frac{\sqrt{x^2+1}\,\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[
x])/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx &= \frac{\sqrt{1+2x+x^2}\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}\\ &= \frac{\sqrt{1+x^2}\,\sqrt{1+2x+x^2}}{1+x}+\frac{\left(2\sqrt{1+2x+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}\\ &= \frac{\sqrt{1+x^2}\,\sqrt{1+2x+x^2}}{1+x}+\frac{\sqrt{1+2x+x^2}\,\sinh^{-1}(x)}{1+x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.56

$$\frac{\sqrt{(x+1)^2}\left(\sqrt{x^2+1}+\sinh^{-1}(x)\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)
```

fricas [A] time = 0.86, size = 22, normalized size = 0.46

$$\sqrt{x^2+1} - \log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

giac [A] time = 0.23, size = 49, normalized size = 1.02

$$-\left(\sqrt{2} - \log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1) - \log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1) + \sqrt{x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

maple [C] time = 0.05, size = 16, normalized size = 0.33

$$\left(\operatorname{arcsinh}(x) + \sqrt{x^2+1}\right)\operatorname{csgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] csgn(1+x)*(arcsinh(x)+(x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)
```

3.12 $\int \frac{1}{\left(-1+x^2\right)^2 \sqrt{-1+x+x^2}} \, dx$

Optimal. Leaf size=70

$$\frac{\sqrt{x^2+x-1}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

```
[Out] -1/8*arctan(1/2*(3+x)/(x^2+x-1)^(1/2))-5/8*arctanh(1/2*(1-3*x)/(x^2+x-1)^(1
/2))+1/2*(x^2+x-1)^(1/2)/(-x^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {976, 1033, 724, 206, 204}

$$\frac{\sqrt{x^2+x-1}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8
 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^2\right)^2 \sqrt{-1+x+x^2}}\,dx &= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}-\frac{1}{4}\int \frac{3+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx\\
&= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}+\frac{1}{8}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{(-1+x)\sqrt{-1+x+x}}\\
&= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}-\frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\frac{-3-x}{\sqrt{-1+x+x^2}}\right)+\frac{5}{4}\mathrm{Subst}\left(\int\right.\\
&= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.09, size = 66, normalized size = 0.94

$$\frac{1}{8}\left(-\frac{4\sqrt{x^2+x-1}}{x^2-1}-\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-5\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] ((-4*Sqrt[-1 + x + x^2])/(-1 + x^2) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])
] - 5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

fricas [A] time = 0.91, size = 82, normalized size = 1.17

$$\frac{2\left(x^2-1\right)\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+5\left(x^2-1\right)\log\left(-x+\sqrt{x^2+x-1}+2\right)-5\left(x^2-1\right)\log\left(-x+\sqrt{x^2}\right.}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^2 - 1)*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5*(x^2 - 1)*log(-x +
sqrt(x^2 + x - 1) + 2) - 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1)) - 4*sqrt(x
^2 + x - 1))/(x^2 - 1)
```

giac [B] time = 0.26, size = 143, normalized size = 2.04

$$\frac{2\left(x-\sqrt{x^2+x-1}\right)^3+3\left(x-\sqrt{x^2+x-1}\right)^2-x+\sqrt{x^2+x-1}-1}{2\left(\left(x-\sqrt{x^2+x-1}\right)^4-2\left(x-\sqrt{x^2+x-1}\right)^2-4x+4\sqrt{x^2+x-1}\right)}+\frac{1}{4}\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+\frac{5}{8}1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(x - sqrt(x^2 + x - 1))^3 + 3*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(x
^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2*(x - sqrt(x^2 + x - 1))^2 -
 4*x + 4*sqrt(x^2 + x - 1)) + 1/4*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5/8*
log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8*log(abs(-x + sqrt(x^2 + x - 1)))
```

maple [A] time = 0.02, size = 84, normalized size = 1.20

$$\frac{5\,\mathrm{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{8}+\frac{\arctan\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{8}+\frac{\sqrt{-x+(x+1)^2-2}}{4x+4}-\frac{\sqrt{3x+(x-1)^2-2}}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x)
```

```
[Out] 1/4/(x+1)*((x+1)^2-2-x)^(1/2)+1/8*arctan(1/2*(-3-x)/((x+1)^2-2-x)^(1/2))+5/
8*arctanh(1/2*(-1+3*x)/((x-1)^2+3*x-2)^(1/2))-1/4/(x-1)*((x-1)^2+3*x-2)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+x-1}\,\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + x - 1)*(x^2 - 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2-1\right)^2\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)^2\,(x+1)^2\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x**2 + x - 1)), x)
```

3.13 $\int \frac{1}{\sqrt{a+bx+cx^2}\,\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=1077

$$-\frac{\sqrt[4]{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-\right.\right.}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2\right.}}}{\left(4ac-\left(b+\sqrt{b^2\right.\right.}}$$

```
[Out] -(cos(2*arctan((2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^(
1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2)))^2)^(1/2)/cos(2*arctan((2*c^2*d-2*
a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2
)/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^(1/4)/(b+2*c*x+(-4*a*c+b^2)
^(1/2))^(1/2)))*EllipticF(sin(2*arctan((2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)
^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d-2*a*(-a*f+c*d)+b
*d*(-4*a*c+b^2)^(1/2))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2))),1/2*(2+2*
(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/2))^(1/2))*(b+2*c
*x+(-4*a*c+b^2)^(1/2))^(3/2)*(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^
(1/4)*(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2
)^(1/2)))^(1/2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*
a*c+b^2)^(1/2))^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((f*x^2+d)*(4*a
*c-(b+(-4*a*c+b^2)^(1/2))^2)^2/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f+d*(b
+(-4*a*c+b^2)^(1/2))^2))^(1/2)*((1-4*(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+
x*(b+(-4*a*c+b^2)^(1/2)))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(4*a^2*f+d*(b+(-4*a*
c+b^2)^(1/2))^2)+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*d+f*(b+(-4*a*c+b^2
)^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2)
)^2))/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2
)^(1/2)))^(1/2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*
a*c+b^2)^(1/2))^(1/2))^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2))
)^(1/4)/(4*a*c-(b+(-4*a*c+b^2)^(1/2))^2)/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2
)/(1-4*(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(b+2
*c*x+(-4*a*c+b^2)^(1/2))/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2))^2)+(2*a+x*(b+(-4
*a*c+b^2)^(1/2)))^2*(4*c^2*d+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c+b
^2)^(1/2))^2/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)
```

Rubi [A] time = 3.10, antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {993, 936, 1103}

$$-\frac{\sqrt[4]{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-\right.\right.}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2\right.}}}{\left(4ac-\left(b+\sqrt{b^2\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

```
[Out] -(((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c -
(b + Sqrt[b^2 - 4*a*c])^2)^2*(d + f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*f +
b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d
 + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
))*Sqrt[(1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 -
4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2
- 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*
f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d -
 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[(
(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[2*a + (b + Sqr
t[b^2 - 4*a*c])*x])/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4
)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^2 - 4*a*c])*(c*d
 + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[b^2*d
+ b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2])/((4*a*c - (b + Sqrt[b^2 -
4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[a +
 b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*
f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2
*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^
2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

Rule 936

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/
((c*f^2 + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sqrt[a + c*x^2]), Subst[Int[1/
Sqrt[1 - ((2*c*d*f + 2*a*e*g)*x^2)/(c*f^2 + a*g^2) + ((c*d^2 + a*e^2)*x^4)/
(c*f^2 + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 993

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (f_.)*(x_)^2]), x
_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r + 2*c*x]*Sqrt[2
*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a
 + (b + r)*x]*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ
[b^2 - 4*a*c, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(\cdots}}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-\cdots}\right.\right.}{\left(\left(b+\sqrt{b^2-4ac}\right)^2 d+4a\cdots\right.}}\right.}{\left(4ac-\left(b\cdots\right.\right.}$$

$$= -\frac{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}\,d-2a(cd-af)}\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\cdots\right.}}{\cdots}$$

Mathematica [C] time = 1.64, size = 600, normalized size = 0.56

$$-\frac{2\sqrt{2}\left(\sqrt{f}\,x-i\sqrt{d}\right)\left(\sqrt{b^2-4ac}-b-2cx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(\sqrt{f}\,x+i\sqrt{d}\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(\sqrt{f}\left(\sqrt{b^2-4ac}+b\right)-2ic\sqrt{d}\right)}}\sqrt{\frac{c\left(-i\sqrt{d}\left(\sqrt{b^2-4ac}+2cx\right)+\sqrt{f}\,(\cdots\right.}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(\sqrt{\cdots}\right.}}}{\sqrt{d+fx^2}\sqrt{a+x(b+cx)}\left(\sqrt{f}\left(\sqrt{b^2-4ac}-b\right)-2ic\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

```
[Out] (-2*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*((-I)*Sqrt[d] + Sqrt[f]*x)*Sqr
t[-((c*Sqrt[b^2 - 4*a*c]*(I*Sqrt[d] + Sqrt[f]*x))/(((-2*I)*c*Sqrt[d] + (b +
 Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqrt[(c*((
-I)*Sqrt[d]*(Sqrt[b^2 - 4*a*c] + 2*c*x) + Sqrt[f]*(-2*a + Sqrt[b^2 - 4*a*c]
*x) + b*((-I)*Sqrt[d] - Sqrt[f]*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*
a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*EllipticF[ArcSin[Sqrt[(((
-2*I)*c*Sqrt[d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]], (c*d - I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)/
(c*d + I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)])/(((-2*I)*c*Sqrt[d] + (-
b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*Sqrt[(I*c*Sqrt[b^2 - 4*a*c]*(Sqrt[d] + I*Sq
rt[f]*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]*Sqrt[d + f*x^2]*Sqrt[a + x*(b + c*x)])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{fx^2+d}}{cfx^4+bfx^3+bdx+\left(cd+af\right)x^2+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)/(c*f*x^4 + b*f*x^3 + b*d*x +
 (c*d + a*f)*x^2 + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{fx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

maple [A] time = 0.25, size = 661, normalized size = 0.61

$$4\Big(bf^2x^2-4cdfx+2\sqrt{-df}\,cfx^2+\sqrt{-4ac+b^2}\,f^2x^2-bdf+2\sqrt{-df}\,bfx-2\sqrt{-df}\,cd-\sqrt{-4ac+b^2}\,df+2\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x)
```

```
[Out] 4*(b*f^2*x^2+2*x^2*c*f*(-d*f)^(1/2)+x^2*f^2*(-4*a*c+b^2)^(1/2)+2*x*b*f*(-d*
f)^(1/2)-4*c*x*f*d+2*x*f*(-4*a*c+b^2)^(1/2)*(-d*f)^(1/2)-b*d*f-2*c*d*(-d*f)
^(1/2)-d*f*(-4*a*c+b^2)^(1/2))*EllipticF((-(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(
1/2)*c+b*f)*(-f*x+(-d*f)^(1/2))/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)
/(f*x+(-d*f)^(1/2)))^(1/2),((f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)*(f*
(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/
2)*c-b*f)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f))^(1/2))*((-d*f)^(1/2)
*(b+2*c*x+(-4*a*c+b^2)^(1/2))*f/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)
/(f*x+(-d*f)^(1/2)))^(1/2)*((-d*f)^(1/2)*(-2*c*x+(-4*a*c+b^2)^(1/2)-b)*f/(f
*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c-b*f)/(f*x+(-d*f)^(1/2)))^(1/2)*(-(f*(-
4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)*(-f*x+(-d*f)^(1/2))/(f*(-4*a*c+b^2)^
(1/2)+2*(-d*f)^(1/2)*c+b*f)/(f*x+(-d*f)^(1/2)))^(1/2)*(c*x^2+b*x+a)^(1/2)*(
f*x^2+d)^(1/2)/(1/c/f*(-f*x+(-d*f)^(1/2))*(f*x+(-d*f)^(1/2))*(-2*c*x+(-4*a*
c+b^2)^(1/2)-b)*(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(-d*f)^(1/2)/(f*(-4*a*c
+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)/(c*f*x^4+b*f*x^3+a*f*x^2+c*d*x^2+b*d*x+a*
d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{fx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f\,x^2+d}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+fx^2}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + f*x**2)*sqrt(a + b*x + c*x**2)), x)
```

3.14 $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sin^{-1}(x+2)$$

```
[Out] -1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x)
/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^
(1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {989, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[
2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x
/Sqrt[-3 - 4*x - x^2]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx\right) - \frac{1}{2}\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$$

$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx + \frac{3}{2}\int$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx + \frac{1}{4}\int -\frac{4x}{\sqrt{-3-4x-x^2}\,(3+4}$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+}{\sqrt{-3-4}}\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\right.\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x}}\right.$$

Mathematica [C] time = 0.39, size = 159, normalized size = 1.62

$$\frac{1}{4}\left(-i\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right) + i\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right) - 2\,\mathrm{s}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] (-2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] +
2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt
[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2
 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/4
```

fricas [A] time = 0.96, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) + 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

giac [B] time = 0.21, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\text{arcsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [B] time = 0.03, size = 341, normalized size = 3.48

$$-\frac{\arcsin(x+2)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)}{3\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(-\text{arctanh}\left(\cdots\right.\right.}{12\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x)
```

```
[Out] -1/2*arcsin(2+x)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2
/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-
3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+
x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^
2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/
2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*
x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x
/(-3/2-x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2-4x-3}}{2x^2+4x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^2-4x-3}}{2x^2+4x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3),x)
```

```
[Out] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2+4x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
[Out] Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

3.15 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx$

Optimal. Leaf size=68

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

```
[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5
075/9*x^9+475/2*x^10+1250/11*x^11
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx = \int \left(48+272x+1064x^2+2624x^3+5099x^4+7131x^5+8232x^6+683\right.$$
$$= 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x}{4}$$

Mathematica [A] time = 0.00, size = 68, normalized size = 1.00

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

fricas [A] time = 0.72, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^9+\frac{3415}{4}x^8+1176x^7+\frac{2377}{2}x^6+\frac{5099}{5}x^5+656x^4+\frac{1064}{3}x^3+136x^2+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

giac [A] time = 0.21, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^{9}+\frac{3415}{4}\,x^{8}+1176\,x^{7}+\frac{2377}{2}\,x^{6}+\frac{5099}{5}\,x^{5}+656\,x^{4}+\frac{1064}{3}\,x^{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^{9}+\frac{3415}{4}x^{8}+1176x^{7}+\frac{2377}{2}x^{6}+\frac{5099}{5}x^{5}+656x^{4}+\frac{1064}{3}x^{3}+136x^{2}+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5
075/9*x^9+475/2*x^10+1250/11*x^11
```

maxima [A] time = 0.46, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^{9}+\frac{3415}{4}\,x^{8}+1176\,x^{7}+\frac{2377}{2}\,x^{6}+\frac{5099}{5}\,x^{5}+656\,x^{4}+\frac{1064}{3}\,x^{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

mupad [B] time = 0.06, size = 54, normalized size = 0.79

$$\frac{1250\,x^{11}}{11}+\frac{475\,x^{10}}{2}+\frac{5075\,x^9}{9}+\frac{3415\,x^8}{4}+1176\,x^7+\frac{2377\,x^6}{2}+\frac{5099\,x^5}{5}+656\,x^4+\frac{1064\,x^3}{3}+136\,x^2+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

sympy [A] time = 0.09, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^{9}}{9}+\frac{3415x^{8}}{4}+1176x^{7}+\frac{2377x^{6}}{2}+\frac{5099x^{5}}{5}+656x^{4}+\frac{1064x^{3}}{3}+136x^{2}+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377*
x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x
```

3.16 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx$

Optimal. Leaf size=56

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

```
[Out] 24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9
*x^9
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx=\int \left(24+100x+322x^2+579x^3+876x^4+804x^5+720x^6+325x^7+2\right.$$

$$=24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{25}{}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

fricas [A] time = 0.72, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9+\frac{325}{8}x^8+\frac{720}{7}x^7+134x^6+\frac{876}{5}x^5+\frac{579}{4}x^4+\frac{322}{3}x^3+50x^2+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

giac [A] time = 0.18, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^{9} + \frac{325}{8}\,x^{8} + \frac{720}{7}\,x^{7} + 134\,x^{6} + \frac{876}{5}\,x^{5} + \frac{579}{4}\,x^{4} + \frac{322}{3}\,x^{3} + 50\,x^{2} + 24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{250}{9}x^{9} + \frac{325}{8}x^{8} + \frac{720}{7}x^{7} + 134x^{6} + \frac{876}{5}x^{5} + \frac{579}{4}x^{4} + \frac{322}{3}x^{3} + 50x^{2} + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x)
```

```
[Out] 24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9
*x^9
```

maxima [A] time = 0.45, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^{9} + \frac{325}{8}\,x^{8} + \frac{720}{7}\,x^{7} + 134\,x^{6} + \frac{876}{5}\,x^{5} + \frac{579}{4}\,x^{4} + \frac{322}{3}\,x^{3} + 50\,x^{2} + 24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.79

$$\frac{250\,x^{9}}{9} + \frac{325\,x^{8}}{8} + \frac{720\,x^{7}}{7} + 134\,x^{6} + \frac{876\,x^{5}}{5} + \frac{579\,x^{4}}{4} + \frac{322\,x^{3}}{3} + 50\,x^{2} + 24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{250x^{9}}{9} + \frac{325x^{8}}{8} + \frac{720x^{7}}{7} + 134x^{6} + \frac{876x^{5}}{5} + \frac{579x^{4}}{4} + \frac{322x^{3}}{3} + 50x^{2} + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)
```

```
[Out] 250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4 +
 322*x**3/3 + 50*x**2 + 24*x
```

3.17 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx$

Optimal. Leaf size=44

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

```
[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx &= \int \left(12+32x+83x^2+85x^3+103x^4+35x^5+50x^6\right)dx\\ &= 12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

fricas [A] time = 0.69, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7+\frac{35}{6}x^6+\frac{103}{5}x^5+\frac{85}{4}x^4+\frac{83}{3}x^3+16x^2+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

giac [**A**] time = 0.21, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^{7}+\frac{35}{6}\,x^{6}+\frac{103}{5}\,x^{5}+\frac{85}{4}\,x^{4}+\frac{83}{3}\,x^{3}+16\,x^{2}+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

maple [**A**] time = 0.00, size = 35, normalized size = 0.80

$$\frac{50}{7}x^{7}+\frac{35}{6}x^{6}+\frac{103}{5}x^{5}+\frac{85}{4}x^{4}+\frac{83}{3}x^{3}+16x^{2}+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

maxima [**A**] time = 0.43, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^{7}+\frac{35}{6}\,x^{6}+\frac{103}{5}\,x^{5}+\frac{85}{4}\,x^{4}+\frac{83}{3}\,x^{3}+16\,x^{2}+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

mupad [**B**] time = 0.03, size = 34, normalized size = 0.77

$$\frac{50\,x^7}{7}+\frac{35\,x^6}{6}+\frac{103\,x^5}{5}+\frac{85\,x^4}{4}+\frac{83\,x^3}{3}+16\,x^2+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

sympy [**A**] time = 0.08, size = 41, normalized size = 0.93

$$\frac{50x^{7}}{7}+\frac{35x^{6}}{6}+\frac{103x^{5}}{5}+\frac{85x^{4}}{4}+\frac{83x^{3}}{3}+16x^{2}+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)
```

```
[Out] 50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*x
```

3.18 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx$

Optimal. Leaf size=30

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

```
[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1657}

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx &= \int \left(6+7x+16x^2+x^3+10x^4\right)\,dx\\ &= 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

fricas [A] time = 0.69, size = 24, normalized size = 0.80

$$2x^5+\frac{1}{4}x^4+\frac{16}{3}x^3+\frac{7}{2}x^2+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

giac [A] time = 0.18, size = 24, normalized size = 0.80

$$2x^5+\frac{1}{4}x^4+\frac{16}{3}x^3+\frac{7}{2}x^2+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2),x)
```

```
[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

maxima [A] time = 0.43, size = 24, normalized size = 0.80

$$2\,x^5 + \frac{1}{4}\,x^4 + \frac{16}{3}\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

mupad [B] time = 0.02, size = 24, normalized size = 0.80

$$2\,x^5 + \frac{x^4}{4} + \frac{16\,x^3}{3} + \frac{7\,x^2}{2} + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2),x)
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

sympy [A] time = 0.06, size = 26, normalized size = 0.87

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)
```

```
[Out] 2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x
```

3.19 $\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

```
[Out] 2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx &= \int \left(\frac{2}{5}+\frac{11(1-x)}{5\left(2+3x+5x^2\right)}\right)dx \\
&= \frac{2x}{5}+\frac{11}{5}\int \frac{1-x}{2+3x+5x^2}\,dx \\
&= \frac{2x}{5}-\frac{11}{50}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{143}{50}\int \frac{1}{2+3x+5x^2}\,dx \\
&= \frac{2x}{5}-\frac{11}{50}\log\left(2+3x+5x^2\right)-\frac{143}{25}\,\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right) \\
&= \frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

fricas [A] time = 0.65, size = 33, normalized size = 0.79

$$\frac{143}{775}\sqrt{31}\,\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

giac [A] time = 0.18, size = 33, normalized size = 0.79

$$\frac{143}{775}\sqrt{31}\,\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{2x}{5}+\frac{143\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{775}-\frac{11\ln\left(5x^2+3x+2\right)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2),x)
```

```
[Out] 2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

maxima [A] time = 0.97, size = 33, normalized size = 0.79

$$\frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

mupad [B] time = 3.39, size = 35, normalized size = 0.83

$$\frac{2\,x}{5}-\frac{11\ln\left(5\,x^2+3\,x+2\right)}{50}+\frac{143\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2),x)
```

```
[Out] (2*x)/5 - (11*log(3*x + 5*x^2 + 2))/50 + (143*31^(1/2)*atan((10*31^(1/2)*x)
/31 + (3*31^(1/2))/31))/775
```

sympy [A] time = 0.14, size = 49, normalized size = 1.17

$$\frac{2x}{5}-\frac{11\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{50}+\frac{143\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] 2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/775
```

3.20 $\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=43

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

```
[Out] 11/155*(7+13*x)/(5*x^2+3*x+2)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2
)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 204}

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{1}{31}\int \frac{41}{2+3x+5x^2}\,dx\\ &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{41}{31}\int \frac{1}{2+3x+5x^2}\,dx\\ &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}-\frac{82}{31}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)\\ &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

fricas [A] time = 0.73, size = 45, normalized size = 1.05

$$\frac{410\sqrt{31}\left(5x^2+3x+2\right)\arctan\left(\frac{1}{31}\sqrt{31}\,(10x+3)\right)+4433x+2387}{4805\left(5x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4
433*x + 2387)/(5*x^2 + 3*x + 2)
```

giac [A] time = 0.21, size = 36, normalized size = 0.84

$$\frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10x+3)\right)+\frac{11\,(13x+7)}{155\left(5x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{961}+\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

```
[Out] (143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*31^(1/2)*arctan(1/31*(10*x+3)*31^
(1/2))
```

maxima [A] time = 0.97, size = 36, normalized size = 0.84

$$\frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{11(13x+7)}{155(5x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3x}{5}+\frac{2}{5}}+\frac{82\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] ((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^(1/2)*atan((10*31^(1/
2)*x)/31 + (3*31^(1/2))/31))/961
```

sympy [A] time = 0.16, size = 42, normalized size = 0.98

$$\frac{143x+77}{775x^2+465x+310}+\frac{82\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31 +
 3*sqrt(31)/31)/961
```

3.21 $\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$

Optimal. Leaf size=64

$$\frac{553(10x+3)}{9610(5x^2+3x+2)}+\frac{11(13x+7)}{310(5x^2+3x+2)^2}+\frac{1106\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

```
[Out] 11/310*(7+13*x)/(5*x^2+3*x+2)^2+553/9610*(3+10*x)/(5*x^2+3*x+2)+1106/29791*
arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1660, 12, 614, 618, 204}

$$\frac{553(10x+3)}{9610(5x^2+3x+2)}+\frac{11(13x+7)}{310(5x^2+3x+2)^2}+\frac{1106\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x
 + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
```

```
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^3}\,dx &= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{553}{5\left(2+3x+5x^2\right)^2}\,dx\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553}{310}\int \frac{1}{\left(2+3x+5x^2\right)^2}\,dx\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{553}{961}\int \frac{1}{2+3x+5x^2}\,dx\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}-\frac{1106}{961}\operatorname{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{1106\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.83

$$
\frac{\frac{31\left(5530x^3+4977x^2+4094x+1141\right)}{\left(5x^2+3x+2\right)^2}+2212\sqrt{31}\,\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{59582}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212*Sqrt
[31]*ArcTan[(3 + 10*x)/Sqrt[31]])/59582
```

fricas [A] time = 0.80, size = 75, normalized size = 1.17

$$
\frac{171430\,x^3+2212\,\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+154287\,x^2+126914\,x+35}{59582\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*a
rctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.17, size = 46, normalized size = 0.72

$$
\frac{1106}{29791}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\left(5\,x^2+3\,x+2\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2
```

maple [A] time = 0.00, size = 47, normalized size = 0.73

$$\frac{1106\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791}+\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{\left(5x^2+3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+11
06/29791*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{1106}{29791}\sqrt{31}\ \arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\,\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

mupad [B] time = 0.05, size = 55, normalized size = 0.86

$$\frac{1106\,\sqrt{31}\ \mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{29791}+\frac{\frac{553\,x^3}{4805}+\frac{4977\,x^2}{48050}+\frac{2047\,x}{24025}+\frac{1141}{48050}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (1106*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((2047*x
)/24025 + (4977*x^2)/48050 + (553*x^3)/4805 + 1141/48050)/((12*x)/25 + (29*
x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.18, size = 63, normalized size = 0.98

$$\frac{5530x^3+4977x^2+4094x+1141}{48050x^4+57660x^3+55738x^2+23064x+7688}+\frac{1106\sqrt{31}\ \mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x*
*2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)
/29791
```

3.22 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=80

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384$$

```
[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*
x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx = \int \left(144+768x+3016x^2+7352x^3+14801x^4+21542x^5+27763x^6+2\right.$$
$$= 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

fricas [A] time = 0.77, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13}+\frac{875}{3}x^{12}+\frac{11525}{11}x^{11}+1571x^{10}+\frac{24859}{9}x^9+3315x^8+\frac{27763}{7}x^7+\frac{10771}{3}x^6+\frac{14801}{5}x^5+1838x^4+\frac{3016}{3}x^3+384$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

giac [A] time = 0.20, size = 64, normalized size = 0.80

$$\frac{2500}{13}\,x^{13}+\frac{875}{3}\,x^{12}+\frac{11525}{11}\,x^{11}+1571\,x^{10}+\frac{24859}{9}\,x^{9}+3315\,x^{8}+\frac{27763}{7}\,x^{7}+\frac{10771}{3}\,x^{6}+\frac{14801}{5}\,x^{5}+1838\,x^{4}+\frac{3016}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.81

$$\frac{2500}{13}x^{13}+\frac{875}{3}x^{12}+\frac{11525}{11}x^{11}+1571x^{10}+\frac{24859}{9}x^{9}+3315x^{8}+\frac{27763}{7}x^{7}+\frac{10771}{3}x^{6}+\frac{14801}{5}x^{5}+1838x^{4}+\frac{3016}{3}x^{3}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x)
```

```
[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*
x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

maxima [A] time = 0.45, size = 64, normalized size = 0.80

$$\frac{2500}{13}\,x^{13}+\frac{875}{3}\,x^{12}+\frac{11525}{11}\,x^{11}+1571\,x^{10}+\frac{24859}{9}\,x^{9}+3315\,x^{8}+\frac{27763}{7}\,x^{7}+\frac{10771}{3}\,x^{6}+\frac{14801}{5}\,x^{5}+1838\,x^{4}+\frac{3016}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.80

$$\frac{2500\,x^{13}}{13}+\frac{875\,x^{12}}{3}+\frac{11525\,x^{11}}{11}+1571\,x^{10}+\frac{24859\,x^{9}}{9}+3315\,x^{8}+\frac{27763\,x^{7}}{7}+\frac{10771\,x^{6}}{3}+\frac{14801\,x^{5}}{5}+1838\,x^{4}+\frac{3016}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

sympy [A] time = 0.10, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^{9}}{9}+3315x^{8}+\frac{27763x^{7}}{7}+\frac{10771x^{6}}{3}+\frac{14801x^{5}}{5}+1838x^{4}+\frac{3016x^{3}}{3}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)
```

```
[Out] 2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 +
3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016*x
**3/3 + 384*x**2 + 144*x
```

3.23 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=66

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

```
[Out] 72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+186
5/9*x^9+40*x^10+500/11*x^11
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1657}

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \int \left(72+276x+914x^2+1615x^3+2693x^4+2694x^5+3108x^6+1863x^7\right.$$
$$= 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}+$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

fricas [A] time = 0.51, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11}+40x^{10}+\frac{1865}{9}x^9+\frac{1863}{8}x^8+444x^7+449x^6+\frac{2693}{5}x^5+\frac{1615}{4}x^4+\frac{914}{3}x^3+138x^2+72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

giac [A] time = 0.21, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^{9}+\frac{1863}{8}\,x^{8}+444\,x^{7}+449\,x^{6}+\frac{2693}{5}\,x^{5}+\frac{1615}{4}\,x^{4}+\frac{914}{3}\,x^{3}+138\,x^{2}+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.83

$$\frac{500}{11}x^{11}+40x^{10}+\frac{1865}{9}x^{9}+\frac{1863}{8}x^{8}+444x^{7}+449x^{6}+\frac{2693}{5}x^{5}+\frac{1615}{4}x^{4}+\frac{914}{3}x^{3}+138x^{2}+72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x)
```

```
[Out] 72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+186
5/9*x^9+40*x^10+500/11*x^11
```

maxima [A] time = 0.44, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^{9}+\frac{1863}{8}\,x^{8}+444\,x^{7}+449\,x^{6}+\frac{2693}{5}\,x^{5}+\frac{1615}{4}\,x^{4}+\frac{914}{3}\,x^{3}+138\,x^{2}+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

mupad [B] time = 0.05, size = 54, normalized size = 0.82

$$\frac{500\,x^{11}}{11}+40\,x^{10}+\frac{1865\,x^{9}}{9}+\frac{1863\,x^{8}}{8}+444\,x^{7}+449\,x^{6}+\frac{2693\,x^{5}}{5}+\frac{1615\,x^{4}}{4}+\frac{914\,x^{3}}{3}+138\,x^{2}+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

sympy [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^{9}}{9}+\frac{1863x^{8}}{8}+444x^{7}+449x^{6}+\frac{2693x^{5}}{5}+\frac{1615x^{4}}{4}+\frac{914x^{3}}{3}+138x^{2}+72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)
```

```
[Out] 500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6 +
 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x
```

3.24 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

```
[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(36+84x+241x^2+236x^3+390x^4+172x^5+321x^6+20x^7+100x^8\right) \\ &= 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

fricas [A] time = 0.98, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9+\frac{5}{2}x^8+\frac{321}{7}x^7+\frac{86}{3}x^6+78x^5+59x^4+\frac{241}{3}x^3+42x^2+36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

giac [A] time = 0.20, size = 44, normalized size = 0.81

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{100}{9}x^9+\frac{5}{2}x^8+\frac{321}{7}x^7+\frac{86}{3}x^6+78x^5+59x^4+\frac{241}{3}x^3+42x^2+36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x)
```

```
[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

maxima [A] time = 0.44, size = 44, normalized size = 0.81

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{100\,x^9}{9}+\frac{5\,x^8}{2}+\frac{321\,x^7}{7}+\frac{86\,x^6}{3}+78\,x^5+59\,x^4+\frac{241\,x^3}{3}+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)
```

```
[Out] 100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x*
*3/3 + 42*x**2 + 36*x
```

3.25 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) \, dx$

Optimal. Leaf size=46

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

```
[Out] 18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) \, dx &= \int \left(18+15x+53x^2+x^3+61x^4-8x^5+20x^6\right) \, dx \\ &= 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 46, normalized size = 1.00

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

fricas [A] time = 1.11, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7-\frac{4}{3}x^6+\frac{61}{5}x^5+\frac{1}{4}x^4+\frac{53}{3}x^3+\frac{15}{2}x^2+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

giac [A] time = 0.19, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^{7} - \frac{4}{3}\,x^{6} + \frac{61}{5}\,x^{5} + \frac{1}{4}\,x^{4} + \frac{53}{3}\,x^{3} + \frac{15}{2}\,x^{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.76

$$\frac{20}{7}x^{7} - \frac{4}{3}x^{6} + \frac{61}{5}x^{5} + \frac{1}{4}x^{4} + \frac{53}{3}x^{3} + \frac{15}{2}x^{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2),x)
```

```
[Out] 18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

maxima [A] time = 0.44, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^{7} - \frac{4}{3}\,x^{6} + \frac{61}{5}\,x^{5} + \frac{1}{4}\,x^{4} + \frac{53}{3}\,x^{3} + \frac{15}{2}\,x^{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

mupad [B] time = 0.03, size = 34, normalized size = 0.74

$$\frac{20\,x^7}{7} - \frac{4\,x^6}{3} + \frac{61\,x^5}{5} + \frac{x^4}{4} + \frac{53\,x^3}{3} + \frac{15\,x^2}{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2),x)
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

sympy [A] time = 0.07, size = 41, normalized size = 0.89

$$\frac{20x^{7}}{7} - \frac{4x^{6}}{3} + \frac{61x^{5}}{5} + \frac{x^{4}}{4} + \frac{53x^{3}}{3} + \frac{15x^{2}}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)
```

```
[Out] 20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x
```

3.26 $\int \frac{(3-x+2x^2)^2}{2+3x+5x^2} \, dx$

Optimal. Leaf size=56

$$\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log\left(5x^2+3x+2\right)}{1250}+\frac{381x}{125}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

```
[Out] 381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*arctan(1/
31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log\left(5x^2+3x+2\right)}{1250}+\frac{381x}{125}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])
/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^2}{2+3x+5x^2}\,dx = \int \left(\frac{381}{125}-\frac{32x}{25}+\frac{4x^2}{5}+\frac{121(3-13x)}{125\left(2+3x+5x^2\right)}\right)dx$$
$$= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{121}{125}\int \frac{3-13x}{2+3x+5x^2}\,dx$$
$$= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\int \frac{3+10x}{2+3x+5x^2}\,dx}{1250}+\frac{8349\int \frac{1}{2+3x+5x^2}\,dx}{1250}$$
$$= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}-\frac{8349}{625}\operatorname{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+1\right.$$
$$= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.95

$$\frac{10x\left(100x^2-240x+1143\right)-4719\log\left(5x^2+3x+2\right)}{3750}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x + 1
00*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

fricas [A] time = 0.71, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3-\frac{16}{25}x^2+\frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{381}{125}x-\frac{1573}{1250}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

giac [A] time = 0.21, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3-\frac{16}{25}x^2+\frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{381}{125}x-\frac{1573}{1250}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{4x^3}{15}-\frac{16x^2}{25}+\frac{381x}{125}+\frac{8349\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19375}-\frac{1573\ln\left(5x^2+3x+2\right)}{1250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

```
[Out] 381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*31^(1/2)*
arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 3.45, size = 45, normalized size = 0.80

$$\frac{381\,x}{125} - \frac{1573\,\ln\left(5\,x^2+3\,x+2\right)}{1250} + \frac{8349\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{19375} - \frac{16\,x^2}{25} + \frac{4\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2),x)
```

```
[Out] (381*x)/125 - (1573*log(3*x + 5*x^2 + 2))/1250 + (8349*31^(1/2)*atan((10*31
^(1/2)*x)/31 + (3*31^(1/2))/31))/19375 - (16*x^2)/25 + (4*x^3)/15
```

sympy [A] time = 0.15, size = 63, normalized size = 1.12

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} - \frac{1573\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{1250} + \frac{8349\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] 4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*log(x**2 + 3*x/5 + 2/5)/1250 + 83
49*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19375
```

3.27 $\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{121(69x+61)}{3875\left(5x^2+3x+2\right)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}+\frac{41932\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

```
[Out] 4/25*x+121/3875*(61+69*x)/(5*x^2+3*x+2)-22/125*ln(5*x^2+3*x+2)+41932/120125
*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(69x+61)}{3875\left(5x^2+3x+2\right)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}+\frac{41932\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 +
10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^2}\,dx &= \frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{1}{31}\int \frac{\frac{4032}{25}-\frac{992x}{25}+\frac{124x^2}{5}}{2+3x+5x^2}\,dx\\
&= \frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{1}{31}\int \left(\frac{124}{25}+\frac{44(86-31x)}{25\left(2+3x+5x^2\right)}\right)dx\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{44}{775}\int \frac{86-31x}{2+3x+5x^2}\,dx\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}-\frac{22}{125}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{20966\int \frac{1}{2+3x+5x^2}\,dx}{3875}\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}-\frac{22}{125}\log\left(2+3x+5x^2\right)-\frac{41932\,\mathrm{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+\right.}{3875}\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{41932\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.94

$$\frac{\frac{3751(69x+61)}{5x^2+3x+2}-21142\log\left(5x^2+3x+2\right)+19220x+41932\sqrt{31}\,\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{120125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (19220*x + (3751*(61 + 69*x))/(2 + 3*x + 5*x^2) + 41932*Sqrt[31]*ArcTan[(3
+ 10*x)/Sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125
```

fricas [A] time = 0.87, size = 78, normalized size = 1.24

$$\frac{96100\,x^3+41932\sqrt{31}\left(5\,x^2+3\,x+2\right)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+57660\,x^2-21142\left(5\,x^2+3\,x+2\right)\log\left(5\,x^2+\right.}{120125\left(5\,x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/120125*(96100*x^3 + 41932*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)
*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 2
97259*x + 228811)/(5*x^2 + 3*x + 2)
```

giac [A] time = 0.17, size = 52, normalized size = 0.83

$$\frac{41932}{120125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\,\left(5\,x^2+3\,x+2\right)}-\frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25}+\frac{41932\sqrt{31}\;\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{120125}-\frac{22\ln\left(5x^2+3x+2\right)}{125}-\frac{11\left(-\frac{759x}{775}-\frac{671}{775}\right)}{25\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

```
[Out] 4/25*x-11/25*(-759/775*x-671/775)/(x^2+3/5*x+2/5)-22/125*ln(5*x^2+3*x+2)+41
932/120125*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 52, normalized size = 0.83

$$\frac{41932}{120125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\,\left(5\,x^2+3\,x+2\right)}-\frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$\frac{4\,x}{25}-\frac{22\,\ln\left(5\,x^2+3\,x+2\right)}{125}+\frac{\frac{8349\,x}{19375}+\frac{7381}{19375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{41932\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (4*x)/25 - (22*log(3*x + 5*x^2 + 2))/125 + ((8349*x)/19375 + 7381/19375)/((
3*x)/5 + x^2 + 2/5) + (41932*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2)
)/31))/120125
```

sympy [A] time = 0.20, size = 65, normalized size = 1.03

$$\frac{4x}{25}+\frac{8349x+7381}{19375x^2+11625x+7750}-\frac{22\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{125}+\frac{41932\sqrt{31}\;\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] 4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x/
5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/120125
```

3.28 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} \, dx$

Optimal. Leaf size=64

$$\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}+\frac{11(45710x+17557)}{240250\left(5x^2+3x+2\right)}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

```
[Out] 121/7750*(61+69*x)/(5*x^2+3*x+2)^2+11/240250*(17557+45710*x)/(5*x^2+3*x+2)+
4330/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 618, 204}

$$\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}+\frac{11(45710x+17557)}{240250\left(5x^2+3x+2\right)}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(2402
50*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^3}\,dx = \frac{121(61+69x)}{7750\left(2+3x+5x^2\right)^2} + \frac{1}{62}\int \frac{\frac{48669}{125}-\frac{1984x}{25}+\frac{248x^2}{5}}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{121(61+69x)}{7750\left(2+3x+5x^2\right)^2} + \frac{11(17557+45710x)}{240250\left(2+3x+5x^2\right)} + \frac{\int \frac{4330}{2+3x+5x^2}\,dx}{1922}$$

$$= \frac{121(61+69x)}{7750\left(2+3x+5x^2\right)^2} + \frac{11(17557+45710x)}{240250\left(2+3x+5x^2\right)} + \frac{2165}{961}\int \frac{1}{2+3x+5x^2}\,dx$$

$$= \frac{121(61+69x)}{7750\left(2+3x+5x^2\right)^2} + \frac{11(17557+45710x)}{240250\left(2+3x+5x^2\right)} - \frac{4330}{961}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx, x, 3 -$$

$$= \frac{121(61+69x)}{7750\left(2+3x+5x^2\right)^2} + \frac{11(17557+45710x)}{240250\left(2+3x+5x^2\right)} + \frac{4330\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.83

$$\frac{11\left(45710x^3+44983x^2+33524x+11183\right)}{48050\left(5x^2+3x+2\right)^2} + \frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
+ (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

fricas [A] time = 0.84, size = 75, normalized size = 1.17

$$\frac{15587110\,x^3 + 216500\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right) + 15339203\,x^2 + 114}{1489550\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/1489550*(15587110*x^3 + 216500*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
+ 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 15339203*x^2 + 11431684*x + 3813403
)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.18, size = 46, normalized size = 0.72

$$\frac{4330}{29791}\sqrt{31}\,\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right) + \frac{11\left(45710\,x^3+44983\,x^2+33524\,x+11183\right)}{48050\left(5\,x^2+3\,x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(5*x^2 + 3*x + 2)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791}+\frac{\frac{50281}{4805}x^{3}+\frac{494813}{48050}x^{2}+\frac{184382}{24025}x+\frac{123013}{48050}}{\left(5x^{2}+3x+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5*
x^2+3*x+2)^2+4330/29791*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.97, size = 56, normalized size = 0.88

$$\frac{4330}{29791}\sqrt{31}\ \arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{11\left(45710\,x^{3}+44983\,x^{2}+33524\,x+11183\right)}{48050\left(25\,x^{4}+30\,x^{3}+29\,x^{2}+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

mupad [B] time = 3.44, size = 55, normalized size = 0.86

$$\frac{4330\,\sqrt{31}\ \mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{29791}+\frac{\frac{50281\,x^{3}}{120125}+\frac{494813\,x^{2}}{1201250}+\frac{184382\,x}{600625}+\frac{123013}{1201250}}{x^{4}+\frac{6\,x^{3}}{5}+\frac{29\,x^{2}}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (4330*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((184382
*x)/600625 + (494813*x^2)/1201250 + (50281*x^3)/120125 + 123013/1201250)/((
12*x)/25 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{502810x^{3}+494813x^{2}+368764x+123013}{1201250x^{4}+1441500x^{3}+1393450x^{2}+576600x+192200}+\frac{4330\sqrt{31}\ \mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (502810*x**3 + 494813*x**2 + 368764*x + 123013)/(1201250*x**4 + 1441500*x**
3 + 1393450*x**2 + 576600*x + 192200) + 4330*sqrt(31)*atan(10*sqrt(31)*x/31
 + 3*sqrt(31)/31)/29791
```

3.29 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$

Optimal. Leaf size=85

$$\frac{16688(10x+3)}{148955\left(5x^2+3x+2\right)}+\frac{11(12060x+4579)}{120125\left(5x^2+3x+2\right)^2}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

```
[Out] 121/11625*(61+69*x)/(5*x^2+3*x+2)^3+11/120125*(4579+12060*x)/(5*x^2+3*x+2)^
2+16688/148955*(3+10*x)/(5*x^2+3*x+2)+66752/923521*arctan(1/31*(3+10*x)*31^
(1/2))*31^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1660, 12, 614, 618, 204}

$$\frac{16688(10x+3)}{148955\left(5x^2+3x+2\right)}+\frac{11(12060x+4579)}{120125\left(5x^2+3x+2\right)^2}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(1201
25*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (
66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{1}{93}\int \frac{\frac{77178}{125}-\frac{2976x}{25}+\frac{372x^2}{5}}{(2+3x+5x^2)^3}\,dx$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{\int \frac{100128}{5(2+3x+5x^2)^2}\,dx}{5766}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688\int \frac{1}{(2+3x+5x^2)^2}\,dx}{4805}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{33376\int}{29}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}-\frac{66752\,\mathrm{Su}}{}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{66752\tan}{2979}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.74

$$\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{446865(5x^2+3x+2)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 1251600
0*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(
29791*Sqrt[31])
```

fricas [A] time = 0.84, size = 105, normalized size = 1.24

$$\frac{387996000\,x^5+581994000\,x^4+666245676\,x^3+1001280\sqrt{31}\left(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\right.}{13852815\left(125\,x^6+225\,x^5+285\,x^4+207\,x^3+11\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*sqrt(31
)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arctan(1/31*
```

```
sqrt(31)*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6 + 2
25*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

giac [A] time = 0.18, size = 56, normalized size = 0.66

$$\frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+56749}{446865\left(5\,x^2+3\,x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(5
*x^2 + 3*x + 2)^3
```

maple [A] time = 0.01, size = 57, normalized size = 0.67

$$\frac{66752\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{923521}+\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{\left(5x^2+3x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x)
```

```
[Out] 125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/1861937
5*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*31^
(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 76, normalized size = 0.89

$$\frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+56749}{446865\left(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(1
25*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

mupad [B] time = 3.47, size = 75, normalized size = 0.88

$$\frac{66752\sqrt{31}\ \mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{923521}+\frac{\frac{33376\,x^5}{148955}+\frac{50064\,x^4}{148955}+\frac{7163932\,x^3}{18619375}+\frac{4260199\,x^2}{18619375}+\frac{1891636\,x}{18619375}+\frac{1259239}{55858125}}{x^6+\frac{9\,x^5}{5}+\frac{57\,x^4}{25}+\frac{207\,x^3}{125}+\frac{114\,x^2}{125}+\frac{36\,x}{125}+\frac{8}{125}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (66752*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/923521 + ((1891
636*x)/18619375 + (4260199*x^2)/18619375 + (7163932*x^3)/18619375 + (50064*
x^4)/148955 + (33376*x^5)/148955 + 1259239/55858125)/((36*x)/125 + (114*x^2
)/125 + (207*x^3)/125 + (57*x^4)/25 + (9*x^5)/5 + x^6 + 8/125)
```

sympy [A] time = 0.23, size = 83, normalized size = 0.98

$$\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{55858125x^6+100544625x^5+127356525x^4+92501055x^3+50942610x^2+16087140x+3574920}+\frac{66752\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)
```

```
[Out] (12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x
+ 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x**3
 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/923521
```

3.30 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+$$

```
[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/
8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+
2250/7*x^14+1000/3*x^15
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \int \left(432+2160x+8568x^2+20576x^3+43083x^4+64529x^5+91349x\right.$$
$$= 432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}$$

Mathematica [A] time = 0.00, size = 96, normalized size = 1.00

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

fricas [A] time = 0.78, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15}+\frac{2250}{7}x^{14}+\frac{27050}{13}x^{13}+\frac{30395}{12}x^{12}+\frac{68583}{11}x^{11}+\frac{75311}{10}x^{10}+\frac{103583}{9}x^9+\frac{94881}{8}x^8+\frac{91349}{7}x^7+\frac{64529}{6}x^6+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

giac [A] time = 0.18, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15}+\frac{2250}{7}x^{14}+\frac{27050}{13}x^{13}+\frac{30395}{12}x^{12}+\frac{68583}{11}x^{11}+\frac{75311}{10}x^{10}+\frac{103583}{9}x^{9}+\frac{94881}{8}x^{8}+\frac{91349}{7}x^{7}+\frac{64529}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

maple [A] time = 0.00, size = 75, normalized size = 0.78

$$\frac{1000}{3}x^{15}+\frac{2250}{7}x^{14}+\frac{27050}{13}x^{13}+\frac{30395}{12}x^{12}+\frac{68583}{11}x^{11}+\frac{75311}{10}x^{10}+\frac{103583}{9}x^{9}+\frac{94881}{8}x^{8}+\frac{91349}{7}x^{7}+\frac{64529}{6}x^{6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x)
```

```
[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/
8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+
2250/7*x^14+1000/3*x^15
```

maxima [A] time = 0.43, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15}+\frac{2250}{7}x^{14}+\frac{27050}{13}x^{13}+\frac{30395}{12}x^{12}+\frac{68583}{11}x^{11}+\frac{75311}{10}x^{10}+\frac{103583}{9}x^{9}+\frac{94881}{8}x^{8}+\frac{91349}{7}x^{7}+\frac{64529}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

mupad [B] time = 0.12, size = 74, normalized size = 0.77

$$\frac{1000\,x^{15}}{3}+\frac{2250\,x^{14}}{7}+\frac{27050\,x^{13}}{13}+\frac{30395\,x^{12}}{12}+\frac{68583\,x^{11}}{11}+\frac{75311\,x^{10}}{10}+\frac{103583\,x^{9}}{9}+\frac{94881\,x^{8}}{8}+\frac{91349\,x^{7}}{7}+\frac{64529\,x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

sympy [A] time = 0.10, size = 92, normalized size = 0.96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^{9}}{9}+\frac{94881x^{8}}{8}+\frac{91349x^{7}}{7}+\frac{64529x^{6}}{6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**11
/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 64529*
x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x
```

3.31 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+21$$

```
[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x
^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+21$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx &= \int \left(216+756x+2610x^2+4483x^3+8292x^4+8619x^5+12016x^6+786\right.\\ &= 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{786}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+21$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

fricas [A] time = 0.80, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^9+\frac{7869}{8}x^8+\frac{12016}{7}x^7+\frac{2873}{2}x^6+\frac{8292}{5}x^5+\frac{4483}{4}x^4+870x^3+378x^2+21$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

giac [A] time = 0.21, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^{9}+\frac{7869}{8}x^{8}+\frac{12016}{7}x^{7}+\frac{2873}{2}x^{6}+\frac{8292}{5}x^{5}+\frac{4483}{4}x^{4}+870x^{3}+37$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^{9}+\frac{7869}{8}x^{8}+\frac{12016}{7}x^{7}+\frac{2873}{2}x^{6}+\frac{8292}{5}x^{5}+\frac{4483}{4}x^{4}+870x^{3}+378x^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x)
```

```
[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x
^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

maxima [A] time = 0.43, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^{9}+\frac{7869}{8}x^{8}+\frac{12016}{7}x^{7}+\frac{2873}{2}x^{6}+\frac{8292}{5}x^{5}+\frac{4483}{4}x^{4}+870x^{3}+37$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.78

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^{9}}{3}+\frac{7869x^{8}}{8}+\frac{12016x^{7}}{7}+\frac{2873x^{6}}{2}+\frac{8292x^{5}}{5}+\frac{4483x^{4}}{4}+870x^{3}+37$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

sympy [A] time = 0.09, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^{9}}{3}+\frac{7869x^{8}}{8}+\frac{12016x^{7}}{7}+\frac{2873x^{6}}{2}+\frac{8292x^{5}}{5}+\frac{4483x^{4}}{4}+870x^{3}+378x^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

```
[Out] 1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 78
69*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*x*
*3 + 378*x**2 + 216*x
```

3.32 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=68

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

```
[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+92
2/9*x^9-6*x^10+200/11*x^11
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(108+216x+711x^2+635x^3+1416x^4+598x^5+1571x^6+83x^7+92\right. \\ &= 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+ \end{aligned}$$

Mathematica [A] time = 0.00, size = 68, normalized size = 1.00

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

fricas [A] time = 0.77, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11}-6x^{10}+\frac{922}{9}x^9+\frac{83}{8}x^8+\frac{1571}{7}x^7+\frac{299}{3}x^6+\frac{1416}{5}x^5+\frac{635}{4}x^4+237x^3+108x^2+108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

giac [A] time = 0.20, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11}-6\,x^{10}+\frac{922}{9}\,x^{9}+\frac{83}{8}\,x^{8}+\frac{1571}{7}\,x^{7}+\frac{299}{3}\,x^{6}+\frac{1416}{5}\,x^{5}+\frac{635}{4}\,x^{4}+237\,x^{3}+108\,x^{2}+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{200}{11}x^{11}-6x^{10}+\frac{922}{9}x^{9}+\frac{83}{8}x^{8}+\frac{1571}{7}x^{7}+\frac{299}{3}x^{6}+\frac{1416}{5}x^{5}+\frac{635}{4}x^{4}+237x^{3}+108x^{2}+108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x)
```

```
[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+92
2/9*x^9-6*x^10+200/11*x^11
```

maxima [A] time = 0.43, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11}-6\,x^{10}+\frac{922}{9}\,x^{9}+\frac{83}{8}\,x^{8}+\frac{1571}{7}\,x^{7}+\frac{299}{3}\,x^{6}+\frac{1416}{5}\,x^{5}+\frac{635}{4}\,x^{4}+237\,x^{3}+108\,x^{2}+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

mupad [B] time = 0.05, size = 54, normalized size = 0.79

$$\frac{200\,x^{11}}{11}-6\,x^{10}+\frac{922\,x^9}{9}+\frac{83\,x^8}{8}+\frac{1571\,x^7}{7}+\frac{299\,x^6}{3}+\frac{1416\,x^5}{5}+\frac{635\,x^4}{4}+237\,x^3+108\,x^2+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

sympy [A] time = 0.08, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^{9}}{9}+\frac{83x^{8}}{8}+\frac{1571x^{7}}{7}+\frac{299x^{6}}{3}+\frac{1416x^{5}}{5}+\frac{635x^{4}}{4}+237x^{3}+108x^{2}+108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)
```

```
[Out] 200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3
+ 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x
```

3.33 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right) dx$

Optimal. Leaf size=56

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

```
[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right) dx &= \int \left(54+27x+180x^2-20x^3+288x^4-83x^5+190x^6-36x^7+40x^8\right) dx\\ &= 54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

fricas [A] time = 0.83, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9-\frac{9}{2}x^8+\frac{190}{7}x^7-\frac{83}{6}x^6+\frac{288}{5}x^5-5x^4+60x^3+\frac{27}{2}x^2+54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

giac [A] time = 0.20, size = 44, normalized size = 0.79

$$\frac{40}{9}\,x^{9} - \frac{9}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{83}{6}\,x^{6} + \frac{288}{5}\,x^{5} - 5\,x^{4} + 60\,x^{3} + \frac{27}{2}\,x^{2} + 54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{40}{9}x^{9} - \frac{9}{2}x^{8} + \frac{190}{7}x^{7} - \frac{83}{6}x^{6} + \frac{288}{5}x^{5} - 5x^{4} + 60x^{3} + \frac{27}{2}x^{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2),x)
```

```
[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

maxima [A] time = 0.43, size = 44, normalized size = 0.79

$$\frac{40}{9}\,x^{9} - \frac{9}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{83}{6}\,x^{6} + \frac{288}{5}\,x^{5} - 5\,x^{4} + 60\,x^{3} + \frac{27}{2}\,x^{2} + 54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.79

$$\frac{40\,x^{9}}{9} - \frac{9\,x^{8}}{2} + \frac{190\,x^{7}}{7} - \frac{83\,x^{6}}{6} + \frac{288\,x^{5}}{5} - 5\,x^{4} + 60\,x^{3} + \frac{27\,x^{2}}{2} + 54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2),x)
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{40x^{9}}{9} - \frac{9x^{8}}{2} + \frac{190x^{7}}{7} - \frac{83x^{6}}{6} + \frac{288x^{5}}{5} - 5x^{4} + 60x^{3} + \frac{27x^{2}}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)
```

```
[Out] 40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x*
*3 + 27*x**2/2 + 54*x
```

3.34 $\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx$

Optimal. Leaf size=70

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}+\frac{328757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

```
[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(
5*x^2+3*x+2)+328757/484375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}+\frac{328757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/2
5 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 +
 3*x + 5*x^2])/31250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^3}{2+3x+5x^2}\,dx = \int \left(\frac{49508}{3125}-\frac{7451x}{625}+\frac{1222x^2}{125}-\frac{84x^3}{25}+\frac{8x^4}{5}-\frac{1331(11+119x)}{3125\left(2+3x+5x^2\right)}\right)dx$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{1331\int\frac{11+119x}{2+3x+5x^2}\,dx}{3125}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\int\frac{3+10x}{2+3x+5x^2}\,dx}{31250}+\frac{328757\int\frac{1}{2+3x+5x^2}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\log\left(2+3x+5x^2\right)}{31250}-\frac{328757\,\mathrm{Su}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}}-\frac{158389\log\left(2+\right.}{3125}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.90

$$\frac{31\left(5x\left(6000x^4-15750x^3+61100x^2-111765x+297048\right)-475167\log\left(5x^2+3x+2\right)\right)+1972542\sqrt{31}\,\tan}{2906250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x
+ 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/2906250
```

fricas [A] time = 0.87, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

giac [A] time = 0.21, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

maple [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}+\frac{49508x}{3125}+\frac{328757\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{484375}-\frac{158389\ln\left(5x^2+3x+2\right)}{31250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

```
[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(
5*x^2+3*x+2)+328757/484375*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2 + 3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.79

$$\frac{49508\,x}{3125} - \frac{158389\,\ln\left(5\,x^2+3\,x+2\right)}{31250} + \frac{328757\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{484375} - \frac{7451\,x^2}{1250} + \frac{1222\,x^3}{375} - \frac{21\,x^4}{25} + \frac{8\,x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2),x)
```

```
[Out] (49508*x)/3125 - (158389*log(3*x + 5*x^2 + 2))/31250 + (328757*31^(1/2)*ata
n((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/484375 - (7451*x^2)/1250 + (1222*x
^3)/375 - (21*x^4)/25 + (8*x^5)/25
```

sympy [A] time = 0.16, size = 76, normalized size = 1.09

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] 8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 15
8389*log(x**2 + 3*x/5 + 2/5)/31250 + 328757*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/484375
```

3.35 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} \, dx$

Optimal. Leaf size=77

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875\left(5x^2+3x+2\right)}-\frac{10769\log\left(5x^2+3x+2\right)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

```
[Out] 1466/625*x-54/125*x^2+8/75*x^3+1331/96875*(443+247*x)/(5*x^2+3*x+2)-10769/6
250*ln(5*x^2+3*x+2)+3819607/3003125*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875\left(5x^2+3x+2\right)}-\frac{10769\log\left(5x^2+3x+2\right)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx &= \frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{1}{31}\int \frac{\frac{372701}{625}-\frac{230981x}{625}+\frac{37882x^2}{125}-\frac{2604x^3}{25}+\frac{248x^4}{5}}{2+3x+5x^2}\,dx\\ &= \frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{1}{31}\int \left(\frac{45446}{625}-\frac{3348x}{125}+\frac{248x^2}{25}+\frac{121(2329-2759x)}{625(2+3x+5x^2)}\right)dx\\ &= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{121\int \frac{2329-2759x}{2+3x+5x^2}\,dx}{19375}\\ &= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}-\frac{10769\int \frac{3+10x}{2+3x+5x^2}\,dx}{6250}+\frac{3819607\int \frac{1}{2+3x\cdots}}{193750}\\ &= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}-\frac{10769\log(2+3x+5x^2)}{6250}-\frac{3819607\,\mathrm{S}\cdots}{\cdots}\\ &= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{3819607\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log(2\cdots}{62\cdots}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.00

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}-\frac{10769\log(5x^2+3x+2)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

fricas [A] time = 0.85, size = 88, normalized size = 1.14

$$\frac{9610000\,x^5-33154500\,x^4+191815600\,x^3+22917642\,\sqrt{31}\,(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+111226\cdots}{18018750\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/18018750*(9610000*x^5 - 33154500*x^4 + 191815600*x^3 + 22917642*sqrt(31)*
(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 111226140*x^2 - 310470
```

```
27*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 145678362*x + 109671738)/(5*x^2
 + 3*x + 2)
```

giac [A] time = 0.21, size = 62, normalized size = 0.81

$$\frac{8}{75}\,x^3-\frac{54}{125}\,x^2+\frac{3819607}{3003125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\left(5\,x^2+3\,x+2\right)}-\frac{10769}{6250}\,\log\left(5\,x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}+\frac{3819607\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{3003125}-\frac{10769\ln\left(5x^2+3x+2\right)}{6250}-\frac{121\left(-\frac{2717x}{775}-\frac{4873}{775}\right)}{625\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

```
[Out] 8/75*x^3-54/125*x^2+1466/625*x-121/625*(-2717/775*x-4873/775)/(x^2+3/5*x+2/
5)-10769/6250*ln(5*x^2+3*x+2)+3819607/3003125*31^(1/2)*arctan(1/31*(10*x+3)
*31^(1/2))
```

maxima [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{8}{75}\,x^3-\frac{54}{125}\,x^2+\frac{3819607}{3003125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\left(5\,x^2+3\,x+2\right)}-\frac{10769}{6250}\,\log\left(5\,x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 3.43, size = 61, normalized size = 0.79

$$\frac{1466\,x}{625}-\frac{10769\,\ln\left(5\,x^2+3\,x+2\right)}{6250}+\frac{\frac{328757\,x}{484375}+\frac{589633}{484375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{3819607\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{3003125}-\frac{54\,x^2}{125}+\frac{8\,x^3}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (1466*x)/625 - (10769*log(3*x + 5*x^2 + 2))/6250 + ((328757*x)/484375 + 589
633/484375)/((3*x)/5 + x^2 + 2/5) + (3819607*31^(1/2)*atan((10*31^(1/2)*x)/
31 + (3*31^(1/2))/31))/3003125 - (54*x^2)/125 + (8*x^3)/75
```

sympy [A] time = 0.19, size = 78, normalized size = 1.01

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}+\frac{328757x+589633}{484375x^2+290625x+193750}-\frac{10769\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{6250}+\frac{3819607\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\right.}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] 8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 + 2
90625*x + 193750) - 10769*log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*sqrt(31)*a
tan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3003125
```

3.36 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=84

$$\frac{121(342840x+188381)}{6006250\left(5x^2+3x+2\right)}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}+\frac{11341176\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

```
[Out] 8/125*x+1331/193750*(443+247*x)/(5*x^2+3*x+2)^2+121/6006250*(188381+342840*
x)/(5*x^2+3*x+2)-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(3+10
*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(342840x+188381)}{6006250\left(5x^2+3x+2\right)}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}+\frac{11341176\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(18838
1 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sq
rt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{1}{62}\int \frac{\frac{4055767}{3125} - \frac{461962x}{625} + \frac{75764x^2}{125} - \frac{5208x^3}{25} + \frac{496x^4}{5}}{(2+3x+5x^2)^2}\,dx$$

$$= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \frac{\frac{2222876}{125} - \frac{207576x}{125} + \frac{15376x^2}{25}}{2+3x+5x^2}\,dx}{1922}$$

$$= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \left(\frac{15376}{125} + \frac{132(16607-1922x)}{125(2+3x+5x^2)}\right)dx}{1922}$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{66\int \frac{16607-1922x}{2+3x+5x^2}\,dx}{120125}$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625}\int \frac{3+10x}{2+3x+5x^2}\,dx + \cdots$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625}\log\left(2+3x+5x^2\right) - \cdots$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{11341176\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}} - \cdots$$

Mathematica [**A**] time = 0.04, size = 78, normalized size = 0.93

$$\frac{\frac{3751(342840x+188381)}{5x^2+3x+2} + \frac{1279091(247x+443)}{(5x^2+3x+2)^2} - 19662060\log\left(5x^2+3x+2\right) + 11916400x + 113411760\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{186193750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381 +
 342840*x))/(2 + 3*x + 5*x^2) + 113411760*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[3
1]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750
```

fricas [A] time = 0.88, size = 118, normalized size = 1.40

$$\frac{59582000\,x^5+71498400\,x^4+1355107960\,x^3+22682352\,\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\right.}{37238750\,(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(31
)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) +
1506812195*x^2 - 3932412*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 +
3*x + 2) + 1011087630*x + 395974315)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.19, size = 62, normalized size = 0.74

$$\frac{11341176}{18619375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{8}{125}\,x+\frac{121\,\left(68568\,x^3+78817\,x^2+53402\,x+21113\right)}{240250\,\left(5\,x^2+3\,x+2\right)^2}-\frac{66}{625}\,\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 66/
625*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{8x}{125}+\frac{11341176\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{18619375}-\frac{66\ln\left(5x^2+3x+2\right)}{625}-\frac{11\left(-\frac{377124}{24025}x^3-\frac{866987}{48050}x^2-\frac{293711}{24025}x-\frac{232243}{48050}\right)}{5\left(5x^2+3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

```
[Out] 8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/4805
0)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*31^(1/2)*arctan
(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.97, size = 72, normalized size = 0.86

$$\frac{11341176}{18619375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{8}{125}\,x+\frac{121\,\left(68568\,x^3+78817\,x^2+53402\,x+21113\right)}{240250\,\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}-\frac{66}{625}\,\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 3.43, size = 71, normalized size = 0.85

$$\frac{8\,x}{125}-\frac{66\,\ln\left(5\,x^2+3\,x+2\right)}{625}+\frac{11341176\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{18619375}+\frac{\frac{4148364\,x^3}{3003125}+\frac{9536857\,x^2}{6006250}+\frac{3230821\,x}{3003125}+\frac{255467}{600625}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (8*x)/125 - (66*log(3*x + 5*x^2 + 2))/625 + (11341176*31^(1/2)*atan((10*31^
(1/2)*x)/31 + (3*31^(1/2))/31))/18619375 + ((3230821*x)/3003125 + (9536857*
x^2)/6006250 + (4148364*x^3)/3003125 + 2554673/6006250)/((12*x)/25 + (29*x^
2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.23, size = 85, normalized size = 1.01

$$\frac{8x}{125}+\frac{8296728x^3+9536857x^2+6461642x+2554673}{6006250x^4+7207500x^3+6967250x^2+2883000x+961000}-\frac{66\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{625}+\frac{11341176\sqrt{31}\operatorname{atan}\left(\right.}{186193}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] 8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**4
 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/5
+ 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18619
375
```

3.37 $\int \frac{\left(2+3x+5x^2\right)^4}{3-x+2x^2}\,dx$

Optimal. Leaf size=84

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}+\frac{1156639\tan^{-1}}{256\sqrt{23}}$$

```
[Out] 122691/128*x-28747/128*x^2-21229/96*x^3+6245/64*x^4+1855/8*x^5+3625/24*x^6+
625/14*x^7+307461/512*ln(2*x^2-x+3)+1156639/5888*arctan(1/23*(1-4*x)*23^(1/
2))*23^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}+\frac{1156639\tan^{-1}}{256\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x
^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{3-x+2x^2}\,dx = \int \left(\frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641(25}{128\,(3-x}\right.$$
$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128}\int \frac{2}{3}$$
$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}\int \frac{}{3}$$
$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}\log$$
$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639\tan}{256\sqrt{}}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.86

$$\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{x\left(120000x^6+406000x^5+623280x^4+262290x^3-594412x^2-603687x+2576511\right)}{2688}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5
+ 120000*x^6))/2688 - (1156639*ArcTan[(-1 + 4*x)/Sqrt[23]])/(256*Sqrt[23])
+ (307461*Log[3 - x + 2*x^2])/512
```

fricas [A] time = 0.80, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7+\frac{3625}{24}x^6+\frac{1855}{8}x^5+\frac{6245}{64}x^4-\frac{21229}{96}x^3-\frac{28747}{128}x^2-\frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{122691}{128}x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

giac [A] time = 0.21, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7+\frac{3625}{24}x^6+\frac{1855}{8}x^5+\frac{6245}{64}x^4-\frac{21229}{96}x^3-\frac{28747}{128}x^2-\frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{122691}{128}x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 64, normalized size = 0.76

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}-\frac{1156639\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{5888}+\frac{307461\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3),x)
```

```
[Out] 625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+12
2691/128*x+307461/512*ln(2*x^2-x+3)-1156639/5888*23^(1/2)*arctan(1/23*(4*x-
1)*23^(1/2))
```

maxima [A] time = 0.98, size = 63, normalized size = 0.75

$$\frac{625}{14}x^7+\frac{3625}{24}x^6+\frac{1855}{8}x^5+\frac{6245}{64}x^4-\frac{21229}{96}x^3-\frac{28747}{128}x^2-\frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{122691}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

mupad [B] time = 3.44, size = 65, normalized size = 0.77

$$\frac{122691\,x}{128}+\frac{307461\,\ln\left(2\,x^2-x+3\right)}{512}-\frac{1156639\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{5888}-\frac{28747\,x^2}{128}-\frac{21229\,x^3}{96}+\frac{6245\,x^4}{64}+\frac{185}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3),x)
```

```
[Out] (122691*x)/128 + (307461*log(2*x^2 - x + 3))/512 - (1156639*23^(1/2)*atan((
4*23^(1/2)*x)/23 - 23^(1/2)/23))/5888 - (28747*x^2)/128 - (21229*x^3)/96 +
(6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14
```

sympy [A] time = 0.17, size = 87, normalized size = 1.04

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{512}-\frac{1156639\sqrt{23}\,\mathrm{at}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)
```

```
[Out] 625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 - 2
8747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639*s
qrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888
```

3.38 $\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} \, dx$

Optimal. Leaf size=70

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

```
[Out] -4795/32*x-829/32*x^2+965/24*x^3+575/16*x^4+25/2*x^5+1331/128*ln(2*x^2-x+3)
-59895/1472*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (5
9895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/
128
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{3-x+2x^2}\,dx = \int \left(-\frac{4795}{32}-\frac{829x}{16}+\frac{965x^2}{8}+\frac{575x^3}{4}+\frac{125x^4}{2}+\frac{1331(11+x)}{32\left(3-x+2x^2\right)}\right)dx$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{32}\int \frac{11+x}{3-x+2x^2}\,dx$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\int \frac{-1+4x}{3-x+2x^2}\,dx+\frac{59895}{128}\int \frac{}{3}$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\log\left(3-x+2x^2\right)-\frac{59895}{64}\text{Subs}$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1331}{128}\log\left(3-x\right.$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.90

$$\frac{1}{384}\left(3993\log\left(2x^2-x+3\right)+4x\left(1200x^4+3450x^3+3860x^2-2487x-14385\right)\right)+\frac{59895\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (59895*ArcTan[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (4*x*(-14385 - 2487*x +
 3860*x^2 + 3450*x^3 + 1200*x^4) + 3993*Log[3 - x + 2*x^2])/384
```

fricas [A] time = 1.02, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

giac [A] time = 0.21, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

maple [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472}+\frac{1331\ln\left(2x^2-x+3\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3),x)
```

```
[Out] 25/2*x^5+575/16*x^4+965/24*x^3-829/32*x^2-4795/32*x+1331/128*ln(2*x^2-x+3)+
59895/1472*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 53, normalized size = 0.76

$$\frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.79

$$\frac{1331\ln\left(2x^2-x+3\right)}{128}-\frac{4795x}{32}+\frac{59895\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3),x)
```

```
[Out] (1331*log(2*x^2 - x + 3))/128 - (4795*x)/32 + (59895*23^(1/2)*atan((4*23^(1
/2)*x)/23 - 23^(1/2)/23))/1472 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16
 + (25*x^5)/2
```

sympy [A] time = 0.15, size = 73, normalized size = 1.04

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{59895\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)
```

```
[Out] 25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*log(
x**2 - x/2 + 3/2)/128 + 59895*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
1472
```

3.39 $\int \frac{(2+3x+5x^2)^2}{3-x+2x^2} \, dx$

Optimal. Leaf size=56

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32}\log\left(2x^2 - x + 3\right) + \frac{51x}{8} + \frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

```
[Out] 51/8*x+85/8*x^2+25/6*x^3-363/32*ln(2*x^2-x+3)+847/368*arctan(1/23*(1-4*x)*2
3^(1/2))*23^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32}\log\left(2x^2 - x + 3\right) + \frac{51x}{8} + \frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*S
qrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule steps

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \int \left(\frac{51}{8}+\frac{85x}{4}+\frac{25x^2}{2}-\frac{121(1+3x)}{8(3-x+2x^2)}\right)dx$$
$$= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{121}{8}\int \frac{1+3x}{3-x+2x^2}\,dx$$
$$= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{847}{32}\int \frac{1}{3-x+2x^2}\,dx$$
$$= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\log\left(3-x+2x^2\right)+\frac{847}{16}\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx, x, -1+4x\right)$$
$$= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}-\frac{363}{32}\log\left(3-x+2x^2\right)$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.93

$$\frac{1}{24}x\left(100x^2+255x+153\right)-\frac{363}{32}\log\left(2x^2-x+3\right)-\frac{847\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt
[23]) - (363*Log[3 - x + 2*x^2])/32
```

fricas [A] time = 0.85, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3+\frac{85}{8}x^2-\frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{51}{8}x-\frac{363}{32}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

giac [A] time = 0.18, size = 43, normalized size = 0.77

$$\frac{25}{6}x^3+\frac{85}{8}x^2-\frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{51}{8}x-\frac{363}{32}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

maple [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{847\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368}-\frac{363\ln\left(2x^2-x+3\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3),x)
```

```
[Out] 25/6*x^3+85/8*x^2+51/8*x-363/32*ln(2*x^2-x+3)-847/368*23^(1/2)*arctan(1/23*
(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

mupad [B] time = 3.44, size = 45, normalized size = 0.80

$$\frac{51\,x}{8}-\frac{363\,\ln\left(2\,x^2-x+3\right)}{32}-\frac{847\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368}+\frac{85\,x^2}{8}+\frac{25\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3),x)
```

```
[Out] (51*x)/8 - (363*log(2*x^2 - x + 3))/32 - (847*23^(1/2)*atan((4*23^(1/2)*x)/
23 - 23^(1/2)/23))/368 + (85*x^2)/8 + (25*x^3)/6
```

sympy [A] time = 0.15, size = 60, normalized size = 1.07

$$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{847\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)
```

```
[Out] 25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*log(x**2 - x/2 + 3/2)/32 - 847*sqrt(23
)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/368
```

3.40 $\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx$

Optimal. Leaf size=42

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

```
[Out] 5/2*x+11/8*ln(2*x^2-x+3)+33/92*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*
x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx &= \int \left(\frac{5}{2}-\frac{11(1-x)}{2\left(3-x+2x^2\right)}\right)dx\\ &= \frac{5x}{2}-\frac{11}{2}\int \frac{1-x}{3-x+2x^2}\,dx\\ &= \frac{5x}{2}+\frac{11}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{33}{8}\int \frac{1}{3-x+2x^2}\,dx\\ &= \frac{5x}{2}+\frac{11}{8}\log\left(3-x+2x^2\right)+\frac{33}{4}\operatorname{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\ &= \frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}-\frac{33\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2
*x^2])/8
```

fricas [A] time = 0.86, size = 33, normalized size = 0.79

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

giac [A] time = 0.21, size = 33, normalized size = 0.79

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

maple [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{5x}{2}-\frac{33\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92}+\frac{11\ln\left(2x^2-x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3),x)
```

```
[Out] 5/2*x+11/8*ln(2*x^2-x+3)-33/92*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.95, size = 33, normalized size = 0.79

$$-\frac{33}{92}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.83

$$\frac{5\,x}{2}+\frac{11\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{33\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3),x)
```

```
[Out] (5*x)/2 + (11*log(2*x^2 - x + 3))/8 - (33*23^(1/2)*atan((4*23^(1/2)*x)/23 -
 23^(1/2)/23))/92
```

sympy [A] time = 0.14, size = 46, normalized size = 1.10

$$\frac{5x}{2}+\frac{11\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{33\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)
```

```
[Out] 5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sqr
t(23)/23)/92
```

3.41 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)+\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

```
[Out] -1/44*ln(2*x^2-x+3)+1/44*ln(5*x^2+3*x+2)+3/506*arctan(1/23*(1-4*x)*23^(1/2)
)*23^(1/2)+13/682*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {980, 634, 618, 204, 628}

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)+\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 980

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2
)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*
f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^2
*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*e^2 - c*d*f - b
*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0
```

]

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx &= \frac{1}{242}\int \frac{-11-22x}{3-x+2x^2}\,dx + \frac{1}{242}\int \frac{88+55x}{2+3x+5x^2}\,dx \\ &= -\left(\frac{1}{44}\int \frac{-1+4x}{3-x+2x^2}\,dx\right) + \frac{1}{44}\int \frac{3+10x}{2+3x+5x^2}\,dx - \frac{3}{44}\int \frac{1}{3-x+2x^2}\,dx \\ &= -\frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right) + \frac{3}{22}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,d \\ &= \frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.00

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{3\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt
[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

fricas [A] time = 0.81, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1}{44}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{44}\log\left(2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

giac [A] time = 0.19, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1}{44}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{44}\log\left(2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

maple [A] time = 0.01, size = 60, normalized size = 0.82

$$\frac{13\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{682}-\frac{3\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506}-\frac{\ln\left(2x^2-x+3\right)}{44}+\frac{\ln\left(5x^2+3x+2\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x)
```

```
[Out] 1/44*ln(5*x^2+3*x+2)+13/682*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))-1/44*ln
(2*x^2-x+3)-3/506*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1}{44}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{44}\log\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

mupad [B] time = 0.19, size = 79, normalized size = 1.08

$$\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{44}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 - 1/44) - log(x + (23^(1
/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 + 1/44) - log(x - (31^(1/2)*1i)/10 + 3
/10)*((31^(1/2)*13i)/1364 - 1/44) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(
1/2)*13i)/1364 + 1/44)
```

sympy [A] time = 0.24, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{44}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{44}-\frac{3\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{506}+\frac{13\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] -log(x**2 - x/2 + 3/2)/44 + log(x**2 + 3*x/5 + 2/5)/44 - 3*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/506 + 13*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sq
rt(31)/31)/682
```

3.42 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=94

$$\frac{65x+4}{682\left(5x^2+3x+2\right)}+\frac{3}{968}\log\left(2x^2-x+3\right)-\frac{3}{968}\log\left(5x^2+3x+2\right)+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

```
[Out] 1/682*(4+65*x)/(5*x^2+3*x+2)+3/968*ln(2*x^2-x+3)-3/968*ln(5*x^2+3*x+2)+7/11
132*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+2891/465124*arctan(1/31*(3+10*x)
*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{682\left(5x^2+3x+2\right)}+\frac{3}{968}\log\left(2x^2-x+3\right)-\frac{3}{968}\log\left(5x^2+3x+2\right)+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sq
rt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
```

```
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx &= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{-1804+1397x-1430x^2}{(3-x+2x^2)(2+3x+5x^2)}\,dx}{7502} \\
&= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{18755-22506x}{3-x+2x^2}\,dx}{1815484} - \frac{\int \frac{-158026+56265x}{2+3x+5x^2}\,dx}{1815484} \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx - \frac{3}{968}\int \frac{3+10x}{2+3x+5x^2}\,dx \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968}\log(3-x+2x^2) - \frac{3}{968}\log(2+3x+5x^2) \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log(
\end{aligned}$$

Mathematica [A] time = 0.08, size = 94, normalized size = 1.00

$$\frac{65x+4}{682(5x^2+3x+2)}+\frac{3}{968}\log(2x^2-x+3)-\frac{3}{968}\log(5x^2+3x+2)-\frac{7\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) - (7*ArcTan[(-1 + 4*x)/Sqrt[23]])/(484*S
qrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
 x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

fricas [A] time = 0.96, size = 117, normalized size = 1.24

$$\frac{132986\sqrt{31}\left(5x^2+3x+2\right)\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)-13454\sqrt{23}\left(5x^2+3x+2\right)\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-}{21395704\left(5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/21395704*(132986*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x +
3)) - 13454*sqrt(23)*(5*x^2 + 3*x + 2)*arctan(1/23*sqrt(23)*(4*x - 1)) - 66
309*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*log(2*
x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)
```

giac [A] time = 0.20, size = 78, normalized size = 0.83

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)-\frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)+\frac{65x+4}{682\left(5x^2+3x+2\right)}-\frac{3}{968}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{2891\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{465124}-\frac{7\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132}+\frac{3\ln\left(2x^2-x+3\right)}{968}-\frac{3\ln\left(5x^2+3x+2\right)}{968}-\frac{-\frac{286x}{31}-}{484\left(x^2+\frac{3}{5}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/484*(-286/31*x-88/155)/(x^2+3/5*x+2/5)-3/968*ln(5*x^2+3*x+2)+2891/465124
*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))+3/968*ln(2*x^2-x+3)-7/11132*23^(1/
2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 78, normalized size = 0.83

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)-\frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)+\frac{65x+4}{682\left(5x^2+3x+2\right)}-\frac{3}{968}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

mupad [B] time = 3.57, size = 95, normalized size = 1.01

$$\frac{\frac{13x}{682}+\frac{2}{1705}}{x^2+\frac{3x}{5}+\frac{2}{5}}+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] ((13*x)/682 + 2/1705)/((3*x)/5 + x^2 + 2/5) + log(x - (23^(1/2)*1i)/4 - 1/4
)*((23^(1/2)*7i)/22264 + 3/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)
*7i)/22264 - 3/968) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/93
0248 + 3/968) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/930248 -
 3/968)
```

sympy [**A**] time = 0.32, size = 102, normalized size = 1.09

$$\frac{65x+4}{3410x^2+2046x+1364}+\frac{3\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}-\frac{3\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{7\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{11132}+\frac{2891\sqrt{31}\,\mathrm{ata}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*lo
g(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124
```

3.43 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$

Optimal. Leaf size=115

$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}+\frac{21605x+7923}{465124\left(5x^2+3x+2\right)}-\frac{\log\left(2x^2-x+3\right)}{21296}+\frac{\log\left(5x^2+3x+2\right)}{21296}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793}{102\ldots}$$

```
[Out] 1/1364*(4+65*x)/(5*x^2+3*x+2)^2+1/465124*(7923+21605*x)/(5*x^2+3*x+2)-1/212
96*ln(2*x^2-x+3)+1/21296*ln(5*x^2+3*x+2)-45/244904*arctan(1/23*(1-4*x)*23^(
1/2))*23^(1/2)+847793/317214568*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}+\frac{21605x+7923}{465124\left(5x^2+3x+2\right)}-\frac{\log\left(2x^2-x+3\right)}{21296}+\frac{\log\left(5x^2+3x+2\right)}{21296}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793}{102\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x +
 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTa
n[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Lo
g[2 + 3*x + 5*x^2]/21296
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
```

```
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\left(2+3x+5x^2\right)^2} + \frac{7923+21605x}{465124\left(2+3x+5x^2\right)} - \frac{\int \frac{-14522420+3833038x-104\ldots}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}}{112560008}$$

$$= \frac{4+65x}{1364\left(2+3x+5x^2\right)^2} + \frac{7923+21605x}{465124\left(2+3x+5x^2\right)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2}\,dx}{27239521936}$$

$$= \frac{4+65x}{1364\left(2+3x+5x^2\right)^2} + \frac{7923+21605x}{465124\left(2+3x+5x^2\right)} - \frac{\int \frac{-1+4x}{3-x+2x^2}\,dx}{21296} + \frac{\int \frac{3+10\ldots}{2+3x+\ldots}}{2129\ldots}$$

$$= \frac{4+65x}{1364\left(2+3x+5x^2\right)^2} + \frac{7923+21605x}{465124\left(2+3x+5x^2\right)} - \frac{\log\left(3-x+2x^2\right)}{21296} + \log\ldots$$

$$= \frac{4+65x}{1364\left(2+3x+5x^2\right)^2} + \frac{7923+21605x}{465124\left(2+3x+5x^2\right)} - \frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{8477\ldots}{1\ldots}$$

Mathematica [**A**] time = 0.16, size = 104, normalized size = 0.90

$$\frac{31\left(-961\log\left(2x^2-x+3\right)+961\log\left(5x^2+3x+2\right)+\frac{44\left(108025x^3+104430x^2+89144x+17210\right)}{\left(5x^2+3x+2\right)^2}\right)+1695586\sqrt{31}\tan^{-1}\left(\ldots\right.}{634429136}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (45*ArcTan[(-1 + 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcTa
n[(3 + 10*x)/Sqrt[31]] + 31*((44*(17210 + 89144*x + 104430*x^2 + 108025*x^3
))/(2 + 3*x + 5*x^2)^2 - 961*Log[3 - x + 2*x^2] + 961*Log[2 + 3*x + 5*x^2])
)/634429136
```

fricas [**A**] time = 1.02, size = 177, normalized size = 1.54

$$\frac{3388960300\,x^3+38998478\,\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\sqrt{31}\left(10\,x+3\right)\right)+2681190\,\sqrt{23}\left(2\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/14591870128*(3388960300*x^3 + 38998478*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2681190*sqrt(23)*(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3276177960*x^
2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) - 685
193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(2*x^2 - x + 3) + 2796625568*x
 + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [**A**] time = 0.18, size = 88, normalized size = 0.77

$$\frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10\,x+3\right)\right)+\frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\left(4\,x-1\right)\right)+\frac{108025\,x^3+104430\,x^2+\ldots}{465124\left(5\,x^2+3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/212
96*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{847793\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{317214568}+\frac{45\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{244904}-\frac{\ln\left(2x^2-x+3\right)}{21296}+\frac{\ln\left(5x^2+3x+2\right)}{21296}+\frac{\frac{108025}{465124}x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25/10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2+
3*x+2)^2+1/21296*ln(5*x^2+3*x+2)+847793/317214568*31^(1/2)*arctan(1/31*(10*
x+3)*31^(1/2))-1/21296*ln(2*x^2-x+3)+45/244904*23^(1/2)*arctan(1/23*(4*x-1)
*23^(1/2))
```

maxima [A] time = 0.96, size = 98, normalized size = 0.85

$$\frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{108025\,x^3+104430\,x^2}{465124\,(25\,x^4+30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*log(5*x^2
+ 3*x + 2) - 1/21296*log(2*x^2 - x + 3)
```

mupad [B] time = 0.18, size = 115, normalized size = 1.00

$$\frac{\frac{4321\,x^3}{465124}+\frac{10443\,x^2}{1162810}+\frac{2026\,x}{264275}+\frac{1721}{1162810}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{212}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 - 1/21296) - log(x -
(23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 + 1/21296) - log(x - (31^(1/2
)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 - 1/21296) + log(x + (31^(1/
2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 + 1/21296) + ((2026*x)/2642
75 + (10443*x^2)/1162810 + (4321*x^3)/465124 + 1721/1162810)/((12*x)/25 + (
29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.36, size = 119, normalized size = 1.03

$$\frac{108025x^3+104430x^2+89144x+17210}{11628100x^4+13953720x^3+13488596x^2+5581488x+1860496}-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296}+45\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x**
3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 + lo
g(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/31721456
8
```

3.44 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=91

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944(2x^2-x+3)}-\frac{30613}{128}\log(2x^2-x+3)-\frac{89359x}{64}-\frac{13292697\tan^{-1}(}{1472\sqrt{23}}$$

```
[Out] -89359/64*x-1185/8*x^2+9775/48*x^3+2125/16*x^4+125/4*x^5-14641/2944*(101+79
*x)/(2*x^2-x+3)-30613/128*ln(2*x^2-x+3)-13292697/33856*arctan(1/23*(1-4*x)*
23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944(2x^2-x+3)}-\frac{30613}{128}\log(2x^2-x+3)-\frac{89359x}{64}-\frac{13292697\tan^{-1}(}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/
Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^2}\,dx = -\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{\frac{832627}{64}-\frac{661181x}{64}-\frac{488267x^2}{32}+\frac{143635x^3}{16}+\frac{213325x^4}{8}+\frac{83375x^5}{4}}{3-x+2x^2}$$

$$= -\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{1}{23}\int\left(-\frac{2055257}{64}-\frac{27255x}{4}+\frac{224825x^2}{16}+\frac{48875x^3}{4}+\frac{14375x}{4}\right.$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{1331}{736}\int \frac{2629}{3-}$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}-\frac{30613}{128}\int \frac{-}{3-}$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}-\frac{30613}{128}\log\left(3\right.$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}-\frac{13292697\tan}{1472\sqrt{}}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 1.00

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944\left(2x^2-x+3\right)}-\frac{30613}{128}\log\left(2x^2-x+3\right)-\frac{89359x}{64}+\frac{13292697\tan^{-1}\left(\frac{4x-}{\sqrt{23}}\right.}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*x)
/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

fricas [A] time = 1.17, size = 98, normalized size = 1.08

$$\frac{12696000\,x^7+47610000\,x^6+74800600\,x^5-20609840\,x^4-413058012\,x^3+79756182\sqrt{23}\left(2\,x^2-x+3\right)\arctan}{203136\left(2\,x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 41305
8012*x^3 + 79756182*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)
```

```
) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*log(2*x^2 - x + 3) - 930684489
*x - 102033129)/(2*x^2 - x + 3)
```

giac [A] time = 0.18, size = 72, normalized size = 0.79

$$\frac{125}{4}\,x^5+\frac{2125}{16}\,x^4+\frac{9775}{48}\,x^3-\frac{1185}{8}\,x^2+\frac{13292697}{33856}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{89359}{64}\,x-\frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 71, normalized size = 0.78

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{13292697\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{33856}-\frac{30613\ln\left(2x^2-x+3\right)}{128}-\frac{133}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x)
```

```
[Out] 125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x-1331/64*(869/92*x+1
111/92)/(x^2-1/2*x+3/2)-30613/128*ln(2*x^2-x+3)+13292697/33856*23^(1/2)*arc
tan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 72, normalized size = 0.79

$$\frac{125}{4}\,x^5+\frac{2125}{16}\,x^4+\frac{9775}{48}\,x^3-\frac{1185}{8}\,x^2+\frac{13292697}{33856}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{89359}{64}\,x-\frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

mupad [B] time = 3.46, size = 72, normalized size = 0.79

$$\frac{13292697\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856}-\frac{30613\,\ln\left(2\,x^2-x+3\right)}{128}-\frac{\frac{1156639\,x}{5888}+\frac{1478741}{5888}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{89359\,x}{64}-\frac{1185\,x^2}{8}+\frac{9775\,x^3}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^2,x)
```

```
[Out] (13292697*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/33856 - (30613*lo
g(2*x^2 - x + 3))/128 - ((1156639*x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/2)
 - (89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/
4
```

sympy [A] time = 0.20, size = 90, normalized size = 0.99

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{-1156639x-1478741}{5888x^2-2944x+8832}-\frac{30613\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{13292697\sqrt{23}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**5/4 + 2125*x**4/16 + 9775*x**3/48 - 1185*x**2/8 - 89359*x/64 + (-115
6639*x - 1478741)/(5888*x**2 - 2944*x + 8832) - 30613*log(x**2 - x/2 + 3/2)
/128 + 13292697*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/33856
```

3.45 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736(2x^2-x+3)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

```
[Out] 915/16*x+175/4*x^2+125/12*x^3-1331/736*(17-45*x)/(2*x^2-x+3)-2057/32*ln(2*x
^2-x+3)+223971/8464*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736(2x^2-x+3)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x +
2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3
- x + 2*x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx &= -\frac{1331(17-45x)}{736(3-x+2x^2)}+\frac{1}{23}\int \frac{-\frac{25195}{16}-\frac{19067x}{16}+\frac{22195x^2}{8}+\frac{13225x^3}{4}+\frac{2875x^4}{2}}{3-x+2x^2}\,dx\\
&= -\frac{1331(17-45x)}{736(3-x+2x^2)}+\frac{1}{23}\int \left(\frac{21045}{16}+\frac{4025x}{2}+\frac{2875x^2}{4}-\frac{121(365+391x)}{8(3-x+2x^2)}\right)dx\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736(3-x+2x^2)}-\frac{121}{184}\int \frac{365+391x}{3-x+2x^2}\,dx\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736(3-x+2x^2)}-\frac{2057}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{223971}{736}\int \frac{}{3-}\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736(3-x+2x^2)}-\frac{2057}{32}\log(3-x+2x^2)+\frac{223971}{368}\,\text{Subst}\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736(3-x+2x^2)}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log(3-x\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.00

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{1331(45x-17)}{736(2x^2-x+3)}-\frac{2057}{32}\log(2x^2-x+3)+\frac{915x}{16}-\frac{223971\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x +
 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[
3 - x + 2*x^2])/32
```

fricas [A] time = 0.73, size = 88, normalized size = 1.14

$$\frac{1058000\,x^5+3914600\,x^4+5173620\,x^3-1343826\,\sqrt{23}\,(2\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+3761190\,x^2-3}{50784\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2
- x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 - x
 + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)
```

giac [A] time = 0.19, size = 62, normalized size = 0.81

$$\frac{125}{12}\,x^3+\frac{175}{4}\,x^2-\frac{223971}{8464}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{915}{16}\,x+\frac{1331\,(45\,x-17)}{736\,\left(2\,x^2-x+3\right)}-\frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

maple [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464}-\frac{2057\ln\left(2x^2-x+3\right)}{32}-\frac{121\left(-\frac{495x}{92}+\frac{187}{92}\right)}{16\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x)
```

```
[Out] 125/12*x^3+175/4*x^2+915/16*x-121/16*(-495/92*x+187/92)/(x^2-1/2*x+3/2)-205
7/32*ln(2*x^2-x+3)-223971/8464*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{125}{12}\,x^3+\frac{175}{4}\,x^2-\frac{223971}{8464}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{915}{16}\,x+\frac{1331\,(45\,x-17)}{736\,\left(2\,x^2-x+3\right)}-\frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

mupad [B] time = 3.42, size = 61, normalized size = 0.79

$$\frac{915\,x}{16}-\frac{2057\,\ln\left(2\,x^2-x+3\right)}{32}+\frac{\frac{59895\,x}{1472}-\frac{22627}{1472}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{223971\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{8464}+\frac{175\,x^2}{4}+\frac{125\,x^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^2,x)
```

```
[Out] (915*x)/16 - (2057*log(2*x^2 - x + 3))/32 + ((59895*x)/1472 - 22627/1472)/(
x^2 - x/2 + 3/2) - (223971*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/
8464 + (175*x^2)/4 + (125*x^3)/12
```

sympy [A] time = 0.19, size = 75, normalized size = 0.97

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}+\frac{59895x-22627}{1472x^2-736x+2208}-\frac{2057\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{223971\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x
+ 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)*x
/23 - sqrt(23)/23)/8464
```

3.46 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{121(19-7x)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

```
[Out] 25/4*x+121/184*(19-7*x)/(2*x^2-x+3)+55/8*ln(2*x^2-x+3)+1859/2116*arctan(1/2
3*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(19-7x)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/
Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^2}\,dx &= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{\frac{163}{4}+\frac{1955x}{4}+\frac{575x^2}{2}}{3-x+2x^2}\,dx\\
&= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(\frac{575}{4}-\frac{11(71-115x)}{2\left(3-x+2x^2\right)}\right)dx\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}-\frac{11}{46}\int \frac{71-115x}{3-x+2x^2}\,dx\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{1859}{184}\int \frac{1}{3-x+2x^2}\,dx\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\log\left(3-x+2x^2\right)+\frac{1859}{92}\,\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-\right.\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$-\frac{121(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}-\frac{1859\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*x
)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

fricas [A] time = 0.97, size = 78, normalized size = 1.24

$$\frac{52900\,x^3-3718\,\sqrt{23}\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-26450\,x^2+29095\left(2\,x^2-x+3\right)\log\left(2\,x^2-x\right.}{4232\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/4232*(52900*x^3 - 3718*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x
 - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 59869*x + 5
2877)/(2*x^2 - x + 3)
```

giac [A] time = 0.17, size = 52, normalized size = 0.83

$$-\frac{1859}{2116}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{25}{4}\,x-\frac{121\,(7\,x-19)}{184\left(2\,x^2-x+3\right)}+\frac{55}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{25x}{4}-\frac{1859\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116}+\frac{55\ln\left(2x^2-x+3\right)}{8}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^2-\frac{1}{2}x+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x)
```

```
[Out] 25/4*x+11/4*(-77/92*x+209/92)/(x^2-1/2*x+3/2)+55/8*ln(2*x^2-x+3)-1859/2116*
23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 52, normalized size = 0.83

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x-\frac{121(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

mupad [B] time = 3.40, size = 52, normalized size = 0.83

$$\frac{25x}{4}+\frac{55\ln\left(2x^2-x+3\right)}{8}-\frac{\frac{847x}{368}-\frac{2299}{368}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{1859\sqrt{23}\ \mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^2,x)
```

```
[Out] (25*x)/4 + (55*log(2*x^2 - x + 3))/8 - ((847*x)/368 - 2299/368)/(x^2 - x/2
+ 3/2) - (1859*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/2116
```

sympy [A] time = 0.19, size = 61, normalized size = 0.97

$$\frac{25x}{4}+\frac{2299-847x}{368x^2-184x+552}+\frac{55\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{1859\sqrt{23}\ \mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

```
[Out] 25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2)
/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

3.47 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^2} \, dx$

Optimal. Leaf size=43

$$-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

```
[Out] -11/46*(5+3*x)/(2*x^2-x+3)-82/529*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 204}

$$-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*
Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx = -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} + \frac{1}{23}\int \frac{41}{3-x+2x^2}\,dx$$
$$= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} + \frac{41}{23}\int \frac{1}{3-x+2x^2}\,dx$$
$$= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} - \frac{82}{23}\,\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx, x, -1+4x\right)$$
$$= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} - \frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{82\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}} - \frac{11(3x+5)}{46\left(2x^2-x+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23
*Sqrt[23])
```

fricas [A] time = 0.95, size = 45, normalized size = 1.05

$$\frac{164\sqrt{23}\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right) - 759\,x - 1265}{1058\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759*
x - 1265)/(2*x^2 - x + 3)
```

giac [A] time = 0.19, size = 36, normalized size = 0.84

$$\frac{82}{529}\sqrt{23}\,\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right) - \frac{11\,(3\,x+5)}{46\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

maple [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{23}\;\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529} + \frac{-\frac{33x}{92}-\frac{55}{92}}{x^2-\frac{1}{2}x+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x)
```

```
[Out] (-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/
2))
```

maxima [A] time = 0.96, size = 36, normalized size = 0.84

$$\frac{82}{529}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{11\,(3\,x+5)}{46\,\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.84

$$\frac{82\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}-\frac{\frac{33\,x}{92}+\frac{55}{92}}{x^2-\frac{x}{2}+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2,x)
```

```
[Out] (82*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/529 - ((33*x)/92 + 55/9
2)/(x^2 - x/2 + 3/2)
```

sympy [A] time = 0.15, size = 42, normalized size = 0.98

$$\frac{-33x-55}{92x^2-46x+138}+\frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)
```

```
[Out] (-33*x - 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - sq
rt(23)/23)/529
```

3.48 $\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx$

Optimal. Leaf size=94

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

```
[Out] 1/506*(13-6*x)/(2*x^2-x+3)-13/968*ln(2*x^2-x+3)+13/968*ln(5*x^2+3*x+2)+241/
256036*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+69/15004*arctan(1/31*(3+10*x)
*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*
Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x
 + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
```

```
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx &= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{\int \frac{-1892-1067x+330x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx}{5566}\\
&= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{\int \frac{-3509+72358x}{3-x+2x^2}\,dx}{1346972}-\frac{\int \frac{-150282-180895x}{2+3x+5x^2}\,dx}{1346972}\\
&= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{241\int \frac{1}{3-x+2x^2}\,dx}{22264}-\frac{13}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx+\frac{13}{968}\int\\
&= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{13}{968}\log\left(3-x+2x^2\right)+\frac{13}{968}\log\left(2+3x+5x^2\right)+\\
&= \frac{13-6x}{506\left(3-x+2x^2\right)}+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}}-\frac{13}{968}\log(3
\end{aligned}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 1.00

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)-\frac{241\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(11132
*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 -
x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

fricas [A] time = 0.93, size = 117, normalized size = 1.24

$$\frac{73002\sqrt{31}\left(2x^2-x+3\right)\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)-14942\sqrt{23}\left(2x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)+2131}{15874232\left(2x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/15874232*(73002*sqrt(31)*(2*x^2 - x + 3)*arctan(1/31*sqrt(31)*(10*x + 3))
 - 14942*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 213187*
(2*x^2 - x + 3)*log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*log(2*x^2 - x
 + 3) - 188232*x + 407836)/(2*x^2 - x + 3)
```

giac [A] time = 0.19, size = 78, normalized size = 0.83

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{6x-13}{506\left(2x^2-x+3\right)}+\frac{13}{968}\log\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{69\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{15004}-\frac{241\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{256036}-\frac{13\ln\left(2x^2-x+3\right)}{968}+\frac{13\ln\left(5x^2+3x+2\right)}{968}-\frac{\frac{66x}{23}-}{484\left(x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

```
[Out] 13/968*ln(5*x^2+3*x+2)+69/15004*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))-1/4
84*(66/23*x-143/23)/(x^2-1/2*x+3/2)-13/968*ln(2*x^2-x+3)-241/256036*23^(1/2
)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 78, normalized size = 0.83

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{6x-13}{506\left(2x^2-x+3\right)}+\frac{13}{968}\log\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

mupad [B] time = 3.58, size = 96, normalized size = 1.02

$$-\frac{\frac{3x}{506}-\frac{13}{1012}}{x^2-\frac{x}{2}+\frac{3}{2}}-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right)+\ln\left(x-\frac{1}{4}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 + 13/968) - log(x -
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 - 13/968) - ((3*x)/506 - 13/
1012)/(x^2 - x/2 + 3/2) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/5
12072 - 13/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/512072 +
13/968)
```

sympy [A] time = 0.32, size = 102, normalized size = 1.09

$$\frac{13-6x}{1012x^2-506x+1518}-\frac{13\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}+\frac{13\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{241\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{256036}+\frac{69\sqrt{31}\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] (13 - 6*x)/(1012*x**2 - 506*x + 1518) - 13*log(x**2 - x/2 + 3/2)/968 + 13*l
og(x**2 + 3*x/5 + 2/5)/968 - 241*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/256036 + 69*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/15004
```

3.49 $\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^2} \, dx$

Optimal. Leaf size=127

$$-\frac{25(117-137x)}{172546\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}+\frac{19\log\left(2x^2-x+3\right)}{10648}-\frac{19\log\left(5x^2+3x+2\right)}{10648}+\frac{2769\cdots}{122\cdots}$$

```
[Out] -25/172546*(117-137*x)/(5*x^2+3*x+2)+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+3*x+
2)+19/10648*ln(2*x^2-x+3)-19/10648*ln(5*x^2+3*x+2)+2769/2816396*arctan(1/23
*(1-4*x)*23^(1/2))*23^(1/2)+12643/5116364*arctan(1/31*(3+10*x)*31^(1/2))*31
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{25(117-137x)}{172546\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}+\frac{19\log\left(2x^2-x+3\right)}{10648}-\frac{19\log\left(5x^2+3x+2\right)}{10648}+\frac{2769\cdots}{122\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2
*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[2
3]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x
 + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
```

```
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{-2321-2299x+990x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx}{5566}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{-303419\ldots}{(3-\ldots}}{\ldots}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{1322827\ldots}{3-\ldots}}{1010\ldots}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \frac{19\int \frac{-1+\ldots}{3-x+\ldots}}{1064\ldots}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \frac{19\log\left(3\ldots\right.}{10\ldots}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \frac{2769\tan\ldots}{12245\ldots}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.83

$$\frac{9659011\log\left(2x^2-x+3\right) - 9659011\log\left(5x^2+3x+2\right) + \frac{31372\left(6850x^3-9275x^2+11154x-4342\right)}{10x^4+x^3+16x^2+7x+6} - 5322018\sqrt{23}\tan^{-1}\left(\ldots\right.}{5413113112}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((31372*(-4342 + 11154*x - 9275*x^2 + 6850*x^3))/(6 + 7*x + 16*x^2 + x^3 +
10*x^4) - 5322018*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 13376294*Sqrt[31]*
ArcTan[(3 + 10*x)/Sqrt[31]] + 9659011*Log[3 - x + 2*x^2] - 9659011*Log[2 +
3*x + 5*x^2])/5413113112
```

fricas [A] time = 0.86, size = 167, normalized size = 1.31

$$\frac{214898200\,x^3 + 13376294\sqrt{31}\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right) - 5322018\sqrt{23}\left(10\,x^4+\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/5413113112*(214898200*x^3 + 13376294*sqrt(31)*(10*x^4 + x^3 + 16*x^2 + 7*
x + 6)*arctan(1/31*sqrt(31)*(10*x + 3)) - 5322018*sqrt(23)*(10*x^4 + x^3 +
16*x^2 + 7*x + 6)*arctan(1/23*sqrt(23)*(4*x - 1)) - 290975300*x^2 - 9659011
*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*log(5*x^2 + 3*x + 2) + 9659011*(10*x^4 +
 x^3 + 16*x^2 + 7*x + 6)*log(2*x^2 - x + 3) + 349923288*x - 136217224)/(10*
x^4 + x^3 + 16*x^2 + 7*x + 6)
```

giac [A] time = 0.19, size = 96, normalized size = 0.76

$$\frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right) - \frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right) + \frac{6850\,x^3-9275\,x^2+11154\ldots}{172546\left(10\,x^4+x^3+16\,x^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 94, normalized size = 0.74

$$\frac{12643\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{5116364}-\frac{2769\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396}+\frac{19\ln\left(2x^2-x+3\right)}{10648}-\frac{19\ln\left(5x^2+3x+2\right)}{10648}-\frac{}{53}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/5324*(-759/31*x+1078/155)/(x^2+3/5*x+2/5)-19/10648*ln(5*x^2+3*x+2)+12643
/5116364*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))+1/5324*(-77/23*x-341/46)/(
x^2-1/2*x+3/2)+19/10648*ln(2*x^2-x+3)-2769/2816396*23^(1/2)*arctan(1/23*(4*
x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 96, normalized size = 0.76

$$\frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{6850\,x^3-9275\,x^2+11}{172546\left(10\,x^4+x^3+16\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

mupad [B] time = 0.18, size = 115, normalized size = 0.91

$$\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{3}}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 + 19/10648) - log(
x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 - 19/10648) - log(x -
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 + 19/10648) + log(x +
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 - 19/10648) + ((507*x)
/78430 - (1855*x^2)/345092 + (685*x^3)/172546 - 2171/862730)/((7*x)/10 + (8
*x^2)/5 + x^3/10 + x^4 + 3/5)
```

sympy [A] time = 0.36, size = 122, normalized size = 0.96

$$\frac{6850x^3-9275x^2+11154x-4342}{1725460x^4+172546x^3+2760736x^2+1207822x+1035276}+\frac{19\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{10648}-\frac{19\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{10648}-\frac{27}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] (6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 2760
736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log(x
**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/5116364
```

3.50 $\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{5765x-9446}{690184\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right.}{468512}$$

```
[Out] 1/690184*(-9446+5765*x)/(5*x^2+3*x+2)^2+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+3
*x+2)^2+1/235352744*(1765599+3996965*x)/(5*x^2+3*x+2)+97/468512*ln(2*x^2-x+
3)-97/468512*ln(5*x^2+3*x+2)-25557/123921424*arctan(1/23*(1-4*x)*23^(1/2))*
23^(1/2)+4464079/6978720496*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{9446-5765x}{690184\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+\right.}{468512}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(9446 - 5765*x)/(690184*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(506*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x
^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*Arc
Tan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/46
8512 - (97*Log[2 + 3*x + 5*x^2])/468512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3}\,dx}{5566}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} - \ldots$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \ldots$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \ldots$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \ldots$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \ldots$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \ldots$$

Mathematica [A] time = 0.08, size = 136, normalized size = 0.92

$$\frac{90x-11}{244904\left(2x^2-x+3\right)} + \frac{164380x+67573}{10232728\left(5x^2+3x+2\right)} + \frac{345x-98}{30008\left(5x^2+3x+2\right)^2} + \frac{97\log\left(2x^2-x+3\right)}{468512} - \frac{97\log\left(5x^2 \ldots\right)}{4685\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-11 + 90*x)/(244904*(3 - x + 2*x^2)) + (-98 + 345*x)/(30008*(2 + 3*x + 5*x
^2)^2) + (67573 + 164380*x)/(10232728*(2 + 3*x + 5*x^2)) + (25557*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt[31
]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 + 3*x
 + 5*x^2])/468512
```

fricas [A] time = 1.07, size = 237, normalized size = 1.60

$$\frac{1253927859800\,x^5 + 679296504260\,x^4 + 2185021181068\,x^3 + 4722995582\sqrt{31}\left(50\,x^6 + 35\,x^5 + 103\,x^4 + 85\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/7383486284768*(1253927859800*x^5 + 679296504260*x^4 + 2185021181068*x^3 +
 4722995582*sqrt(31)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x +
12)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1522737174*sqrt(23)*(50*x^6 + 35*x^5
 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*arctan(1/23*sqrt(23)*(4*x - 1)) +
 1500218514344*x^2 - 1528665583*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^
```

```
2 + 32*x + 12)*log(5*x^2 + 3*x + 2) + 1528665583*(50*x^6 + 35*x^5 + 103*x^4
 + 85*x^3 + 83*x^2 + 32*x + 12)*log(2*x^2 - x + 3) + 1338609358240*x + 2188
80812656)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

giac [A] time = 0.20, size = 110, normalized size = 0.74

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{39969650x^5+2165\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/((5*x^2
+ 3*x + 2)^2*(2*x^2 - x + 3)) - 97/468512*log(5*x^2 + 3*x + 2) + 97/468512*
log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 106, normalized size = 0.72

$$\frac{4464079\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{6978720496}+\frac{25557\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{123921424}+\frac{97\ln\left(2x^2-x+3\right)}{468512}-\frac{97\ln\left(5x^2+3x+2\right)}{468512}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] -25/234256*(-723272/961*x^3-3656422/4805*x^2-14280728/24025*x-2238016/24025
)/(5*x^2+3*x+2)^2-97/468512*ln(5*x^2+3*x+2)+4464079/6978720496*31^(1/2)*arc
tan(1/31*(10*x+3)*31^(1/2))+1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/4
68512*ln(2*x^2-x+3)+25557/123921424*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 118, normalized size = 0.80

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{39969650x^5+2165\cdots}{23535274\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(5*x^2 + 3
*x + 2) + 97/468512*log(2*x^2 - x + 3)
```

mupad [B] time = 3.59, size = 135, normalized size = 0.91

$$\frac{\frac{799393x^5}{235352744}+\frac{4330591x^4}{2353527440}+\frac{69648769x^3}{11767637200}+\frac{23910151x^2}{5883818600}+\frac{1066723x}{294190930}+\frac{158567}{267446300}}{x^6+\frac{7x^5}{10}+\frac{103x^4}{50}+\frac{17x^3}{10}+\frac{83x^2}{50}+\frac{16x}{25}+\frac{6}{25}}+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{97}{468512}+\frac{\sqrt{23}\,25\cdots}{247842\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 + 97/468512) -
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 - 97/468512) +
```

```
((1066723*x)/294190930 + (23910151*x^2)/5883818600 + (69648769*x^3)/1176763
7200 + (4330591*x^4)/2353527440 + (799393*x^5)/235352744 + 158567/267446300
)/((16*x)/25 + (83*x^2)/50 + (17*x^3)/10 + (103*x^4)/50 + (7*x^5)/10 + x^6
+ 6/25) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957440992
 + 97/468512) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957
440992 - 97/468512)
```

sympy [A] time = 0.40, size = 143, normalized size = 0.97

$$\frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{11767637200x^6 + 8237346040x^5 + 24241332632x^4 + 20004983240x^3 + 19534277752x^2 + 7531287808x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*x
 + 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 200049
83240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2 -
x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*atan
(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/6978720496
```

3.51 $\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^3} dx$

Optimal. Leaf size=98

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}+\frac{63799791\tan^{-1}}{16928\sqrt{2}}$$

```
[Out] 2725/8*x+4875/32*x^2+625/24*x^3-14641/5888*(101+79*x)/(2*x^2-x+3)^2+1331/13
5424*(5229+76420*x)/(2*x^2-x+3)-13915/64*ln(2*x^2-x+3)+63799791/389344*arct
an(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}+\frac{63799791\tan^{-1}}{16928\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (637997
91*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2]
)/64
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^3}\,dx = -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{\frac{2173869}{128}-\frac{661181x}{32}-\frac{488267x^2}{16}+\frac{143635x^3}{8}+\frac{213325x^4}{4}+\frac{833}{}}{\left(3-x+2x^2\right)^2}$$

$$= -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}+\frac{\int \frac{-\frac{5460539}{8}-\frac{626865x}{2}+\frac{5170975x^2}{8}+\frac{1124125x^3}{2}+}{3-x+2x^2}}{1058}$$

$$= -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}+\frac{\int\left(\frac{1441525}{4}+\frac{2578875x}{8}+\frac{330625x^2}{4}-\frac{1}{}\right.}{1058}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}-\frac{121\int \frac{1}{}}{}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}-\frac{13915}{64}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}-\frac{13915}{64}\,\mathrm{l}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}+\frac{637997}{1}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 1.00

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}-\frac{63799791\,\mathrm{ta}}{16928}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (637997
91*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2
])/64
```

fricas [A] time = 0.82, size = 128, normalized size = 1.31

$$\frac{486680000\,x^7+2360398000\,x^6+5100406400\,x^5+2157209100\,x^4+24531516180\,x^3-765597492\,\sqrt{23}\left(4\,x^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^4
 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*
arctan(1/23*sqrt(23)*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 72, normalized size = 0.73

$$\frac{625}{24}\,x^3+\frac{4875}{32}\,x^2-\frac{63799791}{389344}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2725}{8}\,x+\frac{1331\left(76420\,x^3-32981\,x^2+102022\,x-4\right.}{67712\left(2\,x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 73, normalized size = 0.74

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}-\frac{63799791\sqrt{23}\;\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{389344}-\frac{13915\ln\left(2x^2-x+3\right)}{64}-\frac{121\left(-\frac{210155}{4232}x^3+\frac{362791}{16928}x^2\right.}{4\left(2x^2-x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x)
```

```
[Out] 625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-56
1121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/3893
44*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.95, size = 82, normalized size = 0.84

$$\frac{625}{24}\,x^3+\frac{4875}{32}\,x^2-\frac{63799791}{389344}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2725}{8}\,x+\frac{1331\left(76420\,x^3-32981\,x^2+102022\,x-4\right.}{67712\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)
```

mupad [B] time = 0.05, size = 81, normalized size = 0.83

$$\frac{2725\,x}{8}-\frac{13915\,\ln\left(2\,x^2-x+3\right)}{64}-\frac{63799791\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{389344}+\frac{4875\,x^2}{32}+\frac{625\,x^3}{24}+\frac{\frac{25428755\,x^3}{67712}-\frac{43897711\,x^2}{270848}}{x^4-x^3+\frac{13}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^3,x)
```

```
[Out] (2725*x)/8 - (13915*log(2*x^2 - x + 3))/64 - (63799791*23^(1/2)*atan((4*23^
(1/2)*x)/23 - 23^(1/2)/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((67895
641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/2708
48)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

sympy [A] time = 0.24, size = 95, normalized size = 0.97

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}+\frac{101715020x^3-43897711x^2+135791282x-6565823}{270848x^4-270848x^3+880256x^2-406272x+609408}-\frac{13915\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{64}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)
```

```
[Out] 625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 1
35791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x +
 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqrt(2
3)*x/23 - sqrt(23)/23)/389344
```

3.52 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} \, dx$

Optimal. Leaf size=84

$$\frac{121(21193-12828x)}{33856(2x^2-x+3)}-\frac{1331(17-45x)}{1472(2x^2-x+3)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

```
[Out] 125/8*x-1331/1472*(17-45*x)/(2*x^2-x+3)^2+121/33856*(21193-12828*x)/(2*x^2-
x+3)+825/32*ln(2*x^2-x+3)+165099/194672*arctan(1/23*(1-4*x)*23^(1/2))*23^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(21193-12828x)}{33856(2x^2-x+3)}-\frac{1331(17-45x)}{1472(2x^2-x+3)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 128
28*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*
Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^3}\,dx = -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{-\frac{40885}{32}-\frac{19067x}{8}+\frac{22195x^2}{4}+\frac{13225x^3}{2}+2875x^4}{\left(3-x+2x^2\right)^2}\,dx$$

$$= -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{\int \frac{\frac{23997}{2}+92575x+\frac{66125x^2}{2}}{3-x+2x^2}\,dx}{1058}$$

$$= -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{\int \left(\frac{66125}{4}-\frac{33(4557-13225x)}{4\left(3-x+2x^2\right)}\right)dx}{1058}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}-\frac{33\int \frac{4557-13225x}{3-x+2x^2}\,dx}{4232}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}-\frac{165099\int \frac{1}{3-x+2x^2}\,dx}{16928}+\frac{825}{32}\int$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{825}{32}\log\left(3-x+2x^2\right)+\frac{1650}{}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}+\frac{825}{32}\mathrm{l}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 1.00

$$-\frac{121(12828x-21193)}{33856\left(2x^2-x+3\right)}+\frac{1331(45x-17)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}-\frac{165099\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 + 1
2828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(84
64*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

fricas [A] time = 0.82, size = 118, normalized size = 1.40

$$\frac{24334000\,x^5-24334000\,x^4+43385176\,x^3-330198\,\sqrt{23}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right.}{389344\left(4\,x^4-4\,x^3+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*x
^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281*x
^2 + 10037775*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) - 12446
818*x + 82485337)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.18, size = 62, normalized size = 0.74

$$-\frac{165099}{194672}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{125}{8}\,x-\frac{121\left(12828\,x^3-27607\,x^2+24146\,x-29639\right)}{16928\left(2\,x^2-x+3\right)^2}+\frac{825}{32}\,\log\left(2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*log
(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{125x}{8}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672}+\frac{825\ln\left(2x^2-x+3\right)}{32}+\frac{-\frac{388047}{4232}x^3+\frac{3340447}{16928}x^2-\frac{1460833}{8464}x+\frac{3586319}{16928}}{\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x)
```

```
[Out] 125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(2
*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(4*x-1)
*23^(1/2))
```

maxima [A] time = 0.96, size = 72, normalized size = 0.86

$$-\frac{165099}{194672}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{125}{8}\,x-\frac{121\left(12828\,x^3-27607\,x^2+24146\,x-29639\right)}{16928\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}+\frac{825}{32}\,\log\left(2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x
+ 9) + 825/32*log(2*x^2 - x + 3)
```

mupad [B] time = 0.05, size = 72, normalized size = 0.86

$$\frac{125\,x}{8}+\frac{825\,\ln\left(2\,x^2-x+3\right)}{32}-\frac{165099\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}-\frac{\frac{388047\,x^3}{16928}-\frac{3340447\,x^2}{67712}+\frac{1460833\,x}{33856}-\frac{3586319}{67712}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^3,x)
```

```
[Out] (125*x)/8 + (825*log(2*x^2 - x + 3))/32 - (165099*23^(1/2)*atan((4*23^(1/2)
*x)/23 - 23^(1/2)/23))/194672 - ((1460833*x)/33856 - (3340447*x^2)/67712 +
(388047*x^3)/16928 - 3586319/67712)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4
)
```

sympy [A] time = 0.23, size = 82, normalized size = 0.98

$$\frac{125x}{8} + \frac{-1552188x^3 + 3340447x^2 - 2921666x + 3586319}{67712x^4 - 67712x^3 + 220064x^2 - 101568x + 152352} + \frac{825\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{165099\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}\right.}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)
```

```
[Out] 125*x/8 + (-1552188*x**3 + 3340447*x**2 - 2921666*x + 3586319)/(67712*x**4
- 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/2)
/32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672
```

3.53 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} \, dx$

Optimal. Leaf size=64

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{55(332x+975)}{8464(2x^2-x+3)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

```
[Out] 121/368*(19-7*x)/(2*x^2-x+3)^2-55/8464*(975+332*x)/(2*x^2-x+3)-4330/12167*a
rctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 618, 204}

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{55(332x+975)}{8464(2x^2-x+3)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x
+ 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^3}\,dx &= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{-\frac{195}{8}+\frac{1955x}{2}+575x^2}{\left(3-x+2x^2\right)^2}\,dx \\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}+\frac{\int \frac{4330}{3-x+2x^2}\,dx}{1058} \\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}+\frac{2165}{529}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}-\frac{4330}{529}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.80

$$
\frac{4330\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{529\sqrt{23}}-\frac{11\left(1660x^3+4045x^2+938x+4909\right)}{4232\left(-2x^2+x-3\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (433
0*ArcTan[(-1 + 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

fricas [A] time = 0.87, size = 75, normalized size = 1.17

$$
-\frac{419980\,x^3-34640\,\sqrt{23}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+1023385\,x^2+237314\,x+}{97336\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] -1/97336*(419980*x^3 - 34640*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.19, size = 46, normalized size = 0.72

$$
\frac{4330}{12167}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{11\left(1660\,x^3+4045\,x^2+938\,x+4909\right)}{4232\left(2\,x^2-x+3\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}+\frac{-\frac{4565}{1058}x^3-\frac{44495}{4232}x^2-\frac{5159}{2116}x-\frac{53999}{4232}}{\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x)
```

```
[Out] 4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+43
30/12167*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{4330}{12167}\sqrt{23}\ \arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{11\left(1660\,x^3+4045\,x^2+938\,x+4909\right)}{4232\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

mupad [B] time = 3.47, size = 56, normalized size = 0.88

$$\frac{4330\,\sqrt{23}\ \mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}-\frac{\frac{4565\,x^3}{4232}+\frac{44495\,x^2}{16928}+\frac{5159\,x}{8464}+\frac{53999}{16928}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^3,x)
```

```
[Out] (4330*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 - ((5159*x)/846
4 + (44495*x^2)/16928 + (4565*x^3)/4232 + 53999/16928)/((13*x^2)/4 - (3*x)/
2 - x^3 + x^4 + 9/4)
```

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{-18260x^3-44495x^2-10318x-53999}{16928x^4-16928x^3+55016x^2-25392x+38088}+\frac{4330\sqrt{23}\ \mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)
```

```
[Out] (-18260*x**3 - 44495*x**2 - 10318*x - 53999)/(16928*x**4 - 16928*x**3 + 550
16*x**2 - 25392*x + 38088) + 4330*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/
23)/12167
```

3.54 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{131(1-4x)}{2116\left(2x^2-x+3\right)}-\frac{11(3x+5)}{92\left(2x^2-x+3\right)^2}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

```
[Out] -11/92*(5+3*x)/(2*x^2-x+3)^2-131/2116*(1-4*x)/(2*x^2-x+3)-262/12167*arctan(
1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1660, 12, 614, 618, 204}

$$-\frac{131(1-4x)}{2116\left(2x^2-x+3\right)}-\frac{11(3x+5)}{92\left(2x^2-x+3\right)^2}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x
^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
```

```
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx &= -\frac{11(5+3x)}{92(3-x+2x^2)^2}+\frac{1}{46}\int \frac{131}{2(3-x+2x^2)^2}\,dx \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2}+\frac{131}{92}\int \frac{1}{(3-x+2x^2)^2}\,dx \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2}-\frac{131(1-4x)}{2116(3-x+2x^2)}+\frac{131}{529}\int \frac{1}{3-x+2x^2}\,dx \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2}-\frac{131(1-4x)}{2116(3-x+2x^2)}-\frac{262}{529}\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= -\frac{11(5+3x)}{92(3-x+2x^2)^2}-\frac{131(1-4x)}{2116(3-x+2x^2)}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.80

$$\frac{\frac{46(524x^3-393x^2+472x-829)}{(-2x^2+x-3)^2}+1048\sqrt{23}\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{48668}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] ((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*Sqrt[23]
*ArcTan[(-1 + 4*x)/Sqrt[23]])/48668
```

fricas [A] time = 0.92, size = 75, normalized size = 1.17

$$\frac{12052\,x^3+524\,\sqrt{23}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-9039\,x^2+10856\,x-19067}{24334\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan
(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 + 13
*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 46, normalized size = 0.72

$$\frac{262}{12167}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{524\,x^3-393\,x^2+472\,x-829}{1058\left(2\,x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(2*x^2 - x + 3)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{262\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}+\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x)
```

```
[Out] 4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*23^
(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{524x^3-393x^2+472x-829}{1058\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.86

$$\frac{262\sqrt{23}\ \mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}+\frac{\frac{131x^3}{1058}-\frac{393x^2}{4232}+\frac{59x}{529}-\frac{829}{4232}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^3,x)
```

```
[Out] (262*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 + ((59*x)/529 -
(393*x^2)/4232 + (131*x^3)/1058 - 829/4232)/((13*x^2)/4 - (3*x)/2 - x^3 + x
^4 + 9/4)
```

sympy [A] time = 0.18, size = 61, normalized size = 0.95

$$\frac{524x^3-393x^2+472x-829}{4232x^4-4232x^3+13754x^2-6348x+9522}+\frac{262\sqrt{23}\ \mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)
```

```
[Out] (524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 - 6
348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167
```

3.55 $\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)}\,dx$

Optimal. Leaf size=115

$$\frac{3625-746x}{256036\left(2x^2-x+3\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}-\frac{119\log\left(2x^2-x+3\right)}{21296}+\frac{119\log\left(5x^2+3x+2\right)}{21296}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}$$

```
[Out] 1/1012*(13-6*x)/(2*x^2-x+3)^2+1/256036*(3625-746*x)/(2*x^2-x+3)-119/21296*l
n(2*x^2-x+3)+119/21296*ln(5*x^2+3*x+2)-53403/129554216*arctan(1/23*(1-4*x)*
23^(1/2))*23^(1/2)+247/330088*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{3625-746x}{256036\left(2x^2-x+3\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}-\frac{119\log\left(2x^2-x+3\right)}{21296}+\frac{119\log\left(5x^2+3x+2\right)}{21296}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2
)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3
 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (11
9*Log[2 + 3*x + 5*x^2])/21296
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
```

```
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2} - \frac{\int \frac{-3652-1936x+990x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{\int \frac{-6551908-7779574x+902660x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}}{61960712}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{\int \frac{-154867174+335151124x}{3-x+2x^2}\,dx}{14994492304}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} + \frac{53403\int \frac{1}{3-x+2x^2}\,dx}{11265584} - \frac{119}{}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{119\log\left(3-x+2x^2\right)}{21296} + \frac{11}{}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} + \frac{247}{1}$$

Mathematica [A] time = 0.16, size = 99, normalized size = 0.86

$$\frac{713\left(-62951\log\left(2x^2-x+3\right)+62951\log\left(5x^2+3x+2\right)-\frac{44\left(1492x^3-7996x^2+7381x-14164\right)}{\left(-2x^2+x-3\right)^2}\right)+3310986\sqrt{23}\tan^{-1}\left(\right.}{8032361392}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (3310986*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3
+ 10*x)/Sqrt[31]] + 713*((-44*(-14164 + 7381*x - 7996*x^2 + 1492*x^3))/(-3
+ x - 2*x^2)^2 - 62951*Log[3 - x + 2*x^2] + 62951*Log[2 + 3*x + 5*x^2]))/80
32361392
```

fricas [A] time = 0.83, size = 177, normalized size = 1.54

$$-\frac{46807024\,x^3-6010498\,\sqrt{31}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)-3310986\,\sqrt{23}\left(4\,x^4-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/8032361392*(46807024*x^3 - 6010498*sqrt(31)*(4*x^4 - 4*x^3 + 13*x^2 - 6*
x + 9)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3310986*sqrt(23)*(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 250850512*x^2 - 448840
63*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(5*x^2 + 3*x + 2) + 44884063*(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 231556732*x - 444353008)
/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 88, normalized size = 0.77

$$\frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{1492\,x^3-7996\,x^2+7381}{256036\left(2\,x^2-x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/21296*
log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{247\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{330088}+\frac{53403\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216}-\frac{119\ln\left(2x^2-x+3\right)}{21296}+\frac{119\ln\left(5x^2+3x+2\right)}{21296}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

```
[Out] 119/21296*ln(5*x^2+3*x+2)+247/330088*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2)
)-1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^2-
119/21296*ln(2*x^2-x+3)+53403/129554216*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/
2))
```

maxima [A] time = 0.97, size = 98, normalized size = 0.85

$$\frac{247}{330088}\sqrt{31}\ \arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{53403}{129554216}\sqrt{23}\ \arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{1492\,x^3-7996\,x^2+7}{256036\left(4\,x^4-4\,x^3+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*log(5*x^2 + 3*x
+ 2) - 119/21296*log(2*x^2 - x + 3)
```

mupad [B] time = 3.58, size = 116, normalized size = 1.01

$$-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\ 1\mathrm{i}}{10}\right)\left(-\frac{119}{21296}+\frac{\sqrt{31}\ 247\mathrm{i}}{660176}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\ 1\mathrm{i}}{10}\right)\left(\frac{119}{21296}+\frac{\sqrt{31}\ 247\mathrm{i}}{660176}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 + 119/21296) - log
(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 - 119/21296) - log(x
- (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 + 119/21296) + log(x
+ (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 - 119/21296) - ((61*x
)/8464 - (1999*x^2)/256036 + (373*x^3)/256036 - 3541/256036)/((13*x^2)/4 -
(3*x)/2 - x^3 + x^4 + 9/4)
```

sympy [A] time = 0.36, size = 122, normalized size = 1.06

$$\frac{-1492x^3+7996x^2-7381x+14164}{1024144x^4-1024144x^3+3328468x^2-1536216x+2304324}-\frac{119\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296}+\frac{119\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] (-1492*x**3 + 7996*x**2 - 7381*x + 14164)/(1024144*x**4 - 1024144*x**3 + 33
28468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119*l
og(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(2
3)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3300
88
```

3.56 $\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=160

$$\frac{-252815x-2328909}{174616552\left(5x^2+3x+2\right)}+\frac{9665-1446x}{512072\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}+\frac{181\,\mathrm{lo}}{}$$

```
[Out] 1/174616552*(-2328909-252815*x)/(5*x^2+3*x+2)+1/1012*(13-6*x)/(2*x^2-x+3)^2
/(5*x^2+3*x+2)+1/512072*(9665-1446*x)/(2*x^2-x+3)/(5*x^2+3*x+2)+181/468512*
ln(2*x^2-x+3)-181/468512*ln(5*x^2+3*x+2)+2038497/2850192752*arctan(1/23*(1-
4*x)*23^(1/2))*23^(1/2)+246757/225120016*arctan(1/31*(3+10*x)*31^(1/2))*31^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{9665-1446x}{512072\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}-\frac{252815x+2328909}{174616552\left(5x^2+3x+2\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}+\frac{181\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(2328909 + 252815*x)/(174616552*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(1012*(3 -
 x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*
(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[2
3]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3
- x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} - \frac{\int \frac{-4081-3168x+1650x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{9665-1446x}{512072\left(3-x+2x^2\right)\left(2+3x+\right.}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \cdots$$

Mathematica [A] time = 0.12, size = 136, normalized size = 0.85

$$\frac{-2923x-1782}{1408198\left(2x^2-x+3\right)} + \frac{1235x-1474}{330088\left(5x^2+3x+2\right)} + \frac{-14x-31}{22264\left(2x^2-x+3\right)^2} + \frac{181\log\left(2x^2-x+3\right)}{468512} - \frac{181\log\left(5x^2+\right.}{46851}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-31 - 14*x)/(22264*(3 - x + 2*x^2)^2) + (-1782 - 2923*x)/(1408198*(3 - x +
 2*x^2)) + (-1474 + 1235*x)/(330088*(2 + 3*x + 5*x^2)) - (2038497*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2 + 3*
x + 5*x^2])/468512
```

fricas [A] time = 0.87, size = 227, normalized size = 1.42

$$-\frac{31725248720\,x^5 + 260524883872\,x^4 - 158204886268\,x^3 - 6004584838\sqrt{31}\left(20\,x^6 - 8\,x^5 + 61\,x^4 + x^3 + 53\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 - 6
004584838*sqrt(31)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*arc
tan(1/31*sqrt(31)*(10*x + 3)) + 3917991234*sqrt(23)*(20*x^6 - 8*x^5 + 61*x^
4 + x^3 + 53*x^2 + 15*x + 18)*arctan(1/23*sqrt(23)*(4*x - 1)) + 67996648469
2*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*log
```

```
(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15
*x + 18)*log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6 - 8*x^
5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

giac [A] time = 0.22, size = 110, normalized size = 0.69

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1011260x^5+830437}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2 + 3
*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*log(5*x^2 + 3*x + 2) + 181/468512*l
og(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 106, normalized size = 0.66

$$\frac{246757\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{225120016}-\frac{2038497\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2850192752}+\frac{181\ln\left(2x^2-x+3\right)}{468512}-\frac{181\ln\left(5x^2+3x+2\right)}{468512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*ln(5*x^2+3*x+2)
+246757/225120016*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))+1/58564*(-128612/
529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3)^2+181/468512*ln
(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 116, normalized size = 0.72

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1011260x^5+830437}{174616552}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6 - 8
*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x + 2)
 + 181/468512*log(2*x^2 - x + 3)
```

mupad [B] time = 3.60, size = 136, normalized size = 0.85

$$-\frac{\frac{50563x^5}{174616552}+\frac{1038047x^4}{436541380}-\frac{5042869x^3}{3492331040}+\frac{21674311x^2}{3492331040}-\frac{294391x}{174616552}+\frac{200677}{79371160}}{x^6-\frac{2x^5}{5}+\frac{61x^4}{20}+\frac{x^3}{20}+\frac{53x^2}{20}+\frac{3x}{4}+\frac{9}{10}}-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1i}{10}\right)\left(\frac{181}{468512}+\frac{\sqrt{31}\,24}{45024}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 - 181/468512
) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 + 181/46
```

```
8512) - ((21674311*x^2)/3492331040 - (294391*x)/174616552 - (5042869*x^3)/3
492331040 + (1038047*x^4)/436541380 + (50563*x^5)/174616552 + 200677/793711
60)/((3*x)/4 + (53*x^2)/20 + x^3/20 + (61*x^4)/20 - (2*x^5)/5 + x^6 + 9/10)
 + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 + 181/468
512) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 - 181
/468512)
```

sympy [A] time = 0.43, size = 143, normalized size = 0.89

$$\frac{-1011260x^5 - 8304376x^4 + 5042869x^3 - 21674311x^2 + 5887820x - 8829788}{3492331040x^6 - 1396932416x^5 + 10651609672x^4 + 174616552x^3 + 9254677256x^2 + 2619248280x + 314309}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] (-1011260*x**5 - 8304376*x**4 + 5042869*x**3 - 21674311*x**2 + 5887820*x -
8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 174616552*
x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2 +
3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/225120016
```

3.57 $\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3} \, dx$

Optimal. Leaf size=181

$$\frac{5(302-35x)}{64009\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{15(7140435x+2618306)}{14886061058\left(5x^2+3x+2\right)}-\frac{5(77020x+223707)}{87308276\left(5x^2+3x+2\right)^2}+\frac{1}{1012\left(2x^2-x+\right.}$$

```
[Out] -5/87308276*(223707+77020*x)/(5*x^2+3*x+2)^2+1/1012*(13-6*x)/(2*x^2-x+3)^2/
(5*x^2+3*x+2)^2+5/64009*(302-35*x)/(2*x^2-x+3)/(5*x^2+3*x+2)^2+15/148860610
58*(2618306+7140435*x)/(5*x^2+3*x+2)+405/1288408*ln(2*x^2-x+3)-405/1288408*
ln(5*x^2+3*x+2)-880575/7838030068*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+27
68835/19191481364*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{5(302-35x)}{64009\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{15(7140435x+2618306)}{14886061058\left(5x^2+3x+2\right)}-\frac{5(77020x+223707)}{87308276\left(5x^2+3x+2\right)^2}+\frac{1}{1012\left(2x^2-x+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(
3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*
x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2
768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x +
 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx = \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} - \frac{\int \frac{-4510-4400x+2310x^2}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx}{11132}$$

$$= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^{2}}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64}$$

Mathematica [A] time = 0.10, size = 151, normalized size = 0.83

$$\frac{405\log(2x^2-x+3)}{1288408} - \frac{405\log(5x^2+3x+2)}{1288408} + \frac{6850x^3-9275x^2+11154x-4342}{345092(10x^4+x^3+16x^2+7x+6)^2} + \frac{5(42842610x^3-571146}{14886061058(10x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 + 1
0*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(14886
061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)) + (880575*ArcTan[(-1 + 4*x)/Sqrt[
23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(6190800
44*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2]
)/1288408
```

fricas [A] time = 0.73, size = 297, normalized size = 1.64

$$\frac{67202918046000\,x^7 - 2238718468800\,x^6 + 186872434930060\,x^5 + 62827256425340\,x^4 + 173919793526820\,x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 186872434930060
*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20*x^7
 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan(1/23
*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 20*x^7
+ 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5*x^2 +
3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 2
36*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*x + 73812
23830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x
^2 + 84*x + 36)
```

giac [A] time = 0.22, size = 116, normalized size = 0.64

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{4284261000\,x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*x +
6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 118, normalized size = 0.65

$$\frac{2768835\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19191481364}+\frac{880575\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{7838030068}+\frac{405\ln\left(2x^2-x+3\right)}{1288408}-\frac{405\ln\left(5x^2+3x\right.}{1288408}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

```
[Out] -25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/24025*x-5423968/24
025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/19191481364*31^(1/
2)*arctan(1/31*(10*x+3)*31^(1/2))+1/644204*(302907/529*x^3-368291/529*x^2+2
501587/2116*x-665819/1058)/(2*x^2-x+3)^2+405/1288408*ln(2*x^2-x+3)+880575/7
838030068*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 138, normalized size = 0.76

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{4284261000\,x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6 + 1
72*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*x^2 +
 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

mupad [B] time = 3.59, size = 155, normalized size = 0.86

$$\frac{\frac{21421305\,x^7}{14886061058}-\frac{356802\,x^6}{7443030529}+\frac{1191332621\,x^5}{297721221160}+\frac{400530769\,x^4}{297721221160}+\frac{1108758087\,x^3}{297721221160}+\frac{938364483\,x^2}{595442442320}+\frac{1254420353\,x}{744303052900}+\frac{235280627}{1488606105800}}{x^8+\frac{x^7}{5}+\frac{321\,x^6}{100}+\frac{43\,x^5}{25}+\frac{39\,x^4}{10}+\frac{59\,x^3}{25}+\frac{241\,x^2}{100}+\frac{21\,x}{25}+\frac{9}{25}}+\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 + 405/128840
8) - log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 - 405/1
288408) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962728
 + 405/1288408) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/383
82962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^2)/59
5442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/297721221160 +
 (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305*x^7)/1
4886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 + (59*x^3)
/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/25)
```

sympy [A] time = 0.47, size = 163, normalized size = 0.90

$$\frac{4284261000x^7-142720800x^6+11913326210x^5+4005307690x^4+11087580870x^3+}{2977212211600x^8+595442442320x^7+9556851199236x^6+5120805003952x^5+11611127625240x^4+702622}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] (4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 11
087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(2977212211600
*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11611
127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 2500858257744*x
+ 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 + 3*x/5
 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/78380
30068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19191481364
```

3.58 $\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^4\,dx$

Optimal. Leaf size=208

$$-\frac{83948353\left(2x^2-x+3\right)^{3/2}x^2}{2293760}+\frac{804243809\left(2x^2-x+3\right)^{3/2}x}{36700160}+\frac{27185733541\left(2x^2-x+3\right)^{3/2}}{440401920}-\frac{359471503(1\cdots}{67\cdots}$$

```
[Out] 27185733541/440401920*(2*x^2-x+3)^(3/2)+804243809/36700160*x*(2*x^2-x+3)^(3
/2)-83948353/2293760*x^2*(2*x^2-x+3)^(3/2)+8325631/1032192*x^3*(2*x^2-x+3)^
(3/2)+4796405/43008*x^4*(2*x^2-x+3)^(3/2)+233225/1536*x^5*(2*x^2-x+3)^(3/2)
+14125/144*x^6*(2*x^2-x+3)^(3/2)+125/4*x^7*(2*x^2-x+3)^(3/2)-8267844569/268
435456*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-359471503/67108864*(1-4*x)*(2
*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7+\frac{14125}{144}\left(2x^2-x+3\right)^{3/2}x^6+\frac{233225\left(2x^2-x+3\right)^{3/2}x^5}{1536}+\frac{4796405\left(2x^2-x+3\right)^{3/2}x^4}{43008}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x +
 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (
83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^
(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3
 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^
7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1342
17728*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4 dx = \frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{20}\int \sqrt{3-x+2x^2}\left(320+1920x+7520x^2+1\right.$$

$$= \frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{360}\int \sqrt{3-x+2x^2}$$

$$= \frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2\right.$$

$$= \frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-\right.$$

$$= \frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3\right.}{15}$$

$$= -\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x}{}$$

$$= \frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x}{1}$$

$$= \frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{8394835}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{80}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{80}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{80}{}$$

Mathematica [A] time = 0.30, size = 85, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+76128080\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2 +
 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 7725962035200*
x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 26043710
39235*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/84557168640
```

fricas [A] time = 0.66, size = 98, normalized size = 0.47

$$\frac{1}{21139292160}\left(1321205760000\,x^9+3486515200000\,x^8+6327795712000\,x^7+7725962035200\,x^6+7612808\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 +
7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856*x
^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x + 3)
 + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.25, size = 93, normalized size = 0.45

$$\frac{1}{21139292160}\,(4\,(8\,(4\,(16\,(20\,(40\,(140\,(160\,(36\,x+95)x+27587)x+4715553)x+185859571)x+2614620113$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 471
5553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x + 1
34438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/268435456*
sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.03, size = 166, normalized size = 0.80

$$\frac{125\left(2x^2-x+3\right)^{\frac{3}{2}}x^7}{4}+\frac{14125\left(2x^2-x+3\right)^{\frac{3}{2}}x^6}{144}+\frac{233225\left(2x^2-x+3\right)^{\frac{3}{2}}x^5}{1536}+\frac{4796405\left(2x^2-x+3\right)^{\frac{3}{2}}x^4}{43008}+\frac{832}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 125/4*x^7*(2*x^2-x+3)^(3/2)+27185733541/440401920*(2*x^2-x+3)^(3/2)+14125/1
44*x^6*(2*x^2-x+3)^(3/2)+233225/1536*x^5*(2*x^2-x+3)^(3/2)+4796405/43008*x^
4*(2*x^2-x+3)^(3/2)+8325631/1032192*x^3*(2*x^2-x+3)^(3/2)-83948353/2293760*
x^2*(2*x^2-x+3)^(3/2)+804243809/36700160*x*(2*x^2-x+3)^(3/2)+8267844569/268
435456*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+359471503/67108864*(4*x-1)*(2
*x^2-x+3)^(1/2)
```

maxima [A] time = 1.01, size = 177, normalized size = 0.85

$$\frac{125}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^7+\frac{14125}{144}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^6+\frac{233225}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{4796405}{43008}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{832}{103}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/4*(2*x^2 - x + 3)^(3/2)*x^7 + 14125/144*(2*x^2 - x + 3)^(3/2)*x^6 + 233
225/1536*(2*x^2 - x + 3)^(3/2)*x^5 + 4796405/43008*(2*x^2 - x + 3)^(3/2)*x^
4 + 8325631/1032192*(2*x^2 - x + 3)^(3/2)*x^3 - 83948353/2293760*(2*x^2 - x
 + 3)^(3/2)*x^2 + 804243809/36700160*(2*x^2 - x + 3)^(3/2)*x + 27185733541/
440401920*(2*x^2 - x + 3)^(3/2) + 359471503/16777216*sqrt(2*x^2 - x + 3)*x
+ 8267844569/268435456*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 359471503
/67108864*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 5.03, size = 221, normalized size = 1.06

$$\frac{8325631\,x^3\left(2\,x^2-x+3\right)^{3/2}}{1032192}-\frac{83948353\,x^2\left(2\,x^2-x+3\right)^{3/2}}{2293760}+\frac{4796405\,x^4\left(2\,x^2-x+3\right)^{3/2}}{43008}+\frac{233225\,x^5\left(2\,x^2-x+\right.}{1536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (8325631*x^3*(2*x^2 - x + 3)^(3/2))/1032192 - (83948353*x^2*(2*x^2 - x + 3)
^(3/2))/2293760 + (4796405*x^4*(2*x^2 - x + 3)^(3/2))/43008 + (233225*x^5*(
2*x^2 - x + 3)^(3/2))/1536 + (14125*x^6*(2*x^2 - x + 3)^(3/2))/144 + (125*x
^7*(2*x^2 - x + 3)^(3/2))/4 - (41987163941*2^(1/2)*log((2*x^2 - x + 3)^(1/2
) + (2^(1/2)*(2*x - 1/2))/2))/1174405120 - (1825528867*(x/2 - 1/8)*(2*x^2 -
 x + 3)^(1/2))/36700160 + (27185733541*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x
+ 45))/7046430720 + (804243809*x*(2*x^2 - x + 3)^(3/2))/36700160 + (6252718
71443*2^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/9395240
960
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)
```

3.59 $\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{531681\left(2x^2-x+3\right)^{3/2}x^2}{71680}-\frac{9627393\left(2x^2-x+3\right)^{3/2}x}{1146880}-\frac{22548119\left(2x^2-x+3\right)^{3/2}}{4587520}-\frac{6766097(1-4x)\sqrt{2x^2-}}{2097152}$$

```
[Out] -22548119/4587520*(2*x^2-x+3)^(3/2)-9627393/1146880*x*(2*x^2-x+3)^(3/2)+531
681/71680*x^2*(2*x^2-x+3)^(3/2)+247435/10752*x^3*(2*x^2-x+3)^(3/2)+8825/448
*x^4*(2*x^2-x+3)^(3/2)+125/16*x^5*(2*x^2-x+3)^(3/2)-155620231/8388608*arcsi
nh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-6766097/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{3/2}x^4+\frac{247435\left(2x^2-x+3\right)^{3/2}x^3}{10752}+\frac{531681\left(2x^2-x+3\right)^{3/2}x^2}{71680}-\frac{96}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2
)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*
(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (
8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 -
(155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3 dx = \frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}\int \sqrt{3-x+2x^2}\left(128+576x+1824x^2+33\right.$$

$$= \frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2} + \frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2} + \frac{1}{224}\int \sqrt{3-x+2x^2}$$

$$= \frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752} + \frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2} + \frac{125}{16}x^5\left(3-x+2x\right.$$

$$= \frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680} + \frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752} + \frac{8825}{448}x^4\left(3-x\right.$$

$$= -\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880} + \frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680} + \frac{247435x^3\left(3\right.}{107}$$

$$= -\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880} + \frac{531681x^2\left(3\right.}{716}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{9627393x}{1}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{9627393x}{1}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{9627393x}{1}$$

Mathematica [A] time = 0.17, size = 75, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+458381\right.}{880803840}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 98721
63456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 3440640000
*x^7) - 16340124255*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/880803840
```

fricas [A] time = 0.90, size = 88, normalized size = 0.53

$$\frac{1}{220200960}\left(3440640000\,x^7+6955008000\,x^6+10958233600\,x^5+11212171264\,x^4+9872163456\,x^3+4583812\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 1121217126
4*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt(2
```

```
*x^2 - x + 3) + 155620231/16777216*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.25, size = 83, normalized size = 0.50

$$\frac{1}{220200960}\left(4\left(8\left(4\left(16\left(100\left(120\left(140x+283\right)x+53507\right)x+5474693\right)x+77126277\right)x+143244129\right)x-4053\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x
+ 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*sqrt(2*x^2 - x +
3) - 155620231/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
 3)) + 1)
```

maple [A] time = 0.01, size = 132, normalized size = 0.80

$$\frac{125\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{5}}{16}+\frac{8825\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{4}}{448}+\frac{247435\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{3}}{10752}+\frac{531681\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{2}}{71680}-\frac{96273}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x)
```

```
[Out] -22548119/4587520*(2*x^2-x+3)^(3/2)+125/16*(2*x^2-x+3)^(3/2)*x^5+8825/448*(
2*x^2-x+3)^(3/2)*x^4+247435/10752*(2*x^2-x+3)^(3/2)*x^3+531681/71680*(2*x^2
-x+3)^(3/2)*x^2-9627393/1146880*(2*x^2-x+3)^(3/2)*x+155620231/8388608*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))+6766097/2097152*(4*x-1)*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.98, size = 143, normalized size = 0.86

$$\frac{125}{16}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{5}+\frac{8825}{448}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{4}+\frac{247435}{10752}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{3}+\frac{531681}{71680}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{2}-\frac{96273}{11468}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 247
435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*x^
2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 - x +
 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 4.69, size = 187, normalized size = 1.13

$$\frac{531681x^{2}\left(2x^{2}-x+3\right)^{3/2}}{71680}+\frac{247435x^{3}\left(2x^{2}-x+3\right)^{3/2}}{10752}+\frac{8825x^{4}\left(2x^{2}-x+3\right)^{3/2}}{448}+\frac{125x^{5}\left(2x^{2}-x+3\right)^{3/2}}{16}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (531681*x^2*(2*x^2 - x + 3)^(3/2))/71680 + (247435*x^3*(2*x^2 - x + 3)^(3/2
))/10752 + (8825*x^4*(2*x^2 - x + 3)^(3/2))/448 + (125*x^5*(2*x^2 - x + 3)^
(3/2))/16 + (875316037*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x -
1/2))/2))/36700160 + (38057219*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1146880 -
 (22548119*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/73400320 - (9627393*x
```

```
*(2*x^2 - x + 3)^(3/2))/1146880 - (1555820211*2^(1/2)*log(2*(2*x^2 - x + 3)
^(1/2) + (2^(1/2)*(4*x - 1))/2))/293601280
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} \left(5x^2 + 3x + 2\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3, x)
```

3.60 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx$

Optimal. Leaf size=124

$$\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{25}{12}\left(2x^2-\right.$$

```
[Out] -2107/3072*(2*x^2-x+3)^(3/2)+769/256*x*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-x
+3)^(3/2)+25/12*x^3*(2*x^2-x+3)^(3/2)+284533/65536*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)+12371/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{}}{1638}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/
3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/1
6 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/(32768*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
```

```
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2 dx = \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int \sqrt{3-x+2x^2}\left(48+144x+123x^2+\frac{945x^3}{2}\right.$$

$$= \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2}+\frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int \sqrt{3-x+2x^2}\left(48\right.$$

$$= \frac{769}{256}x\left(3-x+2x^2\right)^{3/2}+\frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2}+\frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2}+$$

$$= -\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-x+2x^2\right)^{3/2}+\frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2}+$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-x+2x^2\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-x+2x^2\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-x+2x^2\right.$$

Mathematica [A] time = 0.10, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023\right)+853599\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 28467
2*x^4 + 204800*x^5) + 853599*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/196608
```

fricas [A] time = 0.85, size = 78, normalized size = 0.63

$$\frac{1}{49152}\left(204800\,x^5+284672\,x^4+408960\,x^3+365536\,x^2+328204\,x-64023\right)\sqrt{2\,x^2-x+3}+\frac{284533}{131072}\sqrt{2}\log\left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 640
23)*sqrt(2*x^2 - x + 3) + 284533/131072*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.22, size = 73, normalized size = 0.59

$$\frac{1}{49152}\left(4\left(8\left(4\left(16\left(100\,x+139\right)x+3195\right)x+11423\right)x+82051\right)x-64023\right)\sqrt{2\,x^2-x+3}+\frac{284533}{65536}\sqrt{2}\log\left(-2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 6402
3)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 98, normalized size = 0.79

$$\frac{25\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{12}+\frac{63\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{16}+\frac{769\left(2x^2-x+3\right)^{\frac{3}{2}}x}{256}-\frac{284533\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}-\frac{2107\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 25/12*(2*x^2-x+3)^(3/2)*x^3+63/16*(2*x^2-x+3)^(3/2)*x^2+769/256*(2*x^2-x+3)
^(3/2)*x-2107/3072*(2*x^2-x+3)^(3/2)-12371/16384*(4*x-1)*(2*x^2-x+3)^(1/2)-
284533/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.98, size = 109, normalized size = 0.88

$$\frac{25}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{2107}{3072}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{12371}{4096}\sqrt{2x^2-x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/256
*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*sqr
t(2*x^2 - x + 3)*x - 284533/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 12371/16384*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 4.19, size = 153, normalized size = 1.23

$$\frac{63x^2\left(2x^2-x+3\right)^{3/2}}{16}+\frac{25x^3\left(2x^2-x+3\right)^{3/2}}{12}-\frac{29509\sqrt{2}\ \ln\left(\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{8192}-\frac{1283\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (63*x^2*(2*x^2 - x + 3)^(3/2))/16 + (25*x^3*(2*x^2 - x + 3)^(3/2))/12 - (29
509*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/8192 - (1
283*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/256 - (2107*(2*x^2 - x + 3)^(1/2)*(3
2*x^2 - 4*x + 45))/49152 + (769*x*(2*x^2 - x + 3)^(3/2))/256 - (48461*2^(1/
2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/65536
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2, x)
```

3.61 $\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)\,dx$

Optimal. Leaf size=82

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

```
[Out] 73/96*(2*x^2-x+3)^(3/2)+5/8*x*(2*x^2-x+3)^(3/2)-1863/2048*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)-81/512*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (
5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqr
t[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)\,dx = \frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{1}{8}\int\left(1+\frac{73x}{2}\right)\sqrt{3-x+2x^2}\,dx$$
$$=\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{81}{64}\int\sqrt{3-x+2x^2}\,dx$$
$$=-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}$$
$$=-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}$$
$$=-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.67

$$\frac{4\sqrt{2x^2-x+3}\,\left(1920x^3+1376x^2+2684x+3261\right)-5589\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{6144}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2]
*ArcSinh[(1 - 4*x)/Sqrt[23]])/6144
```

fricas [A] time = 1.01, size = 68, normalized size = 0.83

$$\frac{1}{1536}\left(1920\,x^3+1376\,x^2+2684\,x+3261\right)\sqrt{2\,x^2-x+3}+\frac{1863}{4096}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/409
6*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

giac [A] time = 0.24, size = 63, normalized size = 0.77

$$\frac{1}{1536}\,(4\,(8\,(60\,x+43)x+671)x+3261)\sqrt{2\,x^2-x+3}-\frac{1863}{2048}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/2048
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 64, normalized size = 0.78

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x}{8}+\frac{1863\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}+\frac{73\left(2x^2-x+3\right)^{\frac{3}{2}}}{96}+\frac{81\,(4x-1)\sqrt{2x^2-x+3}}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/8*(2*x^2-x+3)^(3/2)*x+73/96*(2*x^2-x+3)^(3/2)+81/512*(4*x-1)*(2*x^2-x+3)^
(1/2)+1863/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 75, normalized size = 0.91

$$\frac{5}{8}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{73}{96}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{81}{128}\sqrt{2x^2-x+3}\,x+\frac{1863}{2048}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{81}{512}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*x
^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/512
*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 3.84, size = 119, normalized size = 1.45

$$\frac{23\sqrt{2}\,\ln\left(\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{256}+\frac{\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^2-x+3}}{8}+\frac{73\sqrt{2x^2-x+3}\,\left(32x^2-4x+45\right)}{1536}+\frac{5x\left(2x^2-x+3\right)^{3/2}}{8}+\frac{1679\sqrt{2}\,\ln\left(2\sqrt{2x^2-x+3}+\frac{\sqrt{2}\,(4x-1)}{2}\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] (23*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/256 + ((x
/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/8 + (73*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*
x + 45))/1536 + (5*x*(2*x^2 - x + 3)^(3/2))/8 + (1679*2^(1/2)*log(2*(2*x^2
- x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/2048
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2), x)
```

3.62 $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=174

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\sqrt{\frac{}{62(}}\right.$$

```
[Out] -1/5*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/155*arctanh(1/62*(6+x*(20-13*
2^(1/2))-7*2^(1/2))*682^(1/2)/(-13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-4
433+3410*2^(1/2))^(1/2)+1/155*arctan(1/62*(6+7*2^(1/2)+x*(20+13*2^(1/2)))*6
82^(1/2)/(13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(4433+3410*2^(1/2))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {989, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\sqrt{\frac{}{62(}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 + (Sqrt[(11*(13 + 10*Sqrt[2]))/31]
*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2])
*x))/Sqrt[3 - x + 2*x^2]])/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))/31]*ArcTanh[(S
qrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt
[3 - x + 2*x^2]])/5
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
```

```
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx &= -\left(\frac{1}{5}\int \frac{-11+11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx\right)+\frac{2}{5}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{1}{5}\sqrt{\frac{2}{23}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)+\frac{\int \frac{121\left(2+\sqrt{2}\right)-121\sqrt{2}\,x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{110\sqrt{2}}-\frac{\int \frac{121\left(2-\sqrt{2}\right)+121\sqrt{}}{\sqrt{3-x+2x^2}\left(2+3x+\right.}}{110\sqrt{2}} \\
&= -\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)-\frac{1}{5}\left(1331\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-907742\left(13-10\sqrt{2}\right)-11x^2}\right. \\
&= -\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+1\right.\right.}{\sqrt{3-x+2x^2}}\right.
\end{aligned}$$

Mathematica [C] time = 0.46, size = 185, normalized size = 1.06

$$-\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)-\frac{1}{5}i\sqrt{\frac{11}{62}}\left(\sqrt{13+i\sqrt{31}}\,\tanh^{-1}\left(\frac{-4i\sqrt{31}\,x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\,\sqrt{2x^2-x+3}}\right)-\sqrt{13-i\sqrt{31}}\,\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -1/5*(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]) - (I/5)*Sqrt[11/62]*(Sqrt[13 + I
*Sqrt[31]]*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286
+ (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - Sqrt[13 - I*Sqrt[31]]*ArcTanh[(6
```

```
3 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sq
rt[3 - x + 2*x^2])])
```

fricas [B] time = 2.11, size = 2016, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/8
9125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19541*x^6 + 40352*x^5
- 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7 - 14488*x^6 + 30919
*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 2764
8) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096*
x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 - 280047*x^6 + 1453472
*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 398
1312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 253000*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*sqrt(31)*(15454*x^7
- 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - sqrt(2)*(1543
8*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x
) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 1000024*x^6 + 3868360*x^5
 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(69479*x^7 - 898236*x^
6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 24
05376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 550*sqrt(31)*sqrt(2)*
(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*
x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x
^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25
*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087819
20*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517
*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*
sqrt(-(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)
*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*sqrt(2)*(2*x^2 - x + 3) + 755*x - 10
00)/x^2) + 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835
344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348
*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 +
4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 1419
1920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 247726
08*x + 18579456)) + 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt
(2) + 20)*arctan(1/89125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19
541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7
 - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 172
80) - 34560*x + 27648) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 19
14360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 -
280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 306892
8*x + 1990656) - 3981312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2)
+ 20) - 253000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385
256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335
*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 54
6048*x - 539136) + 1154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*
sqrt(31)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824
*x^2 - sqrt(2)*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 +
 44928*x^2 - 44928*x) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 10000
24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(6
9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
```

```
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456)) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*s
qrt(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 1/10*sqrt(2)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 5.57
Done
```

maple [B] time = 0.15, size = 2065, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1/52855*(8*(2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(
1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549
362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^
(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+
1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+386*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^
(1/2))^(1/2)-274846*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^
(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^
(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2
```

```
))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(
1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+1/21142*(
8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(
1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-
1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/
(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*
(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+2
18*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))
^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/
2)))*(-775687+549362*2^(1/2))^(1/2)+401698*arctanh(31/2*(8*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2
)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/
2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1
-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(
1/2))^(1/2)+3/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2
^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^
(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(
1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+54
9362*2^(1/2))^(1/2)+520*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-7756
87+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1
0368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)
^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^
(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+465124*arctanh(31/
2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(3
1/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2
^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^
(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

3.63 $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\right.}{\sqrt{2x^2-x+3}}\right.$$

```
[Out] 1/31*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/42284*arctanh(1/31*(419+x*(
973-696*2^(1/2))-277*2^(1/2))*341^(1/2)/(-70517+49942*2^(1/2))^(1/2)/(2*x^2
-x+3)^(1/2))*(-48092594+34060444*2^(1/2))^(1/2)+1/42284*arctan(1/31*(419+27
7*2^(1/2)+x*(973+696*2^(1/2)))*341^(1/2)/(70517+49942*2^(1/2))^(1/2)/(2*x^2
-x+3)^(1/2))*(48092594+34060444*2^(1/2))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {971, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\right.}{\sqrt{2x^2-x+3}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49
942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 277*
Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517
+ 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419
- 277*Sqrt[2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{-\frac{63}{2}+11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{11}{2}\left(85-63\sqrt{2}\right)-\frac{11}{2}\left(41-22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}\left(85+63\sqrt{2}\right)-\frac{11}{2}\left(41+22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,d\ldots}{682\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{1}{248}\left(11\left(99884-70517\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{3751}{4}\left(70517-4994\ldots\right.}\ldots\right.$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} + \frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(4\ldots\right.}{\ldots}\right.$$

Mathematica [C] time = 1.04, size = 214, normalized size = 1.14

$$\frac{\frac{27280\sqrt{2x^2-x+3}\,(10x+3)}{5x^2+3x+2} + i\sqrt{286-22i\sqrt{31}}\left(973\sqrt{31}+1271i\right)\tanh^{-1}\left(\frac{4i\sqrt{31}\,x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\,\sqrt{2x^2-x+3}}\right) - i\sqrt{286+22i\sqrt{31}}\,\left(\ldots\right.}{845680}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((27280*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) + I*Sqrt[286 - (2
2*I)*Sqrt[31]]*(1271*I + 973*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4
*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - I*Sq
rt[286 + (22*I)*Sqrt[31]]*(-1271*I + 973*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
 + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])])/845680
```

fricas [B] time = 2.37, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/186703822445536*(88412*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5
*x^2 + 3*x + 2)*sqrt(70517*sqrt(2) + 99884)*arctan(1/10668926457462302923*(
3096404*sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^
6 + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014
*x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2
- 2633472*x + 1893888) - 3787776*x + 2633472) + 774101*4988406728^(1/4)*sqr
t(341)*(40625*x^7 - 622509*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 -
5992992*x^2 - sqrt(2)*(28204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 +
 6800832*x^3 - 4341024*x^2 - 4810752*x + 3442176) - 6884352*x + 4810752))*s
qrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) + 30285984782473634104*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(3
41)*(84604*x^7 - 121310*x^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624
*x^2 - sqrt(2)*(57082*x^7 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^
3 + 110592*x^2 - 110592*x) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*
(6379*x^7 - 82508*x^6 + 318020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 -
 sqrt(2)*(4365*x^7 - 56468*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207
360*x^2 - 207360*x) - 331776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 9
9884) + 425261673562*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^
6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^
8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209
600*x^2 - 1036800*x) + 3276288*x) + 19330076071*sqrt(31)*(254591*x^8 - 4815
126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689
56928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 -
 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4988406728^(1/4)*sqrt
(24971)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x -
 7)*sqrt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^
2 - x + 3) + 3623566781*x - 4799426200)/x^2) + 344158917982654933*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 88412
*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(7051
7*sqrt(2) + 99884)*arctan(1/10668926457462302923*(3096404*sqrt(24971)*(11*4
988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^6 + 4310846*x^5 - 6853210
*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014*x^7 - 1438653*x^6 + 3014
868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 - 2633472*x + 1893888) -
3787776*x + 2633472) + 774101*4988406728^(1/4)*sqrt(341)*(40625*x^7 - 62250
9*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 - sqrt(2)*(28
204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 - 4341024*x^
2 - 4810752*x + 3442176) - 6884352*x + 4810752))*sqrt(2*x^2 - x + 3)*sqrt(7
0517*sqrt(2) + 99884) - 30285984782473634104*sqrt(31)*sqrt(2)*(28180*x^8 -
254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^
2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sq
rt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(84604*x^7 - 121310*x
^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - sqrt(2)*(57082*x^7
 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x
) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*(6379*x^7 - 82508*x^6 + 3
18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - sqrt(2)*(4365*x^7 - 5646
8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331
776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) - 425261673562*sqrt
(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*
```

```
x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x
^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32
76288*x) - 19330076071*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 -
90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)
*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19
44*x) + 144820224*x))*sqrt((4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(31)
*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sqrt(70517*sqrt(2) + 9
9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4
- 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*
x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3
 - 34615296*x^2 - 24772608*x + 18579456)) - 4988406728^(1/4)*sqrt(24971)*sq
rt(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x +
 199768)*sqrt(70517*sqrt(2) + 99884)*log(199768*(4988406728^(1/4)*sqrt(2497
1)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*s
qrt(70517*sqrt(2) + 99884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x
 + 3) - 3623566781*x + 4799426200)/x^2) + 4988406728^(1/4)*sqrt(24971)*sqrt
(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x + 1
99768)*sqrt(70517*sqrt(2) + 99884)*log(-199768*(4988406728^(1/4)*sqrt(24971
)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^2 - x
+ 3) + 3623566781*x - 4799426200)/x^2) - 6022703949856*sqrt(2*x^2 - x + 3)*
(10*x + 3))/(5*x^2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 17.67Done
```

maple [B] time = 0.23, size = 16357, normalized size = 87.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{\left(5\,x^2 + 3\,x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}}{{\left(5\,x^2+3\,x+2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}}{\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

3.64 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{}{31(}}\right.}{16}$$

```
[Out] 1/62*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+1/84568*(3464+13665*x)*(2*x
^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/115350752*arctanh(1/31*(509587+x*(1235163-872
375*2^(1/2))-362788*2^(1/2))*341^(1/2)/(-112285869463+79399380740*2^(1/2))^
(1/2)/(2*x^2-x+3)^(1/2))*(-76578962973766+54150377664680*2^(1/2))^(1/2)+1/1
15350752*arctan(1/31*(509587+362788*2^(1/2)+x*(1235163+872375*2^(1/2)))*341
^(1/2)/(112285869463+79399380740*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(7657896
2973766+54150377664680*2^(1/2))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {971, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{}{31(}}\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*
x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 7
9399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sq
rt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*Arc
Tanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*
Sqrt[2] + (1235163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/169136
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{-\frac{183}{2}+31x-40x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\int \frac{-213004+\frac{358655x}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{465124}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{121}{4}\left(110061-77456\sqrt{2}\right)-\frac{121}{4}(448}{\sqrt{3-x+2x^2}\,(2+3x+5}}{10232728\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\left(11\left(158798761480-11228\right.\right.}{}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(112285869463+79\right.}}{}$$

Mathematica [C] time = 2.08, size = 299, normalized size = 1.34

$$\frac{5\left(\frac{i\sqrt{286+22i\sqrt{31}}\left(258253\sqrt{31}+1004586i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2} + \frac{2000\left(1364\left(\sqrt{31}+13i\right)\sqrt{2x^2-x+3}\left(68325x^3+58315x^2+51362x+11\right.\right.}{\left(\sqrt{31}-13i\right)\left(\sqrt{3}\right.}\right.}{14418844}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (5*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1004586*I + 258253*Sqrt[31])*ArcTanh[(6
3 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*S
qrt[3 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + (2000*(1364*(13*I + Sqrt[31])*
Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3) - 5*Sqrt[286
- (22*I)*Sqrt[31]]*(-202151*I + 174475*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTan
h[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31
]]*Sqrt[3 - x + 2*x^2])]))/((-13*I + Sqrt[31])*(13*I + Sqrt[31])^2*(-3*I +
Sqrt[31] - (10*I)*x)^2*(3*I + Sqrt[31] + (10*I)*x)^2)))/14418844
```

fricas [B] time = 2.82, size = 2182, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/6505215189695292642599671424O*(14205421276*788032707736935368450^(1/4)*sq
rt(39699690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqr
t(112285869463*sqrt(2) + 158798761480)*arctan(1/861047662213971287591057659
551879544939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^
(3/4)*sqrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 86714302
12*x^4 + 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195
871*x^6 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2
```

```
 - 3352976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*7
88032707736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 411636
7112*x^5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(359
38543*x^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^
3 - 5403919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216
))*sqrt(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) + 24442643
31446112042193970130340819353377000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^
7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(
2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*
x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(396996
90370/160673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*
(104024992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*
x^3 + 214417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5
 - 133050960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152
*x) + 615345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102
233612*x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2
 - 4*sqrt(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116
274240*x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)
*sqrt(112285869463*sqrt(2) + 158798761480) + 4317591252432386621143695850*
sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396
480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2440
47*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
+ 3276288*x) + 1962541478378357555051986175*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(788032707736935368450^(1/
4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053
*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) - 15
0182556985858180945*x^2 - 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) + 4
62807471527848680055*x - 612990028513706861000)/x^2) + 27775731039160364115
840569662963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
+ 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 14205421276*788032707736935368450^(1/4)*sqrt(3969
9690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(11228
5869463*sqrt(2) + 158798761480)*arctan(1/8610476622139712875910576595518795
44939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^(3/4)*s
qrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 8671430212*x^4
+ 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195871*x^6
 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2 - 3352
976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*78803270
7736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 4116367112*x^
5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(35938543*x
^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^3 - 540
3919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216))*sqrt
(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) - 244426433144611
2042193970130340819353377000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(39699690370/1
60673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*(104024
992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*x^3 + 2
14417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5 - 1330
50960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152*x) + 6
15345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102233612*
x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2 - 4*sqr
t(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116274240*
```

```
x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)*sqrt(1
12285869463*sqrt(2) + 158798761480) - 43175912524323866211143695850*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 19625414783783575550519861 75*sqrt(31)*(254591*x^8 - 4815126*x^7 + 3
2303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 -
 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 +
 2268*x^2 - 1944*x) + 144820224*x))*sqrt((788032707736935368450^(1/4)*sqrt(
39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053*x + 513
8) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) + 1501825569
85858180945*x^2 + 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) - 462807471
527848680055*x + 612990028513706861000)/x^2) - 27775731039160364115840569 66
2963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*s
qrt(31)*(3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285
869463*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63
5195045920)*sqrt(112285869463*sqrt(2) + 158798761480)*log(635195045920/1606
73*(788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2
*x^2 - x + 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463
*sqrt(2) + 158798761480) + 150182556985858180945*x^2 + 13485780627301550942
0*sqrt(2)*(2*x^2 - x + 3) - 462807471527848680055*x + 612990028513706861000
)/x^2) - 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*(
3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285869463*sq
rt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63519504592
0)*sqrt(112285869463*sqrt(2) + 158798761480)*log(-635195045920/160673*(7880
32707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2)
 + 158798761480) - 150182556985858180945*x^2 - 134857806273015509420*sqrt(2
)*(2*x^2 - x + 3) + 462807471527848680055*x - 612990028513706861000)/x^2) +
 769228926981280465731680*(68325*x^3 + 58315*x^2 + 51362*x + 11020)*sqrt(2*
x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 48.06Done
```

maple [B] time = 0.37, size = 44343, normalized size = 198.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2-x+3}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^2-x+3}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3,x)
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

3.65 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=231

$$-\frac{56422489\left(2x^2-x+3\right)^{5/2}x^2}{8257536}+\frac{48669967\left(2x^2-x+3\right)^{5/2}x}{22020096}+\frac{2124689283\left(2x^2-x+3\right)^{5/2}}{146800640}-\frac{382121949(1-4x)}{134217}$$

```
[Out] -382121949/134217728*(1-4*x)*(2*x^2-x+3)^(3/2)+2124689283/146800640*(2*x^2-
x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2*x^
2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x^2-x
+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)+625
/24*x^7*(2*x^2-x+3)^(5/2)-606427533063/8589934592*arcsinh(1/23*(1-4*x)*23^(
1/2))*2^(1/2)-26366414481/2147483648*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7+\frac{7625}{96}\left(2x^2-x+3\right)^{5/2}x^6+\frac{95165}{768}\left(2x^2-x+3\right)^{5/2}x^5+\frac{941905\left(2x^2-x+3\right)^{5/2}x^4}{9216}+\frac{10444}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4
*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/1
46800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 -
 x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 +
(941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))
/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2)
)/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^4 dx = \frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{24}\int\left(3-x+2x^2\right)^{3/2}\left(384+2304x+902\right.$$

$$=\frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{528}\int\left(3-x+\right.$$

$$=\frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x\right.$$

$$=\frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3\right.$$

$$=\frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+\frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x$$

$$=-\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+\frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+941$$

$$=\frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+10444$$

$$=\frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}+\frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-5642$$

$$=-\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+\frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}$$

$$=-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+\right.}{134217728}$$

$$=-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+\right.}{134217728}$$

$$=-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+\right.}{134217728}$$

Mathematica [A] time = 0.37, size = 95, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^9+534038708224000x^8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712*
x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 765
```

```
087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651174
400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/2705829396480
```

fricas [A] time = 0.67, size = 108, normalized size = 0.47

$$\frac{1}{676457349120}\left(70464307200000\,x^{11} + 144451829760000\,x^{10} + 349379651174400\,x^{9} + 534038708224000\,x^{8} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 34937965117440
0*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 + 6
75479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 6515199806
3712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 6064275
33063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32
*x^2 + 16*x - 25)
```

giac [A] time = 0.25, size = 103, normalized size = 0.45

$$\frac{1}{676457349120}\,(4\,(8\,(4\,(16\,(20\,(8\,(28\,(160\,(12\,(200\,(20\,x + 41)x + 19833)x + 363785)x + 81213077)x + 23348604$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833)
*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 22049872180
7)*x + 1867353001743)*x + 2035999939491)*x + 3242793881079)*x + 74032009514
181)*sqrt(2*x^2 - x + 3) - 606427533063/8589934592*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.04, size = 185, normalized size = 0.80

$$\frac{625\left(2x^2-x+3\right)^{\frac{5}{2}}x^7}{24}+\frac{7625\left(2x^2-x+3\right)^{\frac{5}{2}}x^6}{96}+\frac{95165\left(2x^2-x+3\right)^{\frac{5}{2}}x^5}{768}+\frac{941905\left(2x^2-x+3\right)^{\frac{5}{2}}x^4}{9216}+\frac{10444117\,(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 625/24*x^7*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)+2124689283/14680
0640*(2*x^2-x+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x
^2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2
*x^2-x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)+606427533063/85899345
92*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+26366414481/2147483648*(4*x-1)*(2
*x^2-x+3)^(1/2)+382121949/134217728*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 1.02, size = 206, normalized size = 0.89

$$\frac{625}{24}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^7+\frac{7625}{96}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^6+\frac{95165}{768}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{941905}{9216}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{10444117}{294912}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 9516
5/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 + 1
```

```
0444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x + 3)
^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/1468006
40*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x - 382
121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2*x^2 -
 x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)
) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{2} + 3x + 2\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**4, x)
```

3.66 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=189

$$\frac{384739 \left(2x^2-x+3\right)^{5/2} x^2}{43008}-\frac{81685 \left(2x^2-x+3\right)^{5/2} x}{114688}-\frac{4625907 \left(2x^2-x+3\right)^{5/2}}{2293760}-\frac{667795(1-4x) \left(2x^2-x+3\right)^{3/2}}{2097152}$$

```
[Out] -667795/2097152*(1-4*x)*(2*x^2-x+3)^(3/2)-4625907/2293760*(2*x^2-x+3)^(5/2)
-81685/114688*x*(2*x^2-x+3)^(5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)+27785/
1536*x^3*(2*x^2-x+3)^(5/2)+725/48*x^4*(2*x^2-x+3)^(5/2)+25/4*x^5*(2*x^2-x+3
)^(5/2)-1059790665/134217728*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4607785
5/33554432*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2} x^5+\frac{725}{48}\left(2x^2-x+3\right)^{5/2} x^4+\frac{27785 \left(2x^2-x+3\right)^{5/2} x^3}{1536}+\frac{384739 \left(2x^2-x+3\right)^{5/2} x^2}{43008}-\frac{81685 \left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 -
 x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (816
85*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/430
08 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2
))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(67108864*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx = \frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}+\frac{1}{20}\int \left(3-x+2x^2\right)^{3/2}\left(160+720x+2280x\right.$$

$$= \frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2}+\frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}+\frac{1}{360}\int \left(3-x+2x\right.$$

$$= \frac{27785x^3\left(3-x+2x^2\right)^{5/2}}{1536}+\frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2}+\frac{25}{4}x^5\left(3-x+\right.$$

$$= \frac{384739x^2\left(3-x+2x^2\right)^{5/2}}{43008}+\frac{27785x^3\left(3-x+2x^2\right)^{5/2}}{1536}+\frac{725}{48}x^4\left(3-\right.$$

$$= -\frac{81685x\left(3-x+2x^2\right)^{5/2}}{114688}+\frac{384739x^2\left(3-x+2x^2\right)^{5/2}}{43008}+\frac{27785x^3\left(\right.}{1}$$

$$= -\frac{4625907\left(3-x+2x^2\right)^{5/2}}{2293760}-\frac{81685x\left(3-x+2x^2\right)^{5/2}}{114688}+\frac{384739x^2\left(\right.}{43}$$

$$= -\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625907\left(3-x+2x^2\right)^{5/2}}{2293760}-\frac{816}{}$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}$$

Mathematica [A] time = 0.23, size = 85, normalized size = 0.45

$$4\sqrt{2x^2-x+3}\left(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 + 3
89257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6 +
328328806400*x^7 + 124780544000*x^8 + 88080384000*x^9) - 111278019825*Sqrt[
2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/14092861440
```

fricas [A] time = 0.99, size = 98, normalized size = 0.52

$$\frac{1}{3523215360}\left(88080384000\,x^9+124780544000\,x^8+328328806400\,x^7+430820229120\,x^6+571298324480\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 43082
0229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 1996
15064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 105979066
5/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
 16*x - 25)
```

giac [A] time = 0.26, size = 93, normalized size = 0.49

$$\frac{1}{3523215360}\left(4\left(8\left(4\left(16\left(20\left(8\left(140\left(160\left(12x+17\right)x+7157\right)x+1314759\right)x+13947713\right)x+238228459\right)x+304\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 131475
9)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 1349635
8003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2)*l
og(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 151, normalized size = 0.80

$$\frac{25\left(2x^2-x+3\right)^{\frac{5}{2}}x^5}{4}+\frac{725\left(2x^2-x+3\right)^{\frac{5}{2}}x^4}{48}+\frac{27785\left(2x^2-x+3\right)^{\frac{5}{2}}x^3}{1536}+\frac{384739\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{43008}-\frac{81685\left(2x^2-\right.}{1146}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x)
```

```
[Out] -4625907/2293760*(2*x^2-x+3)^(5/2)+25/4*(2*x^2-x+3)^(5/2)*x^5+725/48*(2*x^2
-x+3)^(5/2)*x^4+27785/1536*(2*x^2-x+3)^(5/2)*x^3+384739/43008*(2*x^2-x+3)^(
5/2)*x^2-81685/114688*(2*x^2-x+3)^(5/2)*x+1059790665/134217728*2^(1/2)*arcs
inh(4/23*23^(1/2)*(x-1/4))+46077855/33554432*(4*x-1)*(2*x^2-x+3)^(1/2)+6677
95/2097152*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 0.99, size = 172, normalized size = 0.91

$$\frac{25}{4}\left(2x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{27785}{1536}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{384739}{43008}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{81685}{114688}\left(2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/1
536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 - 81
685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/2)
+ 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3)^(3
/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sqrt(2)
*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3,x)
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^2 + 3x + 2\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3, x)
```

3.67 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=147

$$\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499\left(2x^2-x+3\right)^{5/2}x}{10752}+\frac{73861\left(2x^2-x+3\right)^{5/2}}{215040}+\frac{24293(1-4x)\left(2x^2-x+3\right)^{3/2}}{196608}+\frac{558}{}$$

```
[Out] 24293/196608*(1-4*x)*(2*x^2-x+3)^(3/2)+73861/215040*(2*x^2-x+3)^(5/2)+24499
/10752*x*(2*x^2-x+3)^(5/2)+1235/448*x^2*(2*x^2-x+3)^(5/2)+25/16*x^3*(2*x^2-
x+3)^(5/2)+12850997/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+558739/1
048576*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499\left(2x^2-x+3\right)^{5/2}x}{10752}+\frac{73861\left(2x^2-x+3\right)^{5/2}}{215040}+\frac{24293(1-4}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 -
 x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(
3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*
Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2 dx = \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\left(64+192x+239x^2+\right.$$

$$= \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{1}{224}\int \left(3-x+2\right.$$

$$= \frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752} + \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{25}{16}x^3\left(3-x+\right.$$

$$= \frac{73861\left(3-x+2x^2\right)^{5/2}}{215040} + \frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752} + \frac{1235}{448}x^2\left(3-x+\right.$$

$$= \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{73861\left(3-x+2x^2\right)^{5/2}}{215040} + \frac{24499x\left(\right.}{}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{73}{}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{73}{}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{73}{}$$

Mathematica [A] time = 0.14, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}\left(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647\right.}{440401920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 2728413
312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7)
+ 1349354685*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/440401920
```

fricas [A] time = 0.80, size = 88, normalized size = 0.60

$$\frac{1}{110100480}\left(688128000\,x^7+525926400\,x^6+2025840640\,x^5+2061273088\,x^4+2728413312\,x^3+17996471\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x^
4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^2
- x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.26, size = 83, normalized size = 0.56

$$\frac{1}{110100480}\left(4\left(8\left(4\left(16\left(20\left(120\left(140\,x+107\right)x+49459\right)x+1006481\right)x+21315729\right)x+56238973\right)x+40485085\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x +
 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3)
+ 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
 + 1)
```

maple [A] time = 0.01, size = 117, normalized size = 0.80

$$\frac{25\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{3}}{16}+\frac{1235\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{2}}{448}+\frac{24499\left(2x^{2}-x+3\right)^{\frac{5}{2}}x}{10752}-\frac{12850997\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{738}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 73861/215040*(2*x^2-x+3)^(5/2)+25/16*(2*x^2-x+3)^(5/2)*x^3+1235/448*(2*x^2-
x+3)^(5/2)*x^2+24499/10752*(2*x^2-x+3)^(5/2)*x-12850997/4194304*2^(1/2)*arc
sinh(4/23*23^(1/2)*(x-1/4))-558739/1048576*(4*x-1)*(2*x^2-x+3)^(1/2)-24293/
196608*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 0.98, size = 138, normalized size = 0.94

$$\frac{25}{16}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x^{3}+\frac{1235}{448}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x^{2}+\frac{24499}{10752}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x+\frac{73861}{215040}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}-\frac{24293}{49152}\left(2\,x^{2}-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 2449
9/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 2429
3/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 5587
39/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23*sqr
t(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{3}{2}}\left(5x^{2}+3x+2\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)
```

3.68 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right) dx$

Optimal. Leaf size=105

$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}+\frac{107}{240}\left(2x^2-x+3\right)^{5/2}-\frac{179(1-4x)\left(2x^2-x+3\right)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\,\mathrm{s}}{16}$$

```
[Out] -179/1536*(1-4*x)*(2*x^2-x+3)^(3/2)+107/240*(2*x^2-x+3)^(5/2)+5/12*x*(2*x^2
-x+3)^(5/2)-94691/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4117/8192*(1
-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}+\frac{107}{240}\left(2x^2-x+3\right)^{5/2}-\frac{179(1-4x)\left(2x^2-x+3\right)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\,\mathrm{s}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)
^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2)
)/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)dx = \frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{1}{12}\int\left(9+\frac{107x}{2}\right)\left(3-x+2x^2\right)^{3/2}dx$$
$$= \frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{179}{96}\int\left(3-x+2x^2\right)^{3/2}dx$$
$$= -\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right.$$
$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x\right.$$
$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x\right.$$
$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x\right.$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.62

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5+14336x^4+561024x^3+319072x^2+565276x+388341\right)-1420365\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.}{491520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 14336
*x^4 + 204800*x^5) - 1420365*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/491520
```

fricas [A] time = 0.74, size = 78, normalized size = 0.74

$$\frac{1}{122880}\left(204800\,x^5+14336\,x^4+561024\,x^3+319072\,x^2+565276\,x+388341\right)\sqrt{2\,x^2-x+3}+\frac{94691}{65536}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 388
341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.23, size = 73, normalized size = 0.70

$$\frac{1}{122880}\left(4\left(8\left(4\left(16\left(100\,x+7\right)x+4383\right)x+9971\right)x+141319\right)x+388341\right)\sqrt{2\,x^2-x+3}-\frac{94691}{32768}\sqrt{2}\log\left(-2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 38834
1)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 83, normalized size = 0.79

$$\frac{5\left(2x^{2}-x+3\right)^{\frac{5}{2}}x}{12}+\frac{94691\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}+\frac{107\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{240}+\frac{179\left(4x-1\right)\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{1536}+\frac{4117\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x)
```

```
[Out] 5/12*(2*x^2-x+3)^(5/2)*x+107/240*(2*x^2-x+3)^(5/2)+179/1536*(4*x-1)*(2*x^2-
x+3)^(3/2)+4117/8192*(4*x-1)*(2*x^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 104, normalized size = 0.99

$$\frac{5}{12}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x+\frac{107}{240}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}+\frac{179}{384}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x-\frac{179}{1536}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}+\frac{4117}{2048}\sqrt{2\,x^{2}-x+3}\,x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*x
^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x^2
 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 4117/8
192*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{3}{2}}\left(5x^{2}+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2), x)
```

3.69 $\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{1}{100}\sqrt{2x^2-x+3}\,(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)$$

```
[Out] -2203/2000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/100*(49-20*x)*(2*x^2-x+
3)^(1/2)-11/3875*arctanh(1/62*(8+x*(130-69*2^(1/2))-61*2^(1/2))*682^(1/2)/(
-247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-84227+170500*2^(1/2))^(1/2)+11
/3875*arctan(1/62*(8+61*2^(1/2)+x*(130+69*2^(1/2)))*682^(1/2)/(247+500*2^(1
/2))^(1/2)/(2*x^2-x+3)^(1/2))*(84227+170500*2^(1/2))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {977, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{100}\sqrt{2x^2-x+3}\,(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -((49 - 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62
*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(6
2*(-247 + 500*Sqrt[2]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]])/125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx = -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50}\int \frac{-\frac{731}{2}+\frac{1195x}{4}-\frac{2203x^2}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250}\int \frac{-726+3146x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx + \frac{2203\int \frac{1}{\sqrt{3-x}}}{1000}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662\left(16+3\sqrt{2}\right)+2662\left(10-13\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{5500\sqrt{2}} - \frac{\int \frac{2662\left(16-3\sqrt{2}\right)+2662(}{\sqrt{3-x+2x^2}\left(2+3x\right.}}{5500\sqrt{2}}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125}\left(322102\left(1000-247\sqrt{2}\right)\right)\mathrm{S}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-1}$$

Mathematica [C] time = 0.69, size = 310, normalized size = 1.57

$$\frac{400\sqrt{31}\sqrt{2x^2-x+3}\,x-980\sqrt{31}\sqrt{2x^2-x+3}+44\sqrt{286+22i\sqrt{31}}\left(\sqrt{31}-13i\right)\tanh^{-1}\left(\frac{-4i\sqrt{31}\,x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (-980*Sqrt[31]*Sqrt[3 - x + 2*x^2] + 400*Sqrt[31]*x*Sqrt[3 - x + 2*x^2] - 2
203*Sqrt[62]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 44*Sqrt[286 + (22*I)*Sqrt[31]]*(
-13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sq
rt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 44*Sqrt[682*(13 - I*Sqrt[
31])]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (572*I)*Sqrt[286 - (22*I)*Sqrt[31]]*A
rcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqr
t[31]]*Sqrt[3 - x + 2*x^2])])/(2000*Sqrt[31])
```

fricas [B] time = 1.97, size = 2027, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arc
tan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^
6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7
- 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x +
 86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988
*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x
 + 3)*sqrt(247*sqrt(2) + 1000) + 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
```

```
sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/
119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5
- 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 +
 33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*2420
0^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 +
37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*
x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x
))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) + 130900*sqrt(31)*sqrt(2)*(
123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x
^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^
5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 595
0*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781
920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 51
7*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))
*sqrt((24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x -
76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x^2 - x + 3) -
179690*x + 238000)/x^2) + 342125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5338
5560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7
744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*
x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4
661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3461
5296*x^2 - 24772608*x + 18579456)) + 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)
*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arctan(1/10605875*(230*sqrt(10)*(2*24200^
(3/4)*sqrt(31)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x
^3 - 156600*x^2 - sqrt(2)*(28854*x^7 - 90639*x^6 + 200187*x^5 - 262838*x^4
+ 117544*x^3 - 23472*x^2 - 186624*x + 86400) - 172800*x + 186624) + 5*24200
^(1/4)*sqrt(31)*(112238*x^7 - 1817988*x^6 + 10351960*x^5 - 25791248*x^4 + 3
4522560*x^3 - 28368000*x^2 - sqrt(2)*(125839*x^7 - 1864281*x^6 + 9323336*x^
5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - 19989504*x + 10533888) - 2
1067776*x + 19989504))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) - 30107
000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1
549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 3961
04*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53
9136) + 1154304*x - 456192) - sqrt(5/119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)
*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2
 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 + 33771*x^5 - 5576*x^4 - 576*x^3 + 3218
4*x^2 - 32184*x) + 269568*x) + 5*24200^(1/4)*sqrt(31)*(309512*x^7 - 4017952
*x^6 + 15741280*x^5 - 22625280*x^4 + 37693440*x^3 - 13519872*x^2 - sqrt(2)*
(516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46
199808*x^2 - 46199808*x) + 13519872*x))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2
) + 1000) - 130900*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6
- 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8
- 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 120960
0*x^2 - 1036800*x) + 3276288*x) - 5950*sqrt(31)*(254591*x^8 - 4815126*x^7 +
 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(24200^(1/4)*sqrt(10)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) - 58310*
x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x^2) - 342125*sqrt
(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*
x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 97
89*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223
064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^
5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) +
11/36890000*24200^(1/4)*sqrt(10)*sqrt(247*sqrt(2) + 1000)*(247*sqrt(2) - 10
00)*log(1512500/119*(24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x -
 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x
^2 - x + 3) - 179690*x + 238000)/x^2) - 11/36890000*24200^(1/4)*sqrt(10)*sq
```

```
rt(247*sqrt(2) + 1000)*(247*sqrt(2) - 1000)*log(-1512500/119*(24200^(1/4)*s
qrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2)
 + 1000) - 58310*x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x
^2) + 1/100*sqrt(2*x^2 - x + 3)*(20*x - 49) + 2203/4000*sqrt(2)*log(-4*sqrt
(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 10.3Done
```

maple [B] time = 0.05, size = 3460, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x*(2*x^2-x+3)^(1/2)-49/100*(2*x^2-x+3)^(1/2)+2203/2000*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-2/1321375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1
/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*2^(1/
2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692
487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*
(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x
+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+6154*(-8866+6820*2^(1/2))^(1/2)*(
-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41)
)^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^
4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3
*2^(1/2)))+12325786*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1
/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2)
)^(1/2))*2^(1/2)-359414*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*
2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(
1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(
1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-2/
264275*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(2365*2^(1/2)*(-8866+6820*2^(1/2))^(1
/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2
)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2
)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+3338*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1
```

```
/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-2
3*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2
^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2
^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+3192442*arctanh
(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(
1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-5264358*ar
ctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/
2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(
1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2
)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-13/105710*(8*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(
1/2)*2^(1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2
))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2)
)*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)
*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22
379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+386*(-8866
+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775
687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^
2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2
)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-
1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-274846*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)
/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^
(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2
^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2
^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6
820*2^(1/2))^(1/2)+3/10571*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-88
66+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-7
75687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(
1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1
)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1
/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2
)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+218*(-8866+6820*2^(1/2))^(1/2)*(-775687+
549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23
*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*
(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+
2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)
))+401698*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2
^(1/2)-63426*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+
2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)
))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2
)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+9/21142*(8*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8
-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+
549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23
*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*
(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+
2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)
))+520*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/1
```

```
1692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/
(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23
)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+465124*arctanh(31/2*(8*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2
^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8
-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1
/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1
/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/
2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}}{5\,x^2+3\,x+2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}}}{5x^{2} + 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)
```

3.70 $\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=232

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}+\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{11}{62(3169333+2265350}}\right.}{1550}$$

```
[Out] 1/31*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)-2/25*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)+4/155*(4-5*x)*(2*x^2-x+3)^(1/2)-1/48050*arctanh(1/62*(3514+x
*(9440-6477*2^(1/2))-2963*2^(1/2))*682^(1/2)/(-3169333+2265350*2^(1/2))^(1/
2)/(2*x^2-x+3)^(1/2))*(-1080742553+772484350*2^(1/2))^(1/2)+1/48050*arctan(
1/62*(3514+2963*2^(1/2)+x*(9440+6477*2^(1/2)))*682^(1/2)/(3169333+2265350*2
^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1080742553+772484350*2^(1/2))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}+\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{11}{62(3169333+2265350}}\right.}{1550}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/
(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt
[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 226535
0*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x +
2*x^2]])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[1
1/(62*(-3169333 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{\sqrt{3-x+2x^2}\left(-\frac{69}{2}+13x+40x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{3100}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{15500} + \ldots$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{25}\left(2\sqrt{\frac{2}{23}}\right)\text{Subst}\left(\int \ldots\right.$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \ldots$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \ldots$$

Mathematica [C] time = 2.56, size = 530, normalized size = 2.28

$$\frac{62000\sqrt{2x^2-x+3}\,x^2}{10x-i\sqrt{31}+3} + \frac{62000\sqrt{2x^2-x+3}\,x^2}{10x+i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3}\,x}{10x-i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3}\,x}{10x+i\sqrt{31}+3} - 12400\sqrt{2x^2-x+3}\,x + \frac{93000\sqrt{2x^2-x+3}}{10x-i\sqrt{31}+3} + \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (9920*Sqrt[3 - x + 2*x^2] - 12400*x*Sqrt[3 - x + 2*x^2] + (93000*Sqrt[3 - x
 + 2*x^2])/(3 - I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 - I*S
qrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) +
 (93000*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x
+ 2*x^2])/(3 + I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 + I*
Sqrt[31] + 10*x) - 7688*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - (Sqrt[286 + (
22*I)*Sqrt[31]]*(10199*I + 6477*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x -
 (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-
13*I + Sqrt[31]) - (6477*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*Sqrt[3
1] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])])/(13*I + Sqrt[31]) + ((10199*I)*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh
[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
*Sqrt[3 - x + 2*x^2])])/(13*I + Sqrt[31]))/96100
```

fricas [B] time = 2.64, size = 2150, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 1/90746855745853600*(10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(2)*(5*x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/172758074198
807633719789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118
*x^7 - 9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x
^2 - sqrt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 113
67152*x^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) +
1404517*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x
^5 - 70476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 423
2733*x^6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 4669
7472*x + 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(31693
33*sqrt(2) + 4530700) + 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^
8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 9849
6*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*
x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
sqrt(45307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7
 - 12210811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2
 - sqrt(2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 121140
72*x^3 + 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(
1/4)*sqrt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 1
12374720*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 5039126
0*x^5 - 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304
*x))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) + 7590571938849196
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 323035
80*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 1548
8*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268
*x^2 - 1944*x) + 144820224*x))*sqrt(-(1987037073032^(1/4)*sqrt(45307)*sqrt(
62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*s
qrt(3169333*sqrt(2) + 4530700) - 11567627293306*x^2 - 10387257161336*sqrt(2
)*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 5572841103187
343023219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^
5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 -
 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x
 - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x
^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x +
 18579456)) + 10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(2)*(5*
x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/1727580741988076337
19789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118*x^7 -
9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x^2 - sq
rt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 11367152*x
^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) + 1404517
*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x^5 - 70
476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 4232733*x^
6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 46697472*x
+ 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt
(2) + 4530700) - 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
 sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(45
307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7 - 12210
811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2 - sqrt(
2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 12114072*x^3
+ 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(1/4)*sq
rt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 11237472
0*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 50391260*x^5 -
 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304*x))*sq
```

```
rt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) - 7590571938849196*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) - 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
- 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169
333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x^2
 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 557284110318734302321
9*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 2541
46592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^
7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184)
 + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490
880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1857945
6)) + 1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 316
9333*sqrt(2)*(5*x^2 + 3*x + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2)
 + 4530700)*log(113267500/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sq
rt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(31
69333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x
^2 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 1987037073032^(1/4)
*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 3169333*sqrt(2)*(5*x^2 + 3*x
 + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2) + 4530700)*log(-11326750
0/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169333*sqrt(2) + 4530700)
 - 11567627293306*x^2 - 10387257161336*sqrt(2)*(2*x^2 - x + 3) + 3564717798
5494*x - 47214805278800)/x^2) + 3629874229834144*sqrt(2)*(5*x^2 + 3*x + 2)*
log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 644009
9440028320*sqrt(2*x^2 - x + 3)*(13*x + 7))/(5*x^2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{174900625,[8]%%%}+%%%{%%{[-419761500,0]:[1,0,-2]%%},[7]%%%}+%%%{-685
610450,[6]%%%}+%%%{%%{[3246155600,0]:[1,0,-2]%%},[5]%%%}+%%%{1574105625,[4]
%%%}+%%%{%%{[-10885814900,0]:[1,0,-2]%%},[3]%%%}+%%%{-3861805800,[2]%%%}+%%
%{%%{[20372424800,0]:[1,0,-2]%%},[1]%%%}+%%%{21939534400,[0]%%%} / %%%{50,[
8]%%%}+%%%{%%{poly1[-120,0]:[1,0,-2]%%},[7]%%%}+%%%{-196,[6]%%%}+%%%{%%{pol
y1[928,0]:[1,0,-2]%%},[5]%%%}+%%%{450,[4]%%%}+%%%{%%{poly1[-3112,0]:[1,0,-2
]%%},[3]%%%}+%%%{-1104,[2]%%%}+%%%{%%{poly1[5824,0]:[1,0,-2]%%},[1]%%%}+%%%
{6272,[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.16, size = 28185, normalized size = 121.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{\left(5x^2 + 3x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{3/2}}{\left(5x^2 + 3x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{\left(5x^2 + 3x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

3.71 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=223

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{\cdots}{31(36\cdots}}\cdots\right)}{768\cdots}$$

```
[Out] 1/62*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2+3/3844*(277+696*x)*(2*x^2-x
+3)^(1/2)/(5*x^2+3*x+2)-3/5243216*arctanh(1/31*(29367+x*(70517-49942*2^(1/2
))-20575*2^(1/2))*341^(1/2)/(-366990269+259509026*2^(1/2))^(1/2)/(2*x^2-x+3
)^(1/2))*(-250287363458+176985155732*2^(1/2))^(1/2)+3/5243216*arctan(1/31*(
29367+20575*2^(1/2)+x*(70517+49942*2^(1/2)))*341^(1/2)/(366990269+259509026
*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(250287363458+176985155732*2^(1/2))^(1/2
)
```

Rubi [A] time = 0.43, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {971, 1013, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{\cdots}{31(36\cdots}}\cdots\right)}{768\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696
*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 25
9509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))]
*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])
/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*
(-366990269 + 259509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*
Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/7688
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{\left(-\frac{189}{2}+33x\right)\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{\frac{13359}{4}-1353x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{1922}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{-\frac{33}{4}\left(6257-4453\sqrt{2}\right)+\frac{33}{4}\left(26\ldots\right.}{\sqrt{3-x+2x^2}\left(2+3x+\ldots\right.}}{42284\sqrt{2}}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\left(99\left(519018052-36699\ldots\right.\right.}{\ldots}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{3\sqrt{\frac{1}{682}\left(366990269+25\ldots\right.}}{\ldots}$$

Mathematica [C] time = 5.34, size = 1262, normalized size = 5.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (((248000*I)*Sqrt[31]*(3 - x + 2*x^2)^(3/2))/(3*I + Sqrt[31] + (10*I)*x)^2
+ (744000*(3 - x + 2*x^2)^(3/2))/(3 - I*Sqrt[31] + 10*x) + ((248000*I)*Sqrt
[31]*(3 - x + 2*x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x)^2 + (744000*(3 - x + 2*
x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x) + (3*I)*Sqrt[31]*(20*(1199 + (98*I)*Sqr
t[31] - 20*(11 + (2*I)*Sqrt[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(13453 +
(4406*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (352*Sqrt[286 + (22*I)*Sqr
t[31]]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[3
1])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sqrt[
31])) + 558*(20*(27 + (4*I)*Sqrt[31] - 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*
(569 + (88*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 + (22*I)*
Sqrt[31]]*(-81*I + 37*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sq
rt[31])) + (744*Sqrt[31]*(220*(-439 + (497*I)*Sqrt[31] + 20*(69 + (13*I)*Sq
rt[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 - (398*I)*Sqrt[31] + 5*(4
7 - (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 + (22*I)*S
qrt[31]]*(19548 - (4904*I)*Sqrt[31] + (-23345 - (8565*I)*Sqrt[31])*x)*ArcTa
nh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])]))/(11*(-13*I + Sqrt[31])^2*(-3*I + Sqrt[31] - (10
*I)*x)) + (744*Sqrt[31]*(220*(-439 - (497*I)*Sqrt[31] + 20*(69 - (13*I)*Sqr
t[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 + (398*I)*Sqrt[31] + 5*(47
 + (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 - (22*I)*Sq
rt[31]]*(-19548 - (4904*I)*Sqrt[31] + 5*(4669 - (1713*I)*Sqrt[31])*x)*ArcTa
nh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])]))/(11*(13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I
)*x)) + (3*Sqrt[31]*(20*(12549 - (2473*I)*Sqrt[31] + (20*I)*(81*I + 37*Sqrt
[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(38303 - (70731*I)*Sqrt[31])*ArcSinh
[(1 - 4*x)/Sqrt[23]] + (352*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(69*I + 13*Sqrt[
```

```
31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(13*I + Sqrt[31]) + 558*(-20*(-27 + (
4*I)*Sqrt[31] + 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(569 - (88*I)*Sqrt[31])
*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 - (22*I)*Sqrt[31]]*(81*I + 37*Sq
rt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(13*I + Sqrt[31])))/4766560
```

fricas [B] time = 2.61, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/85773071417697924109696*(189113268*134689869150937352^(1/4)*sqrt(1297545
13)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(366990269*
sqrt(2) + 519018052)*arctan(1/1067259092343193675559267622545473*(160895596
12*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(38305160*x^7 - 1
47261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^
2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 +
 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 19
1303424) + 4022389903*134689869150937352^(1/4)*sqrt(341)*(2906601*x^7 - 446
04657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 -
 sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 4905
00864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696
576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 30296387137
4842075642630808980 6504*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(259509026/713)*(sq
rt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(5980372*x^7 - 8582986
*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - sqrt(2)*
(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*x^4 + 9050328*x^3 + 864
0000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134689869150937352^(1/4)*s
qrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 - 29282112*x^4 + 37610208
*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*x^6 + 15927060*x^5 - 20
630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*x) - 22726656*x))*sqrt(
2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 818688798906871280095 4
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 3721312722303960364 07*sqrt(31)*(254591*x^8 - 4815126*x^7 +
32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
- 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
+ 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(134689869150937352^(1/4)*sqrt(1
29754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 9
73*x - 419)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39
11902897144396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 177813768052
01800)/x^2) + 3442771265623205405029895 5565983*sqrt(31)*(2828123*x^8 - 9696
916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37
981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 155
69*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2
585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249
088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 189113268*13468986915093
7352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12
*x + 4)*sqrt(366990269*sqrt(2) + 519018052)*arctan(1/10672590923431936755 59
267622545473*(16089559612*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt
(341)*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 24821
2864*x^3 - 117285552*x^2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 21844820
0*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585
```

```
792) - 271171584*x + 191303424) + 4022389903*134689869150937352^(1/4)*sqrt(
341)*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 69370646
4*x^3 - 436717728*x^2 - sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5
 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760)
 - 493931520*x + 348696576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 5
19018052) - 30296387137484207564263080898065O4*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(259509026/713)*(sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)
*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 +
12517632*x^2 - sqrt(2)*(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*
x^4 + 9050328*x^3 + 8640000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134
689869150937352^(1/4)*sqrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 -
29282112*x^4 + 37610208*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*
x^6 + 15927060*x^5 - 20630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*
x) - 22726656*x))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) -
 818688798906871280O954*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888
*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550
*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1
209600*x^2 - 1036800*x) + 3276288*x) - 372131272230396036407*sqrt(31)*(2545
91*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742193
28*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5
 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((134689869
150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqr
t(2)*(696*x + 277) - 973*x - 419)*sqrt(366990269*sqrt(2) + 519018052) + 435
6437317274441*x^2 + 3911902897144396*sqrt(2)*(2*x^2 - x + 3) - 134249394879
27359*x + 17781376805201800)/x^2) - 3442771265623205405029895556598З*sqrt(3
1)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^
4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789
*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306
4064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5
- 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 3*
134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300*x^
4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2) +
519018052)*log(9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)*sqr
t(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 419)*s
qrt(366990269*sqrt(2) + 519018052) + 4356437317274441*x^2 + 391190289714439
6*sqrt(2)*(2*x^2 - x + 3) - 13424939487927359*x + 17781376805201800)/x^2) +
 3*134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300
*x^4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x
^3 + 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2)
 + 519018052)*log(-9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)
*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 41
9)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39119028971
44396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 17781376805201800)/x^
2) - 22313494125311634784*(11680*x^3 + 10171*x^2 + 8343*x + 2220)*sqrt(2*x^
2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
```

```
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 36.71Done
```

maple [**B**] time = 0.35, size = 81552, normalized size = 365.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{\frac{3}{2}}}{\left(5\,x^2 + 3\,x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{3/2}}{\left(5\,x^2 + 3\,x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{\left(5x^2 + 3x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

3.72 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=254

$$\frac{122595067\left(2x^2-x+3\right)^{7/2}x^2}{19169280}+\frac{112244125\left(2x^2-x+3\right)^{7/2}x}{122683392}+\frac{25250178739\left(2x^2-x+3\right)^{7/2}}{5725224960}-\frac{401135647(1-}{335}$$

```
[Out] -9226119881/2147483648*(1-4*x)*(2*x^2-x+3)^(3/2)-401135647/335544320*(1-4*x
)*(2*x^2-x+3)^(5/2)+25250178739/5725224960*(2*x^2-x+3)^(7/2)+112244125/1226
83392*x*(2*x^2-x+3)^(7/2)+122595067/19169280*x^2*(2*x^2-x+3)^(7/2)+23460839
/532480*x^3*(2*x^2-x+3)^(7/2)+3684995/39936*x^4*(2*x^2-x+3)^(7/2)+1046225/9
984*x^5*(2*x^2-x+3)^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(2*x^2
-x+3)^(7/2)-14641852251147/137438953472*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)-636602271789/34359738368*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{7/2}x^6+\frac{1046225\left(2x^2-x+3\right)^{7/2}x^5}{9984}+\frac{3684995\left(2x^2-x+3\right)^{7/2}x^4}{39936}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1
- 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*
x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (1
12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2
)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*
x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984
 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/
28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^4\,dx = \frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2}+\frac{1}{28}\int \left(3-x+2x^2\right)^{5/2}\left(448+2688x+10528x\right.$$

$$= \frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2}+\frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2}+\frac{1}{728}\int \left(3-x+2\right.$$

$$= \frac{1046225x^5\left(3-x+2x^2\right)^{7/2}}{9984}+\frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2}+\frac{625}{28}x^7\left(3-x\right.$$

$$= \frac{3684995x^4\left(3-x+2x^2\right)^{7/2}}{39936}+\frac{1046225x^5\left(3-x+2x^2\right)^{7/2}}{9984}+\frac{13875}{208}x^6\left(\right.$$

$$= \frac{23460839x^3\left(3-x+2x^2\right)^{7/2}}{532480}+\frac{3684995x^4\left(3-x+2x^2\right)^{7/2}}{39936}+1046225$$

$$= \frac{122595067x^2\left(3-x+2x^2\right)^{7/2}}{19169280}+\frac{23460839x^3\left(3-x+2x^2\right)^{7/2}}{532480}+36849$$

$$= \frac{112244125x\left(3-x+2x^2\right)^{7/2}}{122683392}+\frac{122595067x^2\left(3-x+2x^2\right)^{7/2}}{19169280}+23460$$

$$= \frac{25250178739\left(3-x+2x^2\right)^{7/2}}{5725224960}+\frac{112244125x\left(3-x+2x^2\right)^{7/2}}{122683392}+12259$$

$$= -\frac{401135647(1-4x)\left(3-x+2x^2\right)^{5/2}}{335544320}+\frac{25250178739\left(3-x+2x^2\right)^{7/2}}{5725224960}-$$

$$= -\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648}-\frac{401135647(1-4x)\left(3-x+2x\right.}{335544320}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2\right.}{2147483648}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2\right.}{2147483648}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2\right.}{2147483648}$$

Mathematica [A] time = 0.45, size = 105, normalized size = 0.41

$$4\sqrt{2x^2-x+3}\left(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+20493241166\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 500641740
38215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 4054683822
84161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645588
46345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 137233466
130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 5995838
4968446965*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/562812514467840
```

fricas [A] time = 0.81, size = 118, normalized size = 0.46

$$\frac{1}{140703128616960}\left(25125558681600000\,x^{13}+37398427729920000\,x^{12}+137233466130432000\,x^{11}+20493\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/140703128616960*(25125558681600000*x^13 + 37398427729920000*x^12 + 137233
466130432000*x^11 + 204932411660697600*x^10 + 363646430503501824*x^9 + 4390
64558846345216*x^8 + 530502956133122048*x^7 + 485091164642279424*x^6 + 4054
68382284161024*x^5 + 257786732552566784*x^4 + 142490931553577856*x^3 + 5006
4174038215008*x^2 + 12071614275862524*x + 10820567498568669)*sqrt(2*x^2 - x
 + 3) + 14641852251147/274877906944*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.54, size = 113, normalized size = 0.44

$$\frac{1}{140703128616960}\left(4\left(8\left(4\left(16\left(4\left(8\left(4\left(32\left(12\left(200\left(20\left(240\left(260\,x+387\right)x+340823\right)x+10179103\right)x+36125\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387)
*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 20237081761
67)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113210
402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 1082056749856866
9)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.04, size = 204, normalized size = 0.80

$$\frac{625\left(2x^2-x+3\right)^{\frac{7}{2}}x^7}{28}+\frac{13875\left(2x^2-x+3\right)^{\frac{7}{2}}x^6}{208}+\frac{1046225\left(2x^2-x+3\right)^{\frac{7}{2}}x^5}{9984}+\frac{3684995\left(2x^2-x+3\right)^{\frac{7}{2}}x^4}{39936}+23$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 625/28*x^7*(2*x^2-x+3)^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+25250178739/57
25224960*(2*x^2-x+3)^(7/2)+1046225/9984*x^5*(2*x^2-x+3)^(7/2)+3684995/39936
*x^4*(2*x^2-x+3)^(7/2)+23460839/532480*x^3*(2*x^2-x+3)^(7/2)+122595067/1916
9280*x^2*(2*x^2-x+3)^(7/2)+112244125/122683392*x*(2*x^2-x+3)^(7/2)+14641852
251147/137438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+636602271789/343
59738368*(4*x-1)*(2*x^2-x+3)^(1/2)+401135647/335544320*(4*x-1)*(2*x^2-x+3)^
(5/2)+9226119881/2147483648*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 1.02, size = 235, normalized size = 0.93

$$\frac{625}{28}\left(2x^2-x+3\right)^{\frac{7}{2}}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{\frac{7}{2}}x^6+\frac{1046225}{9984}\left(2x^2-x+3\right)^{\frac{7}{2}}x^5+\frac{3684995}{39936}\left(2x^2-x+3\right)^{\frac{7}{2}}x^4+\frac{23460}{5324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 10
46225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2)*
x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*x^2
 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 2525017
8739/5725224960*(2*x^2 - x + 3)^(7/2) + 401135647/83886080*(2*x^2 - x + 3)^
(5/2)*x - 401135647/335544320*(2*x^2 - x + 3)^(5/2) + 9226119881/536870912*
(2*x^2 - x + 3)^(3/2)*x - 9226119881/2147483648*(2*x^2 - x + 3)^(3/2) + 636
602271789/8589934592*sqrt(2*x^2 - x + 3)*x + 14641852251147/137438953472*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 636602271789/34359738368*sqrt(2*x^
2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2x^2-x+3\right)^{5/2}\left(5x^2+3x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**4, x)
```

3.73 $\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3\,dx$

Optimal. Leaf size=212

$$\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{509257\left(2x^2-x+3\right)^{7/2}x}{294912}-\frac{1696165\left(2x^2-x+3\right)^{7/2}}{2752512}-\frac{57915(1-4x)\left(2x^2-x+3\right)^{5}}{2097152}$$

```
[Out] -6660225/67108864*(1-4*x)*(2*x^2-x+3)^(3/2)-57915/2097152*(1-4*x)*(2*x^2-x+
3)^(5/2)-1696165/2752512*(2*x^2-x+3)^(7/2)+509257/294912*x*(2*x^2-x+3)^(7/2
)+80483/9216*x^2*(2*x^2-x+3)^(7/2)+3823/256*x^3*(2*x^2-x+3)^(7/2)+1175/96*x
^4*(2*x^2-x+3)^(7/2)+125/24*x^5*(2*x^2-x+3)^(7/2)-10569777075/4294967296*ar
csinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-459555525/1073741824*(1-4*x)*(2*x^2-x+
3)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5+\frac{1175}{96}\left(2x^2-x+3\right)^{7/2}x^4+\frac{3823}{256}\left(2x^2-x+3\right)^{7/2}x^3+\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{50925}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*
(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2
097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2
)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x
 + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 -
x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(214748364
8*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3\,dx = \frac{125}{24}x^5\left(3-x+2x^2\right)^{7/2}+\frac{1}{24}\int \left(3-x+2x^2\right)^{5/2}\left(192+864x+2736x^2\right.$$

$$= \frac{1175}{96}x^4\left(3-x+2x^2\right)^{7/2}+\frac{125}{24}x^5\left(3-x+2x^2\right)^{7/2}+\frac{1}{528}\int \left(3-x+2x^2\right.$$

$$= \frac{3823}{256}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1175}{96}x^4\left(3-x+2x^2\right)^{7/2}+\frac{125}{24}x^5\left(3-x+2x\right.$$

$$= \frac{80483x^2\left(3-x+2x^2\right)^{7/2}}{9216}+\frac{3823}{256}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1175}{96}x^4\left(3-x+\right.$$

$$= \frac{509257x\left(3-x+2x^2\right)^{7/2}}{294912}+\frac{80483x^2\left(3-x+2x^2\right)^{7/2}}{9216}+\frac{3823}{256}x^3\left(3-x\right.$$

$$= -\frac{1696165\left(3-x+2x^2\right)^{7/2}}{2752512}+\frac{509257x\left(3-x+2x^2\right)^{7/2}}{294912}+\frac{80483x^2\left(3-\right.}{921}$$

$$= -\frac{57915(1-4x)\left(3-x+2x^2\right)^{5/2}}{2097152}-\frac{1696165\left(3-x+2x^2\right)^{7/2}}{2752512}+\frac{509257x}{}$$

$$= -\frac{6660225(1-4x)\left(3-x+2x^2\right)^{3/2}}{67108864}-\frac{57915(1-4x)\left(3-x+2x^2\right)^{5/2}}{2097152}-$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+2x^2\right)^{3/2}}{67108864}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+2x^2\right)^{3/2}}{67108864}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+2x^2\right)^{3/2}}{67108864}$$

Mathematica [A] time = 0.30, size = 95, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^9+14341894045696x^8+278355\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*x
^2 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 2834753
8538496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x^9
+ 2395786444800*x^10 + 2818572288000*x^11) - 665895955725*Sqrt[2]*ArcSinh[(
1 - 4*x)/Sqrt[23]])/270582939648
```

fricas [A] time = 1.00, size = 108, normalized size = 0.51

$$\frac{1}{67645734912}\left(2818572288000\,x^{11} + 2395786444800\,x^{10} + 12943588589568\,x^{9} + 14341894045696\,x^{8} + 278\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^9
 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 343786139
23840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2 +
4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/85899345
92*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 2
5)
```

giac [A] time = 0.55, size = 103, normalized size = 0.49

$$\frac{1}{67645734912}\left(4\left(8\left(4\left(16\left(4\left(8\left(28\left(32\left(12\left(200\left(20\,x + 17\right)x + 18369\right)x + 244241\right)x + 15169177\right)x + 432549111\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x
+ 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x +
 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*sqrt
(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x
- sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 170, normalized size = 0.80

$$\frac{125\left(2x^{2}-x+3\right)^{\frac{7}{2}}x^{5}}{24}+\frac{1175\left(2x^{2}-x+3\right)^{\frac{7}{2}}x^{4}}{96}+\frac{3823\left(2x^{2}-x+3\right)^{\frac{7}{2}}x^{3}}{256}+\frac{80483\left(2x^{2}-x+3\right)^{\frac{7}{2}}x^{2}}{9216}+\frac{509257\left(2\right.}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x)
```

```
[Out] -1696165/2752512*(2*x^2-x+3)^(7/2)+125/24*(2*x^2-x+3)^(7/2)*x^5+1175/96*(2*
x^2-x+3)^(7/2)*x^4+3823/256*(2*x^2-x+3)^(7/2)*x^3+80483/9216*(2*x^2-x+3)^(7
/2)*x^2+509257/294912*(2*x^2-x+3)^(7/2)*x+10569777075/4294967296*2^(1/2)*ar
csinh(4/23*23^(1/2)*(x-1/4))+459555525/1073741824*(4*x-1)*(2*x^2-x+3)^(1/2)
+57915/2097152*(4*x-1)*(2*x^2-x+3)^(5/2)+6660225/67108864*(4*x-1)*(2*x^2-x+
3)^(3/2)
```

maxima [A] time = 1.00, size = 201, normalized size = 0.95

$$\frac{125}{24}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x^{5}+\frac{1175}{96}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x^{4}+\frac{3823}{256}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x^{3}+\frac{80483}{9216}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x^{2}+\frac{509257}{294912}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 125/24*(2*x^2 - x + 3)^(7/2)*x^5 + 1175/96*(2*x^2 - x + 3)^(7/2)*x^4 + 3823
/256*(2*x^2 - x + 3)^(7/2)*x^3 + 80483/9216*(2*x^2 - x + 3)^(7/2)*x^2 + 509
257/294912*(2*x^2 - x + 3)^(7/2)*x - 1696165/2752512*(2*x^2 - x + 3)^(7/2)
+ 57915/524288*(2*x^2 - x + 3)^(5/2)*x - 57915/2097152*(2*x^2 - x + 3)^(5/2
) + 6660225/16777216*(2*x^2 - x + 3)^(3/2)*x - 6660225/67108864*(2*x^2 - x
```

```
+ 3)^(3/2) + 459555525/268435456*sqrt(2*x^2 - x + 3)*x + 10569777075/429496
7296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824*sqrt(2
*x^2 - x + 3)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)
```

3.74 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=170

$$\frac{305}{144}x^2\left(2x^2-x+3\right)^{7/2}+\frac{8467x\left(2x^2-x+3\right)^{7/2}}{4608}+\frac{23225\left(2x^2-x+3\right)^{7/2}}{43008}-\frac{1547(1-4x)\left(2x^2-x+3\right)^{5/2}}{98304}-1779$$

```
[Out] -177905/3145728*(1-4*x)*(2*x^2-x+3)^(3/2)-1547/98304*(1-4*x)*(2*x^2-x+3)^(5
/2)+23225/43008*(2*x^2-x+3)^(7/2)+8467/4608*x*(2*x^2-x+3)^(7/2)+305/144*x^2
*(2*x^2-x+3)^(7/2)+5/4*x^3*(2*x^2-x+3)^(7/2)-94111745/67108864*arcsinh(1/23
*(1-4*x)*23^(1/2))*2^(1/2)-4091815/16777216*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}+\frac{305}{144}x^2\left(2x^2-x+3\right)^{7/2}+\frac{8467x\left(2x^2-x+3\right)^{7/2}}{4608}+\frac{23225\left(2x^2-x+3\right)^{7/2}}{43008}-\frac{1547(1-4x)(}{98}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 -
x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 +
(23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 +
 (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (9
4111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2 dx = \frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{20}\int \left(3-x+2x^2\right)^{5/2}\left(80+240x+355x^2+\frac{15\ldots}{}\right.$$

$$= \frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{360}\int \left(3-x+2x^2\right)^{5/\ldots}$$

$$= \frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}\ldots$$

$$= \frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x^2\right)\ldots$$

$$= -\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-\ldots\right.}{46\ldots}$$

$$= -\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23\ldots}{}$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{15\ldots}{}$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{15\ldots}{}$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{15\ldots}{}$$

Mathematica [A] time = 0.18, size = 85, normalized size = 0.50

$$\frac{4\sqrt{2x^2-x+3}\left(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+5714\ldots\right.}{4227\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 778
72272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 441631
37536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*ArcSinh[
(1 - 4*x)/Sqrt[23]])/4227858432
```

fricas [A] time = 0.79, size = 98, normalized size = 0.58

$$\frac{1}{1056964608}\left(10569646080\,x^9+2055208960\,x^8+44163137536\,x^7+26401898496\,x^6+75389820928\,x^5+5714\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 26401898
496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 42992644128
*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421772
8*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

giac [A] time = 0.52, size = 93, normalized size = 0.55

$$\frac{1}{1056964608}\,(4\,(8\,(4\,(16\,(4\,(8\,(28\,(160\,(36\,x+7)x+24067)x+402861)x+9202859)x+27904037)x+60837$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)*
x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)*x
 + 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 136, normalized size = 0.80

$$\frac{5\left(2x^2-x+3\right)^{\frac{7}{2}}x^3}{4}+\frac{305\left(2x^2-x+3\right)^{\frac{7}{2}}x^2}{144}+\frac{8467\left(2x^2-x+3\right)^{\frac{7}{2}}x}{4608}+\frac{94111745\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{67108864}+\frac{232}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 23225/43008*(2*x^2-x+3)^(7/2)+5/4*(2*x^2-x+3)^(7/2)*x^3+305/144*(2*x^2-x+3)
^(7/2)*x^2+8467/4608*(2*x^2-x+3)^(7/2)*x+94111745/67108864*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+4091815/16777216*(4*x-1)*(2*x^2-x+3)^(1/2)+1547/9830
4*(4*x-1)*(2*x^2-x+3)^(5/2)+177905/3145728*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 1.03, size = 167, normalized size = 0.98

$$\frac{5}{4}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{305}{144}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{8467}{4608}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x+\frac{23225}{43008}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{1547}{24576}\left(2\,x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/46
08*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/24576
*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786432
*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 4091815/4
194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*sqrt(
23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2, x)
```

3.75 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) dx$

Optimal. Leaf size=128

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{732}{}$$

```
[Out] -31855/98304*(1-4*x)*(2*x^2-x+3)^(3/2)-277/3072*(1-4*x)*(2*x^2-x+3)^(5/2)+1
41/448*(2*x^2-x+3)^(7/2)+5/16*x*(2*x^2-x+3)^(7/2)-16851295/2097152*arcsinh(
1/23*(1-4*x)*23^(1/2))*2^(1/2)-732665/524288*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{732}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-732665*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/3072 + (141*(3
- x + 2*x^2)^(7/2))/448 + (5*x*(3 - x + 2*x^2)^(7/2))/16 - (16851295*ArcSin
h[(1 - 4*x)/Sqrt[23]])/(1048576*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
```

```
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)dx = \frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{1}{16}\int\left(17+\frac{141x}{2}\right)\left(3-x+2x^2\right)^{5/2}dx$$

$$=\frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{277}{128}\int\left(3-x+2x^2\right)^{5/2}dx$$

$$=-\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right.$$

$$=-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3\right.$$

$$=-\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1}{}$$

$$=-\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1}{}$$

$$=-\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1}{}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.59

$$\frac{4\sqrt{2x^2-x+3}\left(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+14895\right.}{44040192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*x
^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 353877195
*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/44040192
```

fricas [A] time = 0.70, size = 88, normalized size = 0.69

$$\frac{1}{11010048}\left(27525120\,x^7-13565952\,x^6+118808576\,x^5-1619968\,x^4+172684416\,x^3+67272352\,x^2+1489574\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/11010048*(27525120*x^7 - 13565952*x^6 + 118808576*x^5 - 1619968*x^4 + 172
684416*x^3 + 67272352*x^2 + 148957444*x + 58536675)*sqrt(2*x^2 - x + 3) + 1
6851295/4194304*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

giac [A] time = 0.39, size = 83, normalized size = 0.65

$$\frac{1}{11010048}\left(4\left(8\left(4\left(16\left(4\left(24\left(140\,x-69\right)x+14503\right)x-791\right)x+1349097\right)x+2102261\right)x+37239361\right)x+5853667\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 1349097
)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295/2
097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 102, normalized size = 0.80

$$\frac{5\left(2x^{2}-x+3\right)^{\frac{7}{2}}x}{16}+\frac{16851295\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2097152}+\frac{141\left(2x^{2}-x+3\right)^{\frac{7}{2}}}{448}+\frac{277\left(4x-1\right)\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{3072}+\frac{31}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(7/2)*x+141/448*(2*x^2-x+3)^(7/2)+277/3072*(4*x-1)*(2*x^2-
x+3)^(5/2)+31855/98304*(4*x-1)*(2*x^2-x+3)^(3/2)+732665/524288*(4*x-1)*(2*x
^2-x+3)^(1/2)+16851295/2097152*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.97, size = 133, normalized size = 1.04

$$\frac{5}{16}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x+\frac{141}{448}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}+\frac{277}{768}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x-\frac{277}{3072}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}+\frac{31855}{24576}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(7/2)*x + 141/448*(2*x^2 - x + 3)^(7/2) + 277/768*(2*x
^2 - x + 3)^(5/2)*x - 277/3072*(2*x^2 - x + 3)^(5/2) + 31855/24576*(2*x^2 -
 x + 3)^(3/2)*x - 31855/98304*(2*x^2 - x + 3)^(3/2) + 732665/131072*sqrt(2*
x^2 - x + 3)*x + 16851295/2097152*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
- 732665/524288*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{5/2}\,\left(5\,x^2+3\,x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{5}{2}}\left(5x^{2}+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2), x)
```

3.76 $\int \frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=222

$$-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{62(250}}{}\right.}{3125}$$

```
[Out] -1/600*(103-60*x)*(2*x^2-x+3)^(3/2)-7216203/1600000*arcsinh(1/23*(1-4*x)*23
^(1/2))*2^(1/2)-1/80000*(226249-99620*x)*(2*x^2-x+3)^(1/2)-121/96875*arctan
(1/62*(196-443*2^(1/2)-x*(690+247*2^(1/2)))*682^(1/2)/(-15457+25000*2^(1/2)
)^(1/2)/(2*x^2-x+3)^(1/2))*(-5270837+8525000*2^(1/2))^(1/2)+121/96875*arcta
nh(1/62*(196-x*(690-247*2^(1/2))+443*2^(1/2))*682^(1/2)/(15457+25000*2^(1/2
))^(1/2)/(2*x^2-x+3)^(1/2))*(5270837+8525000*2^(1/2))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {977, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{62(250}}{}\right.}{3125}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -((226249 - 99620*x)*Sqrt[3 - x + 2*x^2])/80000 - ((103 - 60*x)*(3 - x + 2*
x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2]) -
(121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 + 2
5000*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/
(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x))/
Sqrt[3 - x + 2*x^2]])/3125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx = -\frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{1}{300}\int \frac{\left(-\frac{4731}{2}+\frac{6135x}{4}-\frac{14943x^2}{4}\right)\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{\frac{3205293}{8}-\frac{1133938}{16}}{\sqrt{3-x+2x^2}\,(2}}{300}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{-702768-7602}{\sqrt{3-x+2x^2}\,(2+3}}{150000}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{-702768(108-11}{\sqrt{3-x}}}{3}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\sinh^{-}}{800000\sqrt{}}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\sinh^{-}}{800000\sqrt{}}$$

Mathematica [C] time = 1.05, size = 229, normalized size = 1.03

$$\frac{46464\sqrt{286+22i\sqrt{31}}\left(403-69i\sqrt{31}\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - 46464i\sqrt{286-22i\sqrt{31}}\left(69\sqrt{31}-403\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (620*Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) + 67
1106879*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + 46464*Sqrt[286 + (22*I)*Sqrt
[31]]*(403 - (69*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[
31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (46464*I)*Sq
rt[286 - (22*I)*Sqrt[31]]*(-403*I + 69*Sqrt[31])*ArcTanh[(-63 + I*Sqrt[31]
+ (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^
2])])/148800000
```

fricas [B] time = 2.25, size = 2010, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 121/96875000*6050^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 25000000
00)*arctan(1/254496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) + 2300*(4*6
050^(3/4)*sqrt(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 127
2680*x^3 - 1081800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1
083344*x^4 + 393672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 104
3712) + 5*6050^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 11
1531424*x^4 + 156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 1023330
3*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417
152*x + 37407744) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-7728
50000*sqrt(2) + 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - (4*60
50^(3/4)*sqrt(31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 369
3960*x^3 + 4195584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4
464*x^4 - 49464*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sq
rt(31)*(160956*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*
x^3 - 296027136*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 -
 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x
))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqr
t(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x
^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt
((6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt
(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2)*(2*x^
2 - x + 3) - 431180500*x + 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x
^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096
*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*
x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 9488
7936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^
4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 121/96875000*60
50^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/2
54496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270
*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x
^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3961
44*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2300*(4*6050^(3/4)*sqrt
(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 108
1800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1083344*x^4 + 3
93672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 1043712) + 5*6050
^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 111531424*x^4 +
156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 10233303*x^6 + 485297
68*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417152*x + 374077
44) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2)
+ 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + (4*6050^(3/4)*sqrt(
31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4464*x^4 - 4946
4*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sqrt(31)*(160956
*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x^3 - 29602713
6*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4
 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x))*sqrt(2*x^2
- x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqrt(31)*(254591*
x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*
```

```
x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6050^(1/4)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt(-772850000*s
qrt(2) + 2500000000) - 139919500*x^2 - 125642000*sqrt(2)*(2*x^2 - x + 3) +
431180500*x - 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x^8 - 9696916*
x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814
40*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x
^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851
91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*
x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 121/2213012500000*6050^(1/4)
*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(91506250
00/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 88
8)*sqrt(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2
)*(2*x^2 - x + 3) - 431180500*x + 571100000)/x^2) + 121/2213012500000*6050^
(1/4)*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(-91
50625000/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*
x - 888)*sqrt(-772850000*sqrt(2) + 2500000000) - 139919500*x^2 - 125642000*
sqrt(2)*(2*x^2 - x + 3) + 431180500*x - 571100000)/x^2) + 1/240000*(48000*x
^3 - 106400*x^2 + 412060*x - 802347)*sqrt(2*x^2 - x + 3) + 7216203/3200000*
sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 15.78Done
```

maple [B] time = 0.05, size = 4860, normalized size = 21.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x^3*(2*x^2-x+3)^(1/2)-133/300*x^2*(2*x^2-x+3)^(1/2)+20603/12000*x*(2*x^
2-x+3)^(1/2)-267449/80000*(2*x^2-x+3)^(1/2)+7216203/1600000*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+4/33034375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^
(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(75195*2^
(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11
692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)
^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(
23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)
*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+106294*(-8866+6820*2^(1/2))^(1
/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2
)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2
```

```
)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+108099046*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2
^(1/2))^(1/2))*2^(1/2)-158290154*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2
)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866
+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2
)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))
^(1/2)+6/6606875*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(10915*2^(1/2)*(-8866+6820*
2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+54
9362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^
2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+1036
8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4
/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x
+2^(1/2)+1)*(8+3*2^(1/2)))+14918*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362
*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*
2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*
2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+
1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-505
2938*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-
1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2
)-51565338*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))
/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)
+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-
1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-21/1321375*(8*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-77568
7+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-
23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2
)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/
2)))+6154*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(
1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+3201
6)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2
+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+12325786*arctanh(31/2*(8*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-359414*arctanh(31/2*
(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1
)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x
+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*
2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-37/528550*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2
)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2
)*(2365*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*a
rctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+
2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2
)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2
)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+3338*(-8866+6820*2^(
1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+54936
2*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+2
4*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-
```

```
x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^
(1/2)+1)*(8+3*2^(1/2)))+3192442*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)
+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+
6820*2^(1/2))^(1/2))*2^(1/2)-5264358*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-
8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1
))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1
/2))^(1/2)-63/105710*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/
2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(285*2^(1/2)*(-8866+682
0*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+
549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1
)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10
368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)
^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(
-x+2^(1/2)+1)*(8+3*2^(1/2)))+386*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362
*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*
2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*
2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+
1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-274
846*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)
-1543366*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1
/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/(
(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1
)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)
/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+27/21142*(8*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*
2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549
362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8
+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(64
85*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+
218*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/1169
2487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23
*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(
x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+401698*arctanh(31/2*(8*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1
/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2
^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+
1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-
1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/
(-8866+6820*2^(1/2))^(1/2)+27/21142*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2
^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(
1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/116
92487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(2
3*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*
(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+520*(-8866+6820*2^(1/2))^(1/2)*
(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(
1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41
))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)
^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)
```

```
^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+
3*2^(1/2)))+465124*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/
2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))
^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2
^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1
/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1
+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{5/2}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2),x)
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{5}{2}}}{5x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```

3.77 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=255

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\left(2x^2-x+3\right)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}+\frac{11\sqrt{\frac{11}{31}}\left(224510383+19\right.}{}$$

```
[Out] 4/155*(4-5*x)*(2*x^2-x+3)^(3/2)+1/31*(3+10*x)*(2*x^2-x+3)^(5/2)/(5*x^2+3*x+
2)-4799/5000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/7750*(1277+2240*x)*(2
*x^2-x+3)^(1/2)-11/1201250*arctanh(1/62*(21136+x*(87710-54423*2^(1/2))-3328
7*2^(1/2))*682^(1/2)/(-224510383+194487500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2)
)*(-76558040603+66320237500*2^(1/2))^(1/2)+11/1201250*arctan(1/62*(21136+33
287*2^(1/2)+x*(87710+54423*2^(1/2)))*682^(1/2)/(224510383+194487500*2^(1/2)
)^(1/2)/(2*x^2-x+3)^(1/2))*(76558040603+66320237500*2^(1/2))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\left(2x^2-x+3\right)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}+\frac{11\sqrt{\frac{11}{31}}\left(224510383+19\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^
(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4
799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 +
 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]
))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqr
t[11/(62*(-224510383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (87710
 - 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/38750
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
```

```
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}dx = \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-\frac{75}{2}+15x+80x^2\right)}{2+3x+5x^2}dx$$

$$= \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{\left(87660-54300x-53760x^2\right)\sqrt{3-x+}}{2+3x+5x^2}}{18600}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right.}{31\left(2+3x+5x^2\right)}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right.}{31\left(2+3x+5x^2\right)}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right.}{31\left(2+3x+5x^2\right)}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right.}{31\left(2+3x+5x^2\right)}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right.}{31\left(2+3x+5x^2\right)}$$

Mathematica [C] time = 1.65, size = 685, normalized size = 2.69

$$-\frac{7784100\sqrt{2x^2-x+3}\,x^2 - 5759180\sqrt{2x^2-x+3}\,x - 5577520\sqrt{2x^2-x+3} - 4611839\sqrt{2}\left(5x^2+3x+2\right)\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] -1/4805000*(-5577520*Sqrt[3 - x + 2*x^2] - 5759180*x*Sqrt[3 - x + 2*x^2] +
7784100*x^2*Sqrt[3 - x + 2*x^2] - 1922000*x^3*Sqrt[3 - x + 2*x^2] - 4611839
*Sqrt[2]*(2 + 3*x + 5*x^2)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (11*I)*Sqrt[286 +
 (22*I)*Sqrt[31]]*(5177*I + 8771*Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 +
I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
 - x + 2*x^2])] + (192962*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*S
qrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
x + 2*x^2])] + 113894*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[31]
 + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] + (289443*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[31]
+ 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^
```

```
2])] + 170841*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*
x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]
+ (482405*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22
*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]
 + 284735*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x
- (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(
2 + 3*x + 5*x^2)
```

fricas [B] time = 2.74, size = 2161, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/13227599224357079000000*(38925001324*1464599010050^(1/4)*sqrt(155590)*sqrt
(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) + 388975000)*arctan(1
/29697544706386636381999587 5*(110935670*sqrt(155590)*(4*1464599010050^(3/4)
*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 - 298338397*x^4 + 15
6222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62947113*x^6 + 1354639
29*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - 122404608*x + 714528
00) - 142905600*x + 122404608) + 2411645*1464599010050^(1/4)*sqrt(62)*(3035
566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 + 807721920*x^3 - 57
9888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204950152*x^5 - 4510041
40*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + 268655616) - 5373112
32*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000)
+ 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 70
4270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(87
46*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 +
396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(77795/920561)*
(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374*x^7 - 85793239*x^6
+ 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 601344*x^2 - 4*sqrt(2)*(1
7889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 + 15593832*x^3 + 5
8564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599010050^(1/4)*sqrt(62
)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984960*x^4 + 949582080*
x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036*x^6 + 505509740*x^5
 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296
*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 759924265600
1778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x
^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^
7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 103
6800*x) + 3276288*x) + 345420120727353550*sqrt(31)*(254591*x^8 - 4815126*x^
7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*
x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*
x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(1464599010050^(1/4)*sqrt(15
5590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 387
58*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 6744561519183110*x^2 -
6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2752882252
7278000)/x^2) + 957985313109246334903212 5*sqrt(31)*(2828123*x^8 - 9696916*x
^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3798144
0*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^
4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(258519
1*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x
^3 - 34615296*x^2 - 24772608*x + 18579456)) + 38925001324*1464599010050^(1/
4)*sqrt(155590)*sqrt(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) +
 388975000)*arctan(1/29697544706386636381999587 5*(110935670*sqrt(155590)*(4
*1464599010050^(3/4)*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5
- 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62
947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 -
122404608*x + 71452800) - 142905600*x + 122404608) + 2411645*1464599010050^
```

```
(1/4)*sqrt(62)*(3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4
+ 807721920*x^3 - 579888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204
950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x +
268655616) - 537311232*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*s
qrt(2) + 388975000) - 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x
^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 984
96*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710
*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
 sqrt(77795/920561)*(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374
*x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 60134
4*x^2 - 4*sqrt(2)*(17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^
4 + 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599
010050^(1/4)*sqrt(62)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984
960*x^4 + 949582080*x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036
*x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 7256
77056*x) - 183223296*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 38897
5000) - 7599242656001778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 157
8888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(1
5550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
 + 1209600*x^2 - 1036800*x) + 3276288*x) - 345420120727353550*sqrt(31)*(254
591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219
328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^
5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((14645990
10050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(97
33*x + 29025) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) + 6744
561519183110*x^2 + 6056340956001160*sqrt(2)*(2*x^2 - x + 3) - 2078426100809
4890*x + 27528822527278000)/x^2) - 957985313109246334903212 5*sqrt(31)*(2828
123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493
00096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1
0070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x -
 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 135629
44*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 11*1464599
010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^2 - 224510383*sqr
t(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(224510383*sqrt(2) +
 388975000)*log(14708117187500/920561*(1464599010050^(1/4)*sqrt(155590)*sqr
t(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 38758*x + 19
292)*sqrt(224510383*sqrt(2) + 388975000) + 6744561519183110*x^2 + 605634095
6001160*sqrt(2)*(2*x^2 - x + 3) - 20784261008094890*x + 27528822527278000)/
x^2) - 11*1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^
2 - 224510383*sqrt(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(22
4510383*sqrt(2) + 388975000)*log(-14708117187500/920561*(1464599010050^(1/4
)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 290
25) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 67445615191831
10*x^2 - 6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2
7528822527278000)/x^2) + 634792486776896221210*sqrt(2)*(5*x^2 + 3*x + 2)*lo
g(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 17067869
9669123600*(3100*x^3 - 12555*x^2 + 9289*x + 8996)*sqrt(2*x^2 - x + 3))/(5*x
^2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{15625,[8]%%%}+%%%{%%{[-37500,0]:[1,0,-2]%%},[7]%%%}+%%%{-61250,[6]%%
%}+%%%{%%{[290000,0]:[1,0,-2]%%},[5]%%%}+%%%{140625,[4]%%%}+%%%{%%{[-972500
```

```
,0]:[1,0,-2]%%},[3]%%%}+%%%{-345000,[2]%%%}+%%%{%%{[1820000,0]:[1,0,-2]%%},
[1]%%%}+%%%{1960000,[0]%%%} / %%%{50,[8]%%%}+%%%{%%{poly1[-120,0]:[1,0,-2]%
%},[7]%%%}+%%%{-196,[6]%%%}+%%%{%%{poly1[928,0]:[1,0,-2]%%},[5]%%%}+%%%{450
,[4]%%%}+%%%{%%{poly1[-3112,0]:[1,0,-2]%%},[3]%%%}+%%%{-1104,[2]%%%}+%%%{%%
{poly1[5824,0]:[1,0,-2]%%},[1]%%%}+%%%{6272,[0]%%%} Error: Bad Argument Val
ue
```

maple [B] time = 0.16, size = 40028, normalized size = 156.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(2x^2 - x + 3)^{5/2}}{(5x^2 + 3x + 2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2,x)
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)
```

3.78 $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=281

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{3844\left(5x^2+3x+2\right)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}+\frac{\sqrt{11\left(1+4\sqrt{2}\right)}\left(2\right.}{}$$

```
[Out] 1/62*(3+10*x)*(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2+1/3844*(769+2336*x)*(2*x^2-
x+3)^(3/2)/(5*x^2+3*x+2)-4/125*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/480
50*(11359-12920*x)*(2*x^2-x+3)^(1/2)-1/29791000*arctanh(1/62*(3957722+x*(98
32420-6895071*2^(1/2))-2937349*2^(1/2))*682^(1/2)/(-3531015707557+249885207
1250*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(2937349-1978861*2^(1/2))*(-11+44*2^
(1/2))^(1/2)+1/29791000*arctan(1/62*(3957722+2937349*2^(1/2)+x*(9832420+689
5071*2^(1/2)))*682^(1/2)/(3531015707557+2498852071250*2^(1/2))^(1/2)/(2*x^2
-x+3)^(1/2))*(2937349+1978861*2^(1/2))*(11+44*2^(1/2))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {971, 1054, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{3844\left(5x^2+3x+2\right)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}+\frac{\sqrt{11\left(1+4\sqrt{2}\right)}\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)
^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(
3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sq
rt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(353
1015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[2] + (9832420
 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 197886
1*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 +
 2498852071250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*S
qrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
```

```
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-\frac{195}{2}+35x+40x^2\right)}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{\left(\frac{66735}{4}-7375x-25840x^2\right)\sqrt{3-}}{2+3x+5x^2}}{9610}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

Mathematica [C] time = 2.00, size = 1009, normalized size = 3.59

$$12599950\sqrt{286-22i\sqrt{31}}\ \tanh^{-1}\left(\frac{-4i\sqrt{31}\,x+22x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\ \sqrt{2x^2-x+3}}\right)x^4 - 12290525i\sqrt{682\left(13-i\sqrt{31}\right)}\ \tanh^{-1}\left(\frac{-4i\sqrt{31}\,x+2}{2\sqrt{286-22i\sqrt{}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (153804640*Sqrt[3 - x + 2*x^2] + 474815220*x*Sqrt[3 - x + 2*x^2] + 64020704
0*x^2*Sqrt[3 - x + 2*x^2] + 662597100*x^3*Sqrt[3 - x + 2*x^2] + 1906624*Sqr
t[2]*(2 + 3*x + 5*x^2)^2*ArcSinh[(-1 + 4*x)/Sqrt[23]] - I*Sqrt[286 + (22*I)
*Sqrt[31]]*(-503998*I + 491621*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTanh[(63 +
I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
 - x + 2*x^2])] - (1966484*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*
Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
 x + 2*x^2])] + 2015992*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] - (5899452*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] + 6047976*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] +
 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2
])] - (14257009*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31
] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])] + 14615942*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31]
 + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] - (14748630*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^3*ArcTanh[(-63 + I*Sqrt[
31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2])] + 15119940*Sqrt[286 - (22*I)*Sqrt[31]]*x^3*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] - (12290525*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^4*ArcTanh[(-63 + I*Sqr
t[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x
+ 2*x^2])] + 12599950*Sqrt[286 - (22*I)*Sqrt[31]]*x^4*ArcTanh[(-63 + I*Sqrt
[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
 2*x^2])])/(59582000*(2 + 3*x + 5*x^2)^2)
```

fricas [B] time = 2.98, size = 2240, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/7587141599211748089090757280000*(3184949732636*3868444992270541948232^(1/4
)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*s
qrt(3531015707557*sqrt(2) + 4997704142500)*arctan(1/4535484880629403103991 7
89624695893204150231*(2850690442882*sqrt(1999081657)*(2*386844499227054194 8
232^(3/4)*sqrt(62)*(2627559914*x^7 - 10187615527*x^6 + 21362956024*x^5 - 34
451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - sqrt(2)*(1893366636*x^7
- 7237484076*x^6 + 15226003533*x^5 - 24262105817*x^4 + 12127036096*x^3 - 56
64787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) +
 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(400116332*x^7 - 6149336
082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000
*x^2 - sqrt(2)*(286685371*x^7 - 4395067059*x^6 + 23180544704*x^5 - 52748573
780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432)
- 68184612864*x + 48625837056))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt
(2) + 4997704142500) + 12874924822431853972621854418491567805329688*sqrt(31
)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4
- 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 78
3113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 115
4304*x - 456192) - sqrt(1999081657/828550919)*(sqrt(1999081657)*(2*38684449
92270541948232^(3/4)*sqrt(62)*(9351066298*x^7 - 13433496653*x^6 + 433103458
23*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - sqrt(2)*(683
9273266*x^7 - 9809465289*x^6 + 31524099699*x^5 - 12024617744*x^4 + 13914887
256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3
```

```
868444992270541948232^(1/4)*sqrt(62)*(1427210918*x^7 - 18462714328*x^6 + 71
210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - sqrt
(2)*(1033310523*x^7 - 13365477772*x^6 + 51521534980*x^5 - 66583614528*x^4 +
 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x))*sqrt(
2*x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 45164233298567
21284677540671884*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 -
 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 -
 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600
*x^2 - 1036800*x) + 3276288*x) + 205291969538941876576251848722*sqrt(31)*(2
54591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742
19328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*
x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(38684
44992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(353101570755
7*sqrt(2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553
000872744*sqrt(2)*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588
97968185785200)/x^2) + 146305963891271067870702891119222361424201*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 31849
49732636*3868444992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25
*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(3531015707557*sqrt(2) + 49977041425
00)*arctan(1/4535484880629403103991789624695893204150231*(2850690442882*sqr
t(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(2627559914*x^7 - 10
187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 83207
57400*x^2 - sqrt(2)*(1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24
262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 93380256
00) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^(1/
4)*sqrt(62)*(400116332*x^7 - 6149336082*x^6 + 32552996440*x^5 - 74427496472
*x^4 + 96235107840*x^3 - 61219656000*x^2 - sqrt(2)*(286685371*x^7 - 4395067
059*x^6 + 23180544704*x^5 - 52748573780*x^4 + 68065744032*x^3 - 42544702944
*x^2 - 48625837056*x + 34092306432) - 68184612864*x + 48625837056))*sqrt(2*
x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) - 1287492482243185
3972621854418491567805329688*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(1999081657/8285
50919)*(sqrt(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(93510662
98*x^7 - 13433496653*x^6 + 43310345823*x^5 - 17374572240*x^4 + 20927636280*
x^3 + 18483199488*x^2 - sqrt(2)*(6839273266*x^7 - 9809465289*x^6 + 31524099
699*x^5 - 12024617744*x^4 + 13914887256*x^3 + 14839341696*x^2 - 14839341696
*x) - 18483199488*x) + 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(1
427210918*x^7 - 18462714328*x^6 + 71210222920*x^5 - 92387041920*x^4 + 11948
9780160*x^3 + 68726817792*x^2 - sqrt(2)*(1033310523*x^7 - 13365477772*x^6 +
 51521534980*x^5 - 66583614528*x^4 + 85122955872*x^3 + 53108877312*x^2 - 53
108877312*x) - 68726817792*x))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt(
2) + 4997704142500) - 4516423329856721284677540671884*sqrt(31)*sqrt(2)*(123
408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 +
 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 205291
969538941876576251848722*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((3868444992270541948232^(1/4)*sqrt(1999081657)
*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2141441*x + 1076175) - 321
7616*x - 1065266)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 15599087743
0002205517374*x^2 + 140073440957553000872744*sqrt(2)*(2*x^2 - x + 3) - 4807
```

```
0658146796598026782 6*x + 6366974588979681857852 00)/x^2) - 14630596389127106
78707028911192223614242 01*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^
6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqr
t(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1
080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*
x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^
2 - 24772608*x + 18579456)) + 38684449922705419482 32^(1/4)*sqrt(1999081657)
*sqrt(62)*sqrt(31)*(124942603562500*x^4 + 149931124275000*x^3 + 14493342013
2500*x^2 - 3531015707557*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 59
972449710000*x + 19990816570000)*sqrt(3531015707557*sqrt(2) + 4997704142500
)*log(3123565089062500/828550919*(38684449922705419482 32^(1/4)*sqrt(1999081
657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2141441*x + 1076175) -
 3217616*x - 1065266)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 1559908
77430002205517374*x^2 + 1400734409575530008727 44*sqrt(2)*(2*x^2 - x + 3) -
4807065814679659802678 26*x + 6366974588979681857852 00)/x^2) - 3868444992270
541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*(124942603562500*x^4 + 1
49931124275000*x^3 + 144933420132500*x^2 - 3531015707557*sqrt(2)*(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4) + 59972449710000*x + 19990816570000)*sqrt(35310
15707557*sqrt(2) + 4997704142500)*log(-3123565089062500/828550919*(38684449
92270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)
*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(3531015707557*s
qrt(2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553000
872744*sqrt(2)*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588979
68185785200)/x^2) + 121394265587387969425452116 48*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
+ 16*x - 25) + 868455333936828605411012 80*(97155*x^3 + 93872*x^2 + 69621*x
+ 22552)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 59.56Done
```

maple [B] time = 0.38, size = 119458, normalized size = 425.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{\frac{5}{2}}}{\left(5\,x^2 + 3\,x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{5/2}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3,x)
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{\left(5x^{2}+3x+2\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)
```

3.79 $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{15428243\sqrt{2x^2-x+3}\,x^2}{131072}+\frac{1572007407\sqrt{2x^2-x+3}\,x}{7340032}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}+\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+$$

```
[Out] 2899366573/16777216*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+16493087661/2936
0128*(2*x^2-x+3)^(1/2)+1572007407/7340032*x*(2*x^2-x+3)^(1/2)-15428243/1310
72*x^2*(2*x^2-x+3)^(1/2)-19750457/229376*x^3*(2*x^2-x+3)^(1/2)+686531/6144*
x^4*(2*x^2-x+3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+57375/448*x^6*(2*
x^2-x+3)^(1/2)+625/16*x^7*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+\frac{57375}{448}\sqrt{2x^2-x+3}\,x^6+\frac{2116475\sqrt{2x^2-x+3}\,x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}\,x^4}{6144}-\frac{19750457\sqrt{}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (16493087661*Sqrt[3 - x + 2*x^2])/29360128 + (1572007407*x*Sqrt[3 - x + 2*x
^2])/7340032 - (15428243*x^2*Sqrt[3 - x + 2*x^2])/131072 - (19750457*x^3*Sq
rt[3 - x + 2*x^2])/229376 + (686531*x^4*Sqrt[3 - x + 2*x^2])/6144 + (211647
5*x^5*Sqrt[3 - x + 2*x^2])/10752 + (57375*x^6*Sqrt[3 - x + 2*x^2])/448 + (6
25*x^7*Sqrt[3 - x + 2*x^2])/16 + (2899366573*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
8388608*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{1}{16}\int \frac{256+1536x+6016x^2+14976x^3+28176x^4+37440x^5+}{\sqrt{3-x+2x^2}}$$

$$= \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{1}{224}\int \frac{3584+21504x+84224x^2+20}{\sqrt{}}$$

$$= \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} + \int \frac{43008+258}{}$$

$$= \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{}$$

$$= -\frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57}{4}$$

$$= -\frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + 2$$

$$= \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x+2x^2}}{229376} +$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-103677\right.}{352321536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 758
4175488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 34406400
00*x^7) + 60886698033*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/352321536
```

fricas [A] time = 0.77, size = 88, normalized size = 0.48

$$\frac{1}{88080384}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 9842108416
*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sqrt
(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.50, size = 83, normalized size = 0.45

$$\frac{1}{88080384}\left(4\left(8\left(4\left(16\left(100\left(120\left(140\,x+459\right)x+84659\right)x+4805717\right)x-59251371\right)x-323993103\right)x+47160\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x -
 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x +
 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3)) + 1)
```

maple [A] time = 0.02, size = 147, normalized size = 0.79

$$\frac{625\sqrt{2x^2-x+3}\,x^7}{16}+\frac{57375\sqrt{2x^2-x+3}\,x^6}{448}+\frac{2116475\sqrt{2x^2-x+3}\,x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}\,x^4}{6144}-\frac{19750457}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 16493087661/29360128*(2*x^2-x+3)^(1/2)-2899366573/16777216*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+625/16*x^7*(2*x^2-x+3)^(1/2)+57375/448*x^6*(2*x^2-x+
3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+686531/6144*x^4*(2*x^2-x+3)^(1
/2)-19750457/229376*x^3*(2*x^2-x+3)^(1/2)-15428243/131072*x^2*(2*x^2-x+3)^(
1/2)+1572007407/7340032*x*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 1.00, size = 148, normalized size = 0.80

$$\frac{625}{16}\sqrt{2\,x^2-x+3}\,x^7+\frac{57375}{448}\sqrt{2\,x^2-x+3}\,x^6+\frac{2116475}{10752}\sqrt{2\,x^2-x+3}\,x^5+\frac{686531}{6144}\sqrt{2\,x^2-x+3}\,x^4-\frac{1975045}{229376}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 625/16*sqrt(2*x^2 - x + 3)*x^7 + 57375/448*sqrt(2*x^2 - x + 3)*x^6 + 211647
5/10752*sqrt(2*x^2 - x + 3)*x^5 + 686531/6144*sqrt(2*x^2 - x + 3)*x^4 - 197
50457/229376*sqrt(2*x^2 - x + 3)*x^3 - 15428243/131072*sqrt(2*x^2 - x + 3)*
x^2 + 1572007407/7340032*sqrt(2*x^2 - x + 3)*x - 2899366573/16777216*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16493087661/29360128*sqrt(2*x^2 - x +
3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^4}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^4}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/sqrt(2*x**2 - x + 3), x)
```

3.80 $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}\,x}{8192}-\frac{203373\sqrt{2x^2-x+3}}{32768}+\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x}$$

```
[Out] -9267707/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-203373/32768*(2*x^2-
x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-3387/1024*x^2*(2*x^2-x+3)^(1/2)+
8185/256*x^3*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*x^2-x+3)^(1/2)+125/12*x^5*(2*
x^2-x+3)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1661, 640, 619, 215}

$$\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x+3}\,x^4+\frac{8185}{256}\sqrt{2x^2-x+3}\,x^3-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x+}}{8192}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-203373*Sqrt[3 - x + 2*x^2])/32768 - (372783*x*Sqrt[3 - x + 2*x^2])/8192 -
 (3387*x^2*Sqrt[3 - x + 2*x^2])/1024 + (8185*x^3*Sqrt[3 - x + 2*x^2])/256 +
 (1355*x^4*Sqrt[3 - x + 2*x^2])/48 + (125*x^5*Sqrt[3 - x + 2*x^2])/12 - (92
67707*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{12}\int \frac{96+432x+1368x^2+2484x^3+1545x^4+\frac{6775x^5}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{120}\int \frac{960+4320x+13680x^2-15810x}{\sqrt{3-x+2x^2}}$$

$$= \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{960}\int \frac{7680+3}{}$$

$$= -\frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x}$$

$$= -\frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-}$$

Mathematica [A] time = 0.13, size = 65, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(1024000x^5+2775040x^4+3143040x^3-325152x^2-4473396x-610119\right)-27803121\sqrt{2}\sinh^{-1}}{393216}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 27
75040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/39
3216
```

fricas [A] time = 0.68, size = 78, normalized size = 0.55

$$\frac{1}{98304}\left(1024000\,x^5+2775040\,x^4+3143040\,x^3-325152\,x^2-4473396\,x-610119\right)\sqrt{2\,x^2-x+3}+\frac{9267707}{262144}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x -
 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt(2
*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.52, size = 73, normalized size = 0.51

$$\frac{1}{98304}\left(4\left(8\left(20\left(16\left(100\,x+271\right)x+4911\right)x-10161\right)x-1118349\right)x-610119\right)\sqrt{2\,x^2-x+3}-\frac{9267707}{131072}\sqrt{2}\log\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x - 6
10119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 113, normalized size = 0.79

$$\frac{125\sqrt{2x^2-x+3}\,x^5}{12}+\frac{1355\sqrt{2x^2-x+3}\,x^4}{48}+\frac{8185\sqrt{2x^2-x+3}\,x^3}{256}-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x}}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] -203373/32768*(2*x^2-x+3)^(1/2)+9267707/131072*2^(1/2)*arcsinh(4/23*23^(1/2
)*(x-1/4))+125/12*(2*x^2-x+3)^(1/2)*x^5+1355/48*(2*x^2-x+3)^(1/2)*x^4+8185/
256*(2*x^2-x+3)^(1/2)*x^3-3387/1024*(2*x^2-x+3)^(1/2)*x^2-372783/8192*(2*x^
2-x+3)^(1/2)*x
```

maxima [A] time = 0.98, size = 114, normalized size = 0.80

$$\frac{125}{12}\sqrt{2\,x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2\,x^2-x+3}\,x^4+\frac{8185}{256}\sqrt{2\,x^2-x+3}\,x^3-\frac{3387}{1024}\sqrt{2\,x^2-x+3}\,x^2-\frac{372783}{8192}\sqrt{2\,x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/256
*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/8192*
sqrt(2*x^2 - x + 3)*x + 9267707/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
 1)) - 203373/32768*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^3}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2} + 3x + 2\right)^{3}}{\sqrt{2x^{2} - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/sqrt(2*x**2 - x + 3), x)
```

3.81 $\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=101

$$\frac{655}{96}\sqrt{2x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2x^2-x+3}\,x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{25}{8}\sqrt{2x^2-x+3}\,x^3+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

```
[Out] 30725/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11373/1024*(2*x^2-x+3)^(1
/2)+3443/768*x*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x^2-x+3)^(1/2)+25/8*x^3*(2*x
^2-x+3)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{25}{8}\sqrt{2x^2-x+3}\,x^3+\frac{655}{96}\sqrt{2x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2x^2-x+3}\,x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-11373*Sqrt[3 - x + 2*x^2])/1024 + (3443*x*Sqrt[3 - x + 2*x^2])/768 + (655
*x^2*Sqrt[3 - x + 2*x^2])/96 + (25*x^3*Sqrt[3 - x + 2*x^2])/8 + (30725*ArcS
inh[(1 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8}\int \frac{32+96x+7x^2+\frac{655x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48}\int \frac{192-1389x+\frac{3443x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192}\int \frac{-\frac{7257}{4} - \ldots}{\sqrt{3-x+\ldots}}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+\ldots}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+\ldots}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+\ldots}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}\left(9600x^3+20960x^2+13772x-34119\right)+92175\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*Sq
rt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288
```

fricas [A] time = 0.73, size = 68, normalized size = 0.67

$$\frac{1}{3072}\left(9600\,x^3+20960\,x^2+13772\,x-34119\right)\sqrt{2\,x^2-x+3}+\frac{30725}{8192}\,\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-3\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x -
 25)
```

giac [A] time = 0.49, size = 63, normalized size = 0.62

$$\frac{1}{3072}\,(4\,(40\,(60\,x+131)x+3443)x-34119)\sqrt{2\,x^2-x+3}+\frac{30725}{4096}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.78

$$\frac{25\sqrt{2x^2-x+3}\,x^3}{8}+\frac{655\sqrt{2x^2-x+3}\,x^2}{96}+\frac{3443\sqrt{2x^2-x+3}\,x}{768}-\frac{30725\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{11373\sqrt{2x^2-}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 25/8*(2*x^2-x+3)^(1/2)*x^3+655/96*(2*x^2-x+3)^(1/2)*x^2+3443/768*(2*x^2-x+3
)^(1/2)*x-11373/1024*(2*x^2-x+3)^(1/2)-30725/4096*2^(1/2)*arcsinh(4/23*23^(
1/2)*(x-1/4))
```

maxima [A] time = 0.97, size = 80, normalized size = 0.79

$$\frac{25}{8}\sqrt{2x^2-x+3}\,x^3+\frac{655}{96}\sqrt{2x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2x^2-x+3}\,x-\frac{30725}{4096}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{11373}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*sq
rt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) -
 11373/1024*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5x^2+3x+2\right)^2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/sqrt(2*x**2 - x + 3), x)
```

3.82 $\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=59

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] 17/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+39/16*(2*x^2-x+3)^(1/2)+5/4*x*
(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1
- 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx &= \frac{5}{4}x\sqrt{3-x+2x^2}+\frac{1}{4}\int \frac{-7+\frac{39x}{2}}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}-\frac{17}{32}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}-\frac{17\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{32\sqrt{46}} \\
&= \frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.76

$$\frac{1}{64}\left(4\sqrt{2x^2-x+3}\,(20x+39)+17\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
[Out] (4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/64
```

fricas [A] time = 0.81, size = 58, normalized size = 0.98

$$\frac{1}{16}\sqrt{2\,x^2-x+3}\,(20\,x+39)+\frac{17}{128}\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2*
x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.53, size = 53, normalized size = 0.90

$$\frac{1}{16}\sqrt{2\,x^2-x+3}\,(20\,x+39)+\frac{17}{64}\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 45, normalized size = 0.76

$$\frac{5\sqrt{2x^2-x+3}\;x}{4}-\frac{17\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}+\frac{39\sqrt{2x^2-x+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/4*(2*x^2-x+3)^(1/2)*x+39/16*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 46, normalized size = 0.78

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x-\frac{17}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{39}{16}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 39/16*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)
```

3.83 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$

Optimal. Leaf size=148

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\sqrt{\frac{11}{31(10\sqrt{\cdots}}}\right.$$

```
[Out] -1/682*arctanh(1/31*(7+x*(13-10*2^(1/2))-3*2^(1/2))*341^(1/2)/(-13+10*2^(1/
2))^(1/2)/(2*x^2-x+3)^(1/2))*(-8866+6820*2^(1/2))^(1/2)+1/682*arctan(1/31*(
7+3*2^(1/2)+x*(13+10*2^(1/2)))*341^(1/2)/(13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^
(1/2))*(8866+6820*2^(1/2))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {986, 1029, 204, 206}

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\sqrt{\frac{11}{31(10\sqrt{\cdots}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
```

```
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx = -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{22\sqrt{2}} + \frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{22\sqrt{2}}$$

$$= -\left(\frac{1}{2}\left(11\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-3751\left(13-10\sqrt{2}\right)-11x^2}\,dx, x, \frac{11(\ldots}{\ldots}\right)\right.$$

$$= \sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right.$$

Mathematica [C] time = 0.30, size = 176, normalized size = 1.19

$$-\frac{\sqrt{13+i\sqrt{31}}\left(\sqrt{31}+13i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+\sqrt{13-i\sqrt{31}}\left(\sqrt{31}-13i\right)\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x\ldots}{2\sqrt{286-22i\sqrt{31}}\ldots}\right.}{20\sqrt{682}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] -1/20*(Sqrt[13 + I*Sqrt[31]]*(13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (
-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]
)] + Sqrt[13 - I*Sqrt[31]]*(-13*I + Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-
22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])
])/Sqrt[682]
```

fricas [B] time = 2.25, size = 2002, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/845680*sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sq
rt(2) - 20)*log(1240*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) + 7595*x^2 + 6820*s
qrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) + 1/845680*sqrt(341)*200^(1/
4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-1240*(sqrt
(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3
*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(2)*(2*x^2 - x + 3) + 2
3405*x - 31000)/x^2) - 1/6820*sqrt(341)*200^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*s
qrt(2) + 20)*arctan(1/2762875*(14260*sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 1
8288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^
3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^(1/4)*(18463*x^
7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - sq
rt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 20
```

05920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*sqrt(13*sqrt(2) + 20) + 7843000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(310)*(sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^(1/4)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*sqrt(13*sqrt(2) + 20) + 550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(2)*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) + 89125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 1/6820*sqrt(341)*200^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/2762875*(14260*sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^(1/4)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - sqrt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*sqrt(13*sqrt(2) + 20) - 7843000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(310)*(sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^(1/4)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*sqrt(13*sqrt(2) + 20) - 550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) + 7595*x^2 + 6820*sqrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) - 89125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 7.57
Done
```

maple [B] time = 0.00, size = 684, normalized size = 4.62

$$\frac{\sqrt{\frac{8\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+\frac{3\sqrt{2}\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+8-3\sqrt{2}}\;\sqrt{2}\left(465124\sqrt{2}\,\operatorname{arctanh}\left(\frac{31\sqrt{\frac{8\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+\frac{3\sqrt{2}\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)-866\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 1/21142*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2^(1/2))^(1
/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2
)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2
)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+520*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/
2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23
*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^
(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+465124*arctanh(3
1/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arcta
nh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-
1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2
))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1
))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)),x)
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)
```

3.84 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{682\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+211\right.}{\sqrt{2x^2-x+3}}\right.}{1364}$$

```
[Out] 1/682*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/930248*arctanh(1/31*(2119+
x*(5751-3935*2^(1/2))-1816*2^(1/2))*341^(1/2)/(-2343727+1678700*2^(1/2))^(1
/2)/(2*x^2-x+3)^(1/2))*(-1598421814+1144873400*2^(1/2))^(1/2)+1/930248*arct
an(1/31*(2119+1816*2^(1/2)+x*(5751+3935*2^(1/2)))*341^(1/2)/(2343727+167870
0*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1598421814+1144873400*2^(1/2))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{682\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+211\right.}{\sqrt{2x^2-x+3}}\right.}{1364}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 +
 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*(2
119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1364 -
 (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1
678700*Sqrt[2]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/1364
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
```

```
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{-1826+\frac{2255x}{2}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{121}{2}\left(537-332\sqrt{2}\right)-\frac{121}{2}\left(127-205\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{165044\sqrt{2}} + \frac{\int \frac{\frac{121}{2}(537+\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{1}{496}\left(11\left(3357400-2343727\sqrt{2}\right)\right)\text{Subst}\left(\int \cdots\right.$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{31(2343\cdots}}{\cdots}\right)\cdots}{1364}$$

Mathematica [C] time = 1.00, size = 287, normalized size = 1.53

$$\frac{25\left(\frac{i\sqrt{286+22i\sqrt{31}}\left(224\sqrt{31}+1023i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2} + \frac{10i\left(1364\left(\sqrt{31}+13i\right)(65x+4)\sqrt{2x^2-x+3}-5\sqrt{286-22i\sqrt{31}}\left(787\sqrt{31}-12\cdots\right.\right.}{\left(\sqrt{31}+13i\right)^2\left(10ix+\sqrt{31}+3i\right)\left(5\left(\sqrt{31}-\cdots\right.\right.}\cdots\right)}{116281}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (25*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1023*I + 224*Sqrt[31])*ArcTanh[(63 + I
*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + ((10*I)*(1364*(13*I + Sqrt[31])*(4
+ 65*x)*Sqrt[3 - x + 2*x^2] - 5*Sqrt[286 - (22*I)*Sqrt[31]]*(-1271*I + 787*
Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31
])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/((13*I + Sqrt[
31])^2*(3*I + Sqrt[31] + (10*I)*x)*(-4*I + 8*Sqrt[31] + 5*(-13*I + Sqrt[31]
)*x))))/116281
```

fricas [**B**] time = 2.60, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/263043507934399808*(8422204*563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)
*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2) + 3357400)*arctan(1/710190022151725
4683789*(47876524*sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(2950932*x^7 -
11691762*x^6 + 24397746*x^5 - 40053004*x^4 + 20309552*x^3 - 10145376*x^2 -
sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17801494*x^5 - 27869789*x^4 + 13808040
*x^3 - 6172200*x^2 - 15724800*x + 10596096) - 21192192*x + 15724800) + 5203
97*563606738^(1/4)*sqrt(341)*(226651*x^7 - 3496629*x^6 + 18614024*x^5 - 428
60780*x^4 + 55586592*x^3 - 36274464*x^2 - sqrt(2)*(168871*x^7 - 2579646*x^6
 + 13533020*x^5 - 30582616*x^4 + 39345120*x^3 - 23947200*x^2 - 28449792*x +
 19450368) - 38900736*x + 28449792))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(
2) + 3357400) + 2016023288887690715272*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(33
574/2191)*(sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(10257392*x^7 - 147733
68*x^6 + 47877288*x^5 - 20710528*x^4 + 26321472*x^3 + 17079552*x^2 - sqrt(2
)*(8292238*x^7 - 11867543*x^6 + 37968813*x^5 - 13449840*x^4 + 14570280*x^3
+ 20176128*x^2 - 20176128*x) - 17079552*x) + 520397*563606738^(1/4)*sqrt(34
1)*(795513*x^7 - 10292932*x^6 + 39734380*x^5 - 51864768*x^4 + 68281632*x^3
+ 34255872*x^2 - 8*sqrt(2)*(77213*x^7 - 998548*x^6 + 3846220*x^5 - 4943520*
x^4 + 6215760*x^3 + 4318272*x^2 - 4318272*x) - 34255872*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2343727*sqrt(2) + 3357400) + 421088065768678*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 19140
366625849*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5
+ 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*
x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 14482
0224*x))*sqrt(-(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357
400) - 1731948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 53372285
80187*x - 7069176927400)/x^2) + 22909355532814667219*sqrt(31)*(2828123*x^8
 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x
^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^
5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 948879
36))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4
+ 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 8422204*563606738
^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2)
 + 3357400)*arctan(1/7101900221517254683789*(47876524*sqrt(33574)*(22*56360
6738^(3/4)*sqrt(341)*(2950932*x^7 - 11691762*x^6 + 24397746*x^5 - 40053004*
x^4 + 20309552*x^3 - 10145376*x^2 - sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17
801494*x^5 - 27869789*x^4 + 13808040*x^3 - 6172200*x^2 - 15724800*x + 10596
```

```
096) - 21192192*x + 15724800) + 520397*563606738^(1/4)*sqrt(341)*(226651*x^
7 - 3496629*x^6 + 18614024*x^5 - 42860780*x^4 + 55586592*x^3 - 36274464*x^2
 - sqrt(2)*(168871*x^7 - 2579646*x^6 + 13533020*x^5 - 30582616*x^4 + 393451
20*x^3 - 23947200*x^2 - 28449792*x + 19450368) - 38900736*x + 28449792))*sq
rt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) - 20160232886887690715272
*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549
144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*
x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53913
6) + 1154304*x - 456192) - 2*sqrt(33574/2191)*(sqrt(33574)*(22*563606738^(3
/4)*sqrt(341)*(10257392*x^7 - 14773368*x^6 + 47877288*x^5 - 20710528*x^4 +
26321472*x^3 + 17079552*x^2 - sqrt(2)*(8292238*x^7 - 11867543*x^6 + 3796881
3*x^5 - 13449840*x^4 + 14570280*x^3 + 20176128*x^2 - 20176128*x) - 17079552
*x) + 520397*563606738^(1/4)*sqrt(341)*(795513*x^7 - 10292932*x^6 + 3973438
0*x^5 - 51864768*x^4 + 68281632*x^3 + 34255872*x^2 - 8*sqrt(2)*(77213*x^7 -
 998548*x^6 + 3846220*x^5 - 4943520*x^4 + 6215760*x^3 + 4318272*x^2 - 43182
72*x) - 34255872*x))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) -
421088065768678*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3
293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 1
18051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x
^2 - 1036800*x) + 3276288*x) - 19140366625849*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((563606738^(1/4)*sqrt(335
74)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x
 - 225)*sqrt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028
*sqrt(2)*(2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 22909355
5532814667219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*(16787000*
x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x + 2) + 10072200*x + 6714800)*sqrt(234372
7*sqrt(2) + 3357400)*log(335740000/2191*(563606738^(1/4)*sqrt(33574)*sqrt(3
41)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sq
rt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028*sqrt(2)*(
2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 563606738^(1/4)*sq
rt(33574)*sqrt(341)*sqrt(31)*(16787000*x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x +
 2) + 10072200*x + 6714800)*sqrt(2343727*sqrt(2) + 3357400)*log(-335740000/
2191*(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(s
qrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357400) - 173
1948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 5337228580187*x -
7069176927400)/x^2) + 385694293158944*sqrt(2*x^2 - x + 3)*(65*x + 4))/(5*x^
2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
```

```
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 27.71Done
```

maple [B]   time = 0.01, size = 5225, normalized size = 27.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

3.85 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{1364\left(5x^2+3x+2\right)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\left(5x^2+3x+2\right)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\tan^{-1}\left(\sqrt{31}\ldots\right.}{372\ldots}$$

```
[Out] 1/1364*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+1/1860496*(26794+86265*x)
*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-25/2537716544*arctanh(1/31*(123161+x*(2946
69-208915*2^(1/2))-85754*2^(1/2))*341^(1/2)/(-6414867847+4536374600*2^(1/2)
)^(1/2)/(2*x^2-x+3)^(1/2))*(-4374939871654+3093807477200*2^(1/2))^(1/2)+25/
2537716544*arctan(1/31*(123161+85754*2^(1/2)+x*(294669+208915*2^(1/2)))*341
^(1/2)/(6414867847+4536374600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(4374939871
654+3093807477200*2^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{1364\left(5x^2+3x+2\right)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\left(5x^2+3x+2\right)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\tan^{-1}\left(\sqrt{31}\ldots\right.}{372\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 862
65*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*
Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*Ar
cTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqr
t[2] + (294669 - 208915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/3720992
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
```

```
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5775+\frac{6479x}{2}-2860x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \frac{-\frac{28220225}{2}+}{\sqrt{3-x+2x^2}\,(}}{11256}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \frac{33275}{4}(26103}{}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\left(6875\left(9072\right.\right.}{}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} + \frac{25\sqrt{\frac{1}{682}\left(64\right.}}{}$$

Mathematica [C] time = 6.23, size = 1277, normalized size = 5.73

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I + Sqrt[31])*(3 - I*Sqrt[31]
+ 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 - I*Sqrt[31])*(3 - I*Sqr
t[31] + 10*x)) - (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I - Sqrt[31])
*(3 + I*Sqrt[31] + 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 + I*Sqr
t[31])*(3 + I*Sqrt[31] + 10*x)) - (375*Sqrt[(2*(13 - I*Sqrt[31]))/11]*(11 -
 (2*I)*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 2*(11 - (2*I)*Sqrt[31])*x)/(2*S
qrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(10571*(13*I + Sqrt[31])^2
) - (750*Sqrt[(2*(13 - I*Sqrt[31]))/341]*ArcTanh[(63 - I*Sqrt[31] - 2*(11 -
 (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(9
61*(13*I + Sqrt[31])) + (750*Sqrt[(2*(13 + I*Sqrt[31]))/341]*ArcTanh[(63 +
I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]*Sqrt[
3 - x + 2*x^2])])/(961*(13*I - Sqrt[31])) - (375*Sqrt[(2*(13 + I*Sqrt[31]))
/11]*(11 + (2*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31
])*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(10571*(13*I - S
qrt[31])^2) + (((500*I)/31)*(((-20*(-3 + I*Sqrt[31]) + 10*(27 - (4*I)*Sqrt[
31]))*Sqrt[3 - x + 2*x^2])/((300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[31
])^2)*(-3 + I*Sqrt[31] - 10*x)) + (2*Sqrt[22*(13 - I*Sqrt[31])]*(20*(-3 + I
*Sqrt[31]) + 10*(27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(-3 + I*Sqrt[31])*(27 -
(4*I)*Sqrt[31])))*ArcTanh[(-63 + I*Sqrt[31] - (-10 + 4*(-3 + I*Sqrt[31]))*x
)/(2*Sqrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/((300 - 10*(-3 + I*S
qrt[31]) + 2*(-3 + I*Sqrt[31])^2)*(1200 - 40*(-3 + I*Sqrt[31]) + 8*(-3 + I*
Sqrt[31])^2))))/(Sqrt[31]*(300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[31])
^2)) + (((500*I)/31)*(((20*(3 + I*Sqrt[31]) - 10*(-27 - (4*I)*Sqrt[31]))*Sq
rt[3 - x + 2*x^2])/((300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2)*(3 +
 I*Sqrt[31] + 10*x)) + (2*Sqrt[22*(13 + I*Sqrt[31])]*(-20*(3 + I*Sqrt[31])
- 10*(-27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(3 + I*Sqrt[31])*(-27 - (4*I)*Sqrt
```

```
[31])))*ArcTanh[(63 + I*Sqrt[31] - (10 + 4*(3 + I*Sqrt[31]))*x)/(2*Sqrt[22*
(13 + I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/((300 + 10*(3 + I*Sqrt[31]) + 2*(
3 + I*Sqrt[31])^2)*(1200 + 40*(3 + I*Sqrt[31]) + 8*(3 + I*Sqrt[31])^2))))/(
Sqrt[31]*(300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2))
```

fricas [B] time = 2.37, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2123440000274774263468221 44*(46113488900*4115738902305032^(1/4)*sqrt(226
81873)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867
847*sqrt(2) + 9072749200)*arctan(1/383666830929400953005832237394876 9*(6468
8701796*sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*(160344708*x^7
- 615873378*x^6 + 1294230774*x^5 - 2070733376*x^4 + 1037098288*x^3 - 489164
544*x^2 - sqrt(2)*(112700446*x^7 - 434839553*x^6 + 912850886*x^5 - 14661276
91*x^4 + 735661560*x^3 - 350098200*x^2 - 799200000*x + 567316224) - 1134632
448*x + 799200000) + 703138063*4115738902305032^(1/4)*sqrt(341)*(12162569*x
^7 - 186616851*x^6 + 985490056*x^5 - 2246141620*x^4 + 2900382048*x^3 - 1823
848416*x^2 - sqrt(2)*(8564099*x^7 - 131508024*x^6 + 695288980*x^5 - 1587105
104*x^4 + 2050714080*x^3 - 1296806400*x^2 - 1457077248*x + 1033108992) - 20
66217984*x + 1457077248))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 907
2749200) + 10891187458641059311133302222822312*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(45363746/479849)*(sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*
(576322648*x^7 - 827050092*x^6 + 2660713572*x^5 - 1032439232*x^4 + 12116047
68*x^3 + 1213394688*x^2 - sqrt(2)*(403157522*x^7 - 578844217*x^6 + 18641293
47*x^5 - 735062160*x^4 + 873708120*x^3 + 823986432*x^2 - 823986432*x) - 121
3394688*x) + 703138063*4115738902305032^(1/4)*sqrt(341)*(43684647*x^7 - 565
067708*x^6 + 2178643220*x^5 - 2819241792*x^4 + 3618371808*x^3 + 2197767168*
x^2 - 2*sqrt(2)*(15328963*x^7 - 198290348*x^6 + 764653220*x^5 - 990717120*x
^4 + 1276256160*x^3 + 755350272*x^2 - 755350272*x) - 2197767168*x))*sqrt(2*
x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) + 168363055004367262339 3
22*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) + 765286613656214828815 1*sqrt(31)*(254591*x^8 - 4815126*x^7
 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x
^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x
^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4115738902305032^(1/4)*sqrt(
22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012)
 - 93199*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187
753*x^2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*
x - 2091880597633419400)/x^2) + 12376349384819385580833297980 4799*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 46113
488900*4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(2)*(25*x^4 + 30
*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867847*sqrt(2) + 9072749200)*arctan(1/38
36668309294009530058322373948769*(64688701796*sqrt(22681873)*(11*4115738902
305032^(3/4)*sqrt(341)*(160344708*x^7 - 615873378*x^6 + 1294230774*x^5 - 20
70733376*x^4 + 1037098288*x^3 - 489164544*x^2 - sqrt(2)*(112700446*x^7 - 43
4839553*x^6 + 912850886*x^5 - 1466127691*x^4 + 735661560*x^3 - 350098200*x^
2 - 799200000*x + 567316224) - 1134632448*x + 799200000) + 703138063*411573
```

```
8902305032^(1/4)*sqrt(341)*(12162569*x^7 - 186616851*x^6 + 985490056*x^5 -
2246141620*x^4 + 2900382048*x^3 - 1823848416*x^2 - sqrt(2)*(8564099*x^7 - 1
31508024*x^6 + 695288980*x^5 - 1587105104*x^4 + 2050714080*x^3 - 1296806400
*x^2 - 1457077248*x + 1033108992) - 2066217984*x + 1457077248))*sqrt(2*x^2
- x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) - 108911874586410593111333302
222822312*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x
^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
+ 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*
x - 539136) + 1154304*x - 456192) - 2*sqrt(45363746/479849)*(sqrt(22681873)
*(11*4115738902305032^(3/4)*sqrt(341)*(576322648*x^7 - 827050092*x^6 + 2660
713572*x^5 - 1032439232*x^4 + 1211604768*x^3 + 1213394688*x^2 - sqrt(2)*(40
3157522*x^7 - 578844217*x^6 + 1864129347*x^5 - 735062160*x^4 + 873708120*x^
3 + 823986432*x^2 - 823986432*x) - 1213394688*x) + 703138063*41157389023050
32^(1/4)*sqrt(341)*(43684647*x^7 - 565067708*x^6 + 2178643220*x^5 - 2819241
792*x^4 + 3618371808*x^3 + 2197767168*x^2 - 2*sqrt(2)*(15328963*x^7 - 19829
0348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 -
 755350272*x) - 2197767168*x))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2)
+ 9072749200) - 168363055004367262339322*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 7652866136562148288
151*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087
81920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 +
517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x
))*sqrt((4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sq
rt(2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(
2*x^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) - 123763
49384819385580833279804799*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*
x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*s
qrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 +
 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 466120
0*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*
x^2 - 24772608*x + 18579456)) - 25*4115738902305032^(1/4)*sqrt(22681873)*sq
rt(341)*sqrt(31)*(226818730000*x^4 + 272182476000*x^3 + 263109726800*x^2 -
6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 108872990400*x +
 36290996800)*sqrt(6414867847*sqrt(2) + 9072749200)*log(1134093650000000/47
9849*(4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sqrt(
2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(2*x
^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) + 25*411573
8902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*(226818730000*x^4 + 2721
82476000*x^3 + 263109726800*x^2 - 6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4) + 108872990400*x + 36290996800)*sqrt(6414867847*sqrt(2) + 9
072749200)*log(-1134093650000000/479849*(4115738902305032^(1/4)*sqrt(226818
73)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 931
99*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187753*x^
2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*x - 20
91880597633419400)/x^2) - 114133005406879362464*(431325*x^3 + 392765*x^2 +
341572*x + 59044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
```

```
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 42.09Done
```

maple [B] time = 0.01, size = 13040, normalized size = 58.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

3.86 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{111315\sqrt{2x^2-x+3}\,x^2}{2048}-\frac{8992487\sqrt{2x^2-x+3}\,x}{16384}-\frac{31009685\sqrt{2x^2-x+3}}{65536}-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}+\frac{625}{24}\sqrt{2x^2-x}$$

```
[Out] -310445587/262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-14641/1472*(101+79
*x)/(2*x^2-x+3)^(1/2)-31009685/65536*(2*x^2-x+3)^(1/2)-8992487/16384*x*(2*x
^2-x+3)^(1/2)-111315/2048*x^2*(2*x^2-x+3)^(1/2)+79425/512*x^3*(2*x^2-x+3)^(
1/2)+10075/96*x^4*(2*x^2-x+3)^(1/2)+625/24*x^5*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{24}\sqrt{2x^2-x+3}\,x^5+\frac{10075}{96}\sqrt{2x^2-x+3}\,x^4+\frac{79425}{512}\sqrt{2x^2-x+3}\,x^3-\frac{111315\sqrt{2x^2-x+3}\,x^2}{2048}-\frac{8992487\sqrt{2x^2-x}}{16384}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2
*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 -
 x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3
 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(
1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{2821893}{256} - \frac{661181x}{128} - \frac{488267x^2}{64} + \frac{143635x^3}{32} + \frac{213325x^4}{16} + \frac{83375}{8}\cdots}{\sqrt{3-x+2x^2}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{1}{138}\int \frac{\frac{8465679}{64} - \frac{1983543x}{32} - \frac{1464801x^2}{16} + \cdots}{\sqrt{3-x\cdots}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \int \frac{\frac{42328395}{32} - \cdots}{\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\cdots$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\cdots$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x\cdots}{\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x\cdots}{\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x\cdots}{\cdots}$$

Mathematica [A] time = 0.37, size = 95, normalized size = 0.57

$$\sqrt{2x^2-x+3}\left(\frac{625x^5}{24} + \frac{10075x^4}{96} + \frac{79425x^3}{512} - \frac{111315x^2}{2048} - \frac{14641(79x+101)}{1472(2x^2-x+3)} - \frac{8992487x}{16384} - \frac{31009685}{65536}\right) + \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] Sqrt[3 - x + 2*x^2]*(-31009685/65536 - (8992487*x)/16384 - (111315*x^2)/204
8 + (79425*x^3)/512 + (10075*x^4)/96 + (625*x^5)/24 - (14641*(101 + 79*x))/
```

```
(1472*(3 - x + 2*x^2))) + (310445587*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(131072*
Sqrt[2])
```

fricas [A] time = 0.91, size = 112, normalized size = 0.67

$$\frac{21420745503\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(235520000\,x^7+831\right.}{3617}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^6
+ 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 885930
5979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.25, size = 82, normalized size = 0.49

$$-\frac{310445587}{262144}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(46\,(4\,(40\,(20\,(16\,(100\,x+353)x+8707)x+31341)x}{452198}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
+ 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x -
 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x +
3)
```

maple [A] time = 0.03, size = 166, normalized size = 1.00

$$\frac{625x^7}{12\sqrt{2x^2-x+3}}+\frac{8825x^6}{48\sqrt{2x^2-x+3}}+\frac{217675x^5}{768\sqrt{2x^2-x+3}}+\frac{52235x^4}{1024\sqrt{2x^2-x+3}}-\frac{4734827x^3}{8192\sqrt{2x^2-x+3}}-\frac{18367831x^2}{32768\sqrt{2x^2-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -1217267299/524288/(2*x^2-x+3)^(1/2)+1234044515/12058624*(4*x-1)/(2*x^2-x+3
)^(1/2)+310445587/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+217675/768*
x^5/(2*x^2-x+3)^(1/2)+52235/1024*x^4/(2*x^2-x+3)^(1/2)-4734827/8192*x^3/(2*
x^2-x+3)^(1/2)+625/12*x^7/(2*x^2-x+3)^(1/2)+8825/48*x^6/(2*x^2-x+3)^(1/2)-1
8367831/32768*x^2/(2*x^2-x+3)^(1/2)-310445587/131072*x/(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.98, size = 148, normalized size = 0.89

$$\frac{625\,x^7}{12\,\sqrt{2\,x^2-x+3}}+\frac{8825\,x^6}{48\,\sqrt{2\,x^2-x+3}}+\frac{217675\,x^5}{768\,\sqrt{2\,x^2-x+3}}+\frac{52235\,x^4}{1024\,\sqrt{2\,x^2-x+3}}-\frac{4734827\,x^3}{8192\,\sqrt{2\,x^2-x+3}}-\frac{183678}{32768\,\sqrt{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 625/12*x^7/sqrt(2*x^2 - x + 3) + 8825/48*x^6/sqrt(2*x^2 - x + 3) + 217675/7
68*x^5/sqrt(2*x^2 - x + 3) + 52235/1024*x^4/sqrt(2*x^2 - x + 3) - 4734827/8
192*x^3/sqrt(2*x^2 - x + 3) - 18367831/32768*x^2/sqrt(2*x^2 - x + 3) + 3104
45587/262144*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 2953101993/1507328*
x/sqrt(2*x^2 - x + 3) - 3653899049/1507328/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^4}{\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2),x)
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^4}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)
```

3.87 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{1825}{64}\sqrt{2x^2-x+3}\,x^2+\frac{15565}{512}\sqrt{2x^2-x+3}\,x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{125}{16}\sqrt{2x^2-x+3}\,x^3+\frac{11\cdots}{}$$

```
[Out] 1168881/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1331/368*(17-45*x)/(2*x
^2-x+3)^(1/2)-181561/2048*(2*x^2-x+3)^(1/2)+15565/512*x*(2*x^2-x+3)^(1/2)+1
825/64*x^2*(2*x^2-x+3)^(1/2)+125/16*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{125}{16}\sqrt{2x^2-x+3}\,x^3+\frac{1825}{64}\sqrt{2x^2-x+3}\,x^2+\frac{15565}{512}\sqrt{2x^2-x+3}\,x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{11\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2]
)/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])
/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23
]])/(4096*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{110285}{64}-\frac{19067x}{32}+\frac{22195x^2}{16}+\frac{13225x^3}{8}+\frac{2875x^4}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}+\frac{125}{16}x^3\sqrt{3-x+2x^2}+\frac{1}{92}\int \frac{-\frac{110285}{8}-\frac{19067x}{4}+\frac{18515x^2}{4}+\frac{12592}{8}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}+\frac{1825}{64}x^2\sqrt{3-x+2x^2}+\frac{125}{16}x^3\sqrt{3-x+2x^2}+\frac{1}{552}\int \frac{-\frac{330855}{4}}{}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}+\frac{15565}{512}x\sqrt{3-x+2x^2}+\frac{1825}{64}x^2\sqrt{3-x+2x^2}+\frac{125}{16}x^3\sqrt{3-x}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}-\frac{181561\sqrt{3-x+2x^2}}{2048}+\frac{15565}{512}x\sqrt{3-x+2x^2}+\frac{1825}{64}x^2\sqrt{3-x}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}-\frac{181561\sqrt{3-x+2x^2}}{2048}+\frac{15565}{512}x\sqrt{3-x+2x^2}+\frac{1825}{64}x^2\sqrt{3-x}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}-\frac{181561\sqrt{3-x+2x^2}}{2048}+\frac{15565}{512}x\sqrt{3-x+2x^2}+\frac{1825}{64}x^2\sqrt{3-x}$$

Mathematica [A] time = 0.23, size = 65, normalized size = 0.52

$$\frac{\frac{4\left(736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965\right)}{\sqrt{2x^2-x+3}}-26884263\sqrt{2}\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{188416}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] ((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 736
000*x^5))/Sqrt[3 - x + 2*x^2] - 26884263*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/188416
```

fricas [A] time = 0.94, size = 102, normalized size = 0.82

$$\frac{26884263\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(736000x^5+2318400x\right.}{376832\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/376832*(26884263*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(736000*x^5 + 2318400*x^4 + 2624760*x
^3 - 5754186*x^2 + 16138403*x - 15423965)*sqrt(2*x^2 - x + 3))/(2*x^2 - x +
 3)
```

giac [A] time = 0.28, size = 72, normalized size = 0.58

$$\frac{1168881}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(20(40(20x+63)x+2853)x-125091)x+16138403}{47104\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
+ 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x -
15423965)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 132, normalized size = 1.06

$$\frac{125x^5}{8\sqrt{2x^2-x+3}}+\frac{1575x^4}{32\sqrt{2x^2-x+3}}+\frac{14265x^3}{256\sqrt{2x^2-x+3}}-\frac{125091x^2}{1024\sqrt{2x^2-x+3}}+\frac{1168881x}{4096\sqrt{2x^2-x+3}}-\frac{1168881\sqrt{2}\ \mathrm{arc}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5130399/16384/(2*x^2-x+3)^(1/2)+5392543/376832*(4*x-1)/(2*x^2-x+3)^(1/2)-1
168881/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+125/8/(2*x^2-x+3)^(1/2)*
x^5+1575/32/(2*x^2-x+3)^(1/2)*x^4+14265/256/(2*x^2-x+3)^(1/2)*x^3-125091/10
24/(2*x^2-x+3)^(1/2)*x^2+1168881/4096/(2*x^2-x+3)^(1/2)*x
```

maxima [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{125x^5}{8\sqrt{2x^2-x+3}}+\frac{1575x^4}{32\sqrt{2x^2-x+3}}+\frac{14265x^3}{256\sqrt{2x^2-x+3}}-\frac{125091x^2}{1024\sqrt{2x^2-x+3}}-\frac{1168881}{8192}\sqrt{2}\ \mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 125/8*x^5/sqrt(2*x^2 - x + 3) + 1575/32*x^4/sqrt(2*x^2 - x + 3) + 14265/256
*x^3/sqrt(2*x^2 - x + 3) - 125091/1024*x^2/sqrt(2*x^2 - x + 3) - 1168881/81
92*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16138403/47104*x/sqrt(2*x^2 -
 x + 3) - 15423965/47104/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

3.88 $\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] -223/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+121/92*(19-7*x)/(2*x^2-x+3)
^(1/2)+415/32*(2*x^2-x+3)^(1/2)+25/8*x*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 +
(25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2
])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{3/2}}\,dx &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{\frac{1173}{16}+\frac{1955x}{8}+\frac{575x^2}{4}}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{25}{8}x\sqrt{3-x+2x^2}+\frac{1}{46}\int \frac{-138+\frac{9545x}{8}}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}+\frac{223}{64}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}+\frac{223\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx\right.}{64\sqrt{46}}\\ &= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 55, normalized size = 0.67

$$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (223*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

fricas [A] time = 0.88, size = 92, normalized size = 1.12

$$\frac{5129\sqrt{2}\left(2x^2-x+3\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+8\left(4600x^3+16790x^2-9421\right.}{5888\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*sq
rt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.24, size = 62, normalized size = 0.76

$$-\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(230(20x+73)x-9421)x+47027}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 98, normalized size = 1.20

$$\frac{25x^3}{4\sqrt{2x^2-x+3}}+\frac{365x^2}{16\sqrt{2x^2-x+3}}-\frac{223x}{64\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{15761}{256\sqrt{2x^2-x+3}}-\frac{13713}{5888\sqrt{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 25/4/(2*x^2-x+3)^(1/2)*x^3+365/16/(2*x^2-x+3)^(1/2)*x^2-223/64/(2*x^2-x+3)^
(1/2)*x+15761/256/(2*x^2-x+3)^(1/2)-13713/5888*(4*x-1)/(2*x^2-x+3)^(1/2)+22
3/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 80, normalized size = 0.98

$$\frac{25\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{365\,x^2}{16\,\sqrt{2\,x^2-x+3}}+\frac{223}{128}\,\sqrt{2}\ \operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{9421\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{47027}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sqr
t(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) + 47
027/736/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^2}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2} + 3x + 2\right)^{2}}{\left(2x^{2} - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)
```

3.89 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

```
[Out] -5/4*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11/23*(5+3*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 619, 215}

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{115}{4\sqrt{3-x+2x^2}}\,dx\\ &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5}{2}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\ &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{2\sqrt{46}}\\ &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 45, normalized size = 1.00

$$\frac{5\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(2*Sqrt[2])
```

fricas [B] time = 0.80, size = 82, normalized size = 1.82

$$\frac{115\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)-88\,\sqrt{2\,x^2-x+3}\,(3\,x+5)}{184\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x +
 3)
```

giac [A] time = 0.23, size = 53, normalized size = 1.18

$$-\frac{5}{4}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{11\,(3\,x+5)}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23*
(3*x + 5)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 64, normalized size = 1.42

$$-\frac{5x}{2\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4}-\frac{17}{8\sqrt{2x^2-x+3}}+\frac{\frac{49x}{46}-\frac{49}{184}}{\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/2/(2*x^2-x+3)^(1/2)*x-17/8/(2*x^2-x+3)^(1/2)+49/184*(4*x-1)/(2*x^2-x+3)^
(1/2)+5/4*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.95, size = 46, normalized size = 1.02

$$\frac{5}{4}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{33\,x}{23\,\sqrt{2\,x^2-x+3}}-\frac{55}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3)
- 55/23/sqrt(2*x^2 - x + 3)
```

mupad [B] time = 0.23, size = 87, normalized size = 1.93

$$\frac{5\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{4}+\frac{3\,(2\,x-12)}{23\,\sqrt{2\,x^2-x+3}}-\frac{10\,\left(\frac{11\,x}{2}+\frac{3}{2}\right)}{23\,\sqrt{2\,x^2-x+3}}+\frac{16\,x-4}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] (5*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/4 + (3*(2*
x - 12))/(23*(2*x^2 - x + 3)^(1/2)) - (10*((11*x)/2 + 3/2))/(23*(2*x^2 - x
+ 3)^(1/2)) + (16*x - 4)/(23*(2*x^2 - x + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{5x^2+3x+2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

3.90 $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx$

Optimal. Leaf size=176

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{22}\sqrt{\frac{1}{682}}$$

```
[Out] 1/253*(13-6*x)/(2*x^2-x+3)^(1/2)-1/15004*arctanh(1/31*(61+x*(69-65*2^(1/2))
-4*2^(1/2))*341^(1/2)/(-247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-168454+
341000*2^(1/2))^(1/2)+1/15004*arctan(1/31*(61+4*2^(1/2)+x*(69+65*2^(1/2)))*
341^(1/2)/(247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(168454+341000*2^(1/2)
)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{22}\sqrt{\frac{1}{682}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTa
n[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)
)/Sqrt[3 - x + 2*x^2]])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[
11/(31*(-247 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]])/22
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
```

```
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{2783}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}\left(3+8\sqrt{2}\right)-\frac{2783}{2}\left(13-5\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}\left(3-8\sqrt{2}\right)-\ldots}{\sqrt{3-x+2x^2}\,(\ldots}}{61226\ldots}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8}\left(253\left(1000-247\sqrt{2}\right)\right)\operatorname{Subst}\left(\int \frac{\ldots}{-\frac{240097759}{4}\,(\ldots}\right.$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\ldots\right.}}}{\ldots}\right.$$

Mathematica [C] time = 1.29, size = 202, normalized size = 1.15

$$\frac{-\frac{27280(6x-13)}{\sqrt{2x^2-x+3}} - 23\sqrt{682\left(13+i\sqrt{31}\right)}\left(13\sqrt{31}+69i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - 23\sqrt{682\left(13-i\sqrt{31}\right)}\,(\ldots}{6901840}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] ((-27280*(-13 + 6*x))/Sqrt[3 - x + 2*x^2] - 23*Sqrt[682*(13 + I*Sqrt[31])]*
(69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(
```

```
2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - 23*Sqrt[682*(13 - I*S
qrt[31])]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]])/6901840
```

fricas [B] time = 2.34, size = 2083, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/50921775520*(339388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*
sqrt(247*sqrt(2) + 1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(
2*x^2 - x + 3)*(22*50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x
^4 + 235088*x^3 - 46944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5
- 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 17
2800) + 5*50^(1/4)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 +
 24624288*x^3 - 10862496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^
5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19
989504*x + 10533888))*sqrt(247*sqrt(2) + 1000) + 933317000*sqrt(31)*sqrt(2)
*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*
x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5
+ 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x -
456192) - 2*sqrt(310/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3
/4)*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 10298
88*x^2 - sqrt(2)*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*
x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 667694
8*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(
2)*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689
984*x^2 + 1689984*x) - 46199808*x))*sqrt(247*sqrt(2) + 1000) + 65450*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 2975*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*
x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 -
 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1
44820224*x))*sqrt(-(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 81
1580*sqrt(2)*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2) + 10605875*sqrt(31
)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4
 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*
x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064
064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 -
 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339
388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*sqrt(247*sqrt(2) +
1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*
50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 4
6944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 2343
60*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^(1/4)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 +
17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888
))*sqrt(247*sqrt(2) + 1000) - 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(31
0/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 3
48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(4
6522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 +
269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^
```

```
5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502
244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x
) - 46199808*x))*sqrt(247*sqrt(2) + 1000) - 65450*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 2975*sqrt(
31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4
 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 -
 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((
sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 3
8) + x - 75)*sqrt(247*sqrt(2) + 1000) + 903805*x^2 + 811580*sqrt(2)*(2*x^2
- x + 3) - 2785195*x + 3689000)/x^2) - 10605875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 23*sqrt(341)*50^(1/4)*s
qrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2 - x + 3) - 1000*x + 3000)*s
qrt(247*sqrt(2) + 1000)*log(3100000/119*(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(1
0)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 10
00) + 903805*x^2 + 811580*sqrt(2)*(2*x^2 - x + 3) - 2785195*x + 3689000)/x^
2) + 23*sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2
 - x + 3) - 1000*x + 3000)*sqrt(247*sqrt(2) + 1000)*log(-3100000/119*(sqrt(
341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) +
x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 811580*sqrt(2)*(2*x^2 - x +
 3) + 2785195*x - 3689000)/x^2) + 201271840*sqrt(2*x^2 - x + 3)*(6*x - 13))
/(2*x^2 - x + 3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 9.49Done
```

maple [B] time = 0.03, size = 718, normalized size = 4.08

$$\frac{\sqrt{\frac{8\left(x+\sqrt{2}-1\right)^2}{\left(-x+\sqrt{2}+1\right)^2}+\frac{3\sqrt{2}\,\left(x+\sqrt{2}-1\right)^2}{\left(-x+\sqrt{2}+1\right)^2}+8-3\sqrt{2}}\;\sqrt{2}\left(1712502\sqrt{2}\;\operatorname{arctanh}\left(\frac{31\sqrt{\frac{8\left(x+\sqrt{2}-1\right)^2}{\left(-x+\sqrt{2}+1\right)^2}+\frac{3\sqrt{2}\,\left(x+\sqrt{2}-1\right)^2}{\left(-x+\sqrt{2}+1\right)^2}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)-66\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/465124*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(2197*2^(1/2)*(-8866+6820*2^(1/2))^
(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1
/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1
/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1
/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1
/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+
1)*(8+3*2^(1/2)))+3070*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^
(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(
-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379
*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+1712502*arcta
nh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-6617446*
arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2
^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1
/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+1/22/(2*x^2-x+3)^(1/2)-3/5
06*(4*x-1)/(2*x^2-x+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)),x)
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^{2}-x+3\right)^{\frac{3}{2}}\left(5x^{2}+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

3.91 $\displaystyle\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=211

$$-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\tan^{-1}\left(\sqrt{31(12\cdots}\right.}{300\cdots}$$

```
[Out] 1/345092*(-6315+2306*x)/(2*x^2-x+3)^(1/2)+1/682*(4+65*x)/(5*x^2+3*x+2)/(2*x
^2-x+3)^(1/2)-1/20465456*arctanh(1/31*(12611+x*(45519-29065*2^(1/2))-16454*
2^(1/2))*341^(1/2)/(-129694447+103775000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*
(-88451612854+70774550000*2^(1/2))^(1/2)+1/20465456*arctan(1/31*(12611+1645
4*2^(1/2)+x*(45519+29065*2^(1/2)))*341^(1/2)/(129694447+103775000*2^(1/2))^
(1/2)/(2*x^2-x+3)^(1/2))*(88451612854+70774550000*2^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\tan^{-1}\left(\sqrt{31(12\cdots}\right.}{300\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(6315 - 2306*x)/(345092*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*Sqrt[3 - x
+ 2*x^2]*(2 + 3*x + 5*x^2)) + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
+ (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/30008 - (Sqrt[(-1296944
47 + 103775000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*S
qrt[2]))]*(12611 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x +
 2*x^2]])/30008
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
```

```
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

<u>Rule 1029</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

<u>Rule 1035</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

<u>Rule 1060</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

<u>Rubi steps</u>

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} - \frac{\int \frac{-1782+\frac{3333x}{2}-2860x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} - \cdots$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} - \cdots$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} + \cdots$$

Mathematica [C] time = 1.53, size = 740, normalized size = 3.51

$$100\left(\frac{682\left(\left(22-4i\sqrt{31}\right)x+i\sqrt{31}+52\right)}{\left(\sqrt{31}+13i\right)\left(10ix+\sqrt{31}+3i\right)\sqrt{2x^2-x+3}} + \frac{682\left(\left(22+4i\sqrt{31}\right)x-i\sqrt{31}+52\right)}{\left(\sqrt{31}-13i\right)\left(-10ix+\sqrt{31}-3i\right)\sqrt{2x^2-x+3}} + \frac{22\left(2\left(11\sqrt{31}-62i\right)x+52\sqrt{31}+31i\right)}{\left(\sqrt{31}+13i\right)\sqrt{2x^2-x+3}} + \cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (100*((682*(52 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x))/((13*I + Sqrt[31])*
(3*I + Sqrt[31] + (10*I)*x)*Sqrt[3 - x + 2*x^2]) + (682*(52 - I*Sqrt[31] +
(22 + (4*I)*Sqrt[31])*x))/((-13*I + Sqrt[31])*(-3*I + Sqrt[31] - (10*I)*x)*
Sqrt[3 - x + 2*x^2]) + (22*(31*I + 52*Sqrt[31] + 2*(-62*I + 11*Sqrt[31])*x)
)/((13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + (22*(-31*I + 52*Sqrt[31] + 2*(6
2*I + 11*Sqrt[31])*x))/((-13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + ((575*I)*
Sqrt[682*(13 + I*Sqrt[31])]*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31
])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sqrt[3
1])^2 + (155*(44*(16353 + (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 345*Sqrt[
286 + (22*I)*Sqrt[31]]*(-29 + (17*I)*Sqrt[31] + 10*(11 + (2*I)*Sqrt[31])*x)
*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*
Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(22*(-13*I + Sqrt[31])^3*(-3*I + Sqrt[31]
 - (10*I)*x)) + (155*(44*(16353 - (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 3
45*Sqrt[286 - (22*I)*Sqrt[31]]*(29 + (17*I)*Sqrt[31] + (-110 + (20*I)*Sqrt[
31])*x)*ArcTanh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(22*(13*I + Sqrt[31])^3*(3*I + Sqr
t[31] + (10*I)*x)) - ((575*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*S
qrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
 x + 2*x^2])]))/(13*I + Sqrt[31])^2))/2674463
```

fricas [B] time = 2.53, size = 2173, normalized size = 10.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/298892470388411098707 20*(35183643812*3446160200^(1/4)*sqrt(20755)*sqrt(34
1)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sqrt(2) + 20755
0000)*arctan(1/243787105524753264092412 5*(59193260*sqrt(20755)*(11*34461602
00^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^5 - 311740976
*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 - 65709803*x^6
+ 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2 - 126316800*
x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(1/4)*sqrt(341
)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4 + 425065248*x
^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106575580*x^5 -
236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x + 143534592) -
287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 2075
50000) + 692040815683170556133300 0*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7
 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2
)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x
^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(41510/3
97951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x^7 - 96938292
*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 38700288*x^2 - sqrt(
2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^4 + 79054920*
x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446160200^(1/4)*
sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358592*x^4 + 506
287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948*x^6 + 128512
220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 173705472*x) - 14
4903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 207550000) + 11691
2097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329
3072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118
051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2
 - 1036800*x) + 3276288*x) + 5314186228782025*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 18
949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 - 3
49243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 1587470711561
000)/x^2) + 7864100178217847228787 5*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5
3385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2
 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 55
68*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8
 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3
4615296*x^2 - 24772608*x + 18579456)) + 35183643812*3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sq
rt(2) + 207550000)*arctan(1/243787105524753264092412 5*(59193260*sqrt(20755)
*(11*3446160200^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^
5 - 311740976*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 -
65709803*x^6 + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2
 - 126316800*x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(
1/4)*sqrt(341)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4
+ 425065248*x^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106
575580*x^5 - 236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x +
143534592) - 287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*s
qrt(2) + 207550000) - 692040815683170556133300 0*sqrt(31)*sqrt(2)*(28180*x^8
 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496
*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x
^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2
*sqrt(41510/397951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x
^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028
8*x^2 - sqrt(2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^
```

```
4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446
160200^(1/4)*sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358
592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948
*x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737
05472*x) - 144903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 20755
0000) - 116912097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578
888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15
550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
+ 1209600*x^2 - 1036800*x) + 3276288*x) - 5314186228782025*sqrt(31)*(254591
*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328
*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((3446160200^
(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x +
 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 38893032433
2445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 1198540387228555*x + 1
587470711561000)/x^2) - 786410017821784722878755*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 23*3446160200^(1/4)*sqr
t(20755)*sqrt(341)*sqrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^
2 - 129694447*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 12
45300000)*sqrt(129694447*sqrt(2) + 207550000)*log(1037750000000/397951*(344
6160200^(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
6137*x + 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 388
930324332445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 11985403872285
55*x + 1587470711561000)/x^2) - 23*3446160200^(1/4)*sqrt(20755)*sqrt(341)*s
qrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^2 - 129694447*sqrt(2
)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 1245300000)*sqrt(12969
4447*sqrt(2) + 207550000)*log(-1037750000000/397951*(3446160200^(1/4)*sqrt(
20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 1
8949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 -
349243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 158747071156
1000)/x^2) + 86612402022768160*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sq
rt(2*x^2 - x + 3))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 36.15Done
```

maple [B] time = 0.10, size = 5942, normalized size = 28.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

3.92 $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}+\frac{3\sqrt{\frac{}{6}}}{}$$

```
[Out] 1/941410976*(-4353943+6508666*x)/(2*x^2-x+3)^(1/2)+1/1364*(4+65*x)/(5*x^2+3
*x+2)^2/(2*x^2-x+3)^(1/2)+5/1860496*(7318+17315*x)/(5*x^2+3*x+2)/(2*x^2-x+3
)^(1/2)-3/55829763968*arctanh(1/31*(5538393+x*(13785797-9662095*2^(1/2))-41
23702*2^(1/2))*341^(1/2)/(-13874275807943+9819738650000*2^(1/2))^(1/2)/(2*x
^2-x+3)^(1/2))*(-9462256101017126+6697061759300000*2^(1/2))^(1/2)+3/5582976
3968*arctan(1/31*(5538393+4123702*2^(1/2)+x*(13785797+9662095*2^(1/2)))*341
^(1/2)/(13874275807943+9819738650000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(946
2256101017126+6697061759300000*2^(1/2))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}+\frac{3\sqrt{\frac{}{6}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*S
qrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqr
t[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 98197386500
00*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2
]))]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/
682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538
393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/81861824
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
```

```
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\left(2+3x\right.}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \cdots$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \cdots$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \cdots$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \cdots$$

Mathematica [C] time = 2.21, size = 231, normalized size = 0.94

$$\frac{69\sqrt{286-22i\sqrt{31}}\left(13785797\sqrt{31}+14026539i\right)\tan^{-1}\left(\frac{-2\left(2\sqrt{31}+11i\right)x+\sqrt{31}+63i}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-69i\sqrt{286+22i\sqrt{31}}\left(13785\cdots\right.}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((27280*(22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*
x^4 + 162716650*x^5))/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + 69*Sqrt[2
86 - (22*I)*Sqrt[31]]*(14026539*I + 13785797*Sqrt[31])*ArcTan[(63*I + Sqrt[
31] - 2*(11*I + 2*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2])] - (69*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(-14026539*I + 13785797*Sqrt[
31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/25681691425280
```

fricas [B] time = 3.04, size = 2263, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/33652296632397026886019646994897920*(920746859815884*1928545343086076450^
(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^
```

```
3 + 83*x^2 + 32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arcta
n(1/225227015528909794375187692534722839169237 5*(2800589462980*sqrt(1963947
730)*(22*1928545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*
x^6 + 59872788262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2
 - sqrt(2)*(5311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 6800681868
3*x^4 + 33985514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) -
52334913024*x + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(3
41)*(560592897*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 +
134890825824*x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^
6 + 32499503540*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 -
 68177562624*x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 -
x + 3)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 63935410859819554532
31134497759874144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424
919)*(sqrt(1963947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824
*x^7 - 37618468196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x
^3 + 51583129344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 8845761
3411*x^5 - 33685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 4176467481
6*x) - 51583129344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(19
98926311*x^7 - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 16744
6420704*x^3 + 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 +
3285070260*x^5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259
776*x) - 96037622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) +
19639477300000) + 2282926923240949861309948624550*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 1037694056
01861357332270392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt(-(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(
341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*
x - 1471975)*sqrt(13874275807943*sqrt(2) + 19639477300000) - 16051926912456
8199977215*x^2 - 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) + 4946614
21179791799929785*x - 655180690304359999907000)/x^2) + 72653875977067675604
899255656362206183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 920746859815884*1928545343086076450^(1/4)*sqrt(19
63947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 +
32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arctan(1/225227015
528909794375187692534722839169237 5*(2800589462980*sqrt(1963947730)*(22*1928
545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*x^6 + 5987278
8262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2 - sqrt(2)*(5
311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 68006818683*x^4 + 33985
514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) - 52334913024*x
 + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(56059289
7*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 + 134890825824*
x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^6 + 324995035
40*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 - 68177562624*
x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 - x + 3)*sqrt(1
3874275807943*sqrt(2) + 19639477300000) - 63935410859819554532311344977598 7
4144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*
x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048
```

```
*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424919)*(sqrt(19
63947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824*x^7 - 376184
68196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x^3 + 51583129
344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 88457613411*x^5 - 33
685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 41764674816*x) - 515831
29344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(1998926311*x^7
- 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 167446420704*x^3 +
 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 + 3285070260*x^
5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259776*x) - 9603
7622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) + 1963947730000
0) - 22829269232409498613099486245 50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*
x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - s
qrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 16
67952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 10376940560186135733227
0392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 +
108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^
7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1448202
24*x))*sqrt((1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 1471975)*s
qrt(13874275807943*sqrt(2) + 19639477300000) + 160519269124568199977215*x^2
 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 494661421179791799929
785*x + 655180690304359999907000)/x^2) - 72653875977067675604899255656362 20
6183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5
+ 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2
692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1
8579456)) + 69*1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31
)*(981973865000000*x^6 + 687381705500000*x^5 + 2022866161900000*x^4 + 16693
55570500000*x^3 + 1630076615900000*x^2 - 13874275807943*sqrt(2)*(50*x^6 + 3
5*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) + 628463273600000*x + 235673
727600000)*sqrt(13874275807943*sqrt(2) + 19639477300000)*log(17675529570000
00000/3471424919*(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 14719
75)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 1605192691245681999772 1
5*x^2 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 4946614211797917
99929785*x + 655180690304359999907000)/x^2) - 69*1928545343086076450^(1/4)*
sqrt(1963947730)*sqrt(341)*sqrt(31)*(981973865000000*x^6 + 687381705500000*
x^5 + 2022866161900000*x^4 + 1669355570500000*x^3 + 1630076615900000*x^2 -
13874275807943*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12) + 628463273600000*x + 235673727600000)*sqrt(13874275807943*sqrt(2) +
19639477300000)*log(-1767552957000000000/3471424919*(1928545343086076450^(1
/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(29954
31*x + 1523456) - 4518887*x - 1471975)*sqrt(13874275807943*sqrt(2) + 196394
77300000) - 160519269124568199977215*x^2 - 144139751866959199979540*sqrt(2)
*(2*x^2 - x + 3) + 494661421179791799929785*x - 655180690304359999907000)/x
^2) + 357466584630058815949259 20*(162716650*x^5 + 86411405*x^4 + 277167774*
x^3 + 175833195*x^2 + 161806828*x + 22374044)*sqrt(2*x^2 - x + 3))/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
```

```
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 69.9Done
```

maple [B] time = 0.21, size = 18981, normalized size = 77.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

3.93 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{38375}{384}\sqrt{2x^2-x+3}\,x^2+\frac{526075\sqrt{2x^2-x+3}\,x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x+7409)}{101568\sqrt{2x^2-x+3}}-\frac{14641(79x}{4416\,(2x^2-}$$

```
[Out] -14641/4416*(101+79*x)/(2*x^2-x+3)^(3/2)+16955197/16384*arcsinh(1/23*(1-4*x
)*23^(1/2))*2^(1/2)+1331/101568*(7409+116368*x)/(2*x^2-x+3)^(1/2)-1308645/4
096*(2*x^2-x+3)^(1/2)+526075/3072*x*(2*x^2-x+3)^(1/2)+38375/384*x^2*(2*x^2-
x+3)^(1/2)+625/32*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{32}\sqrt{2x^2-x+3}\,x^3+\frac{38375}{384}\sqrt{2x^2-x+3}\,x^2+\frac{526075\sqrt{2x^2-x+3}\,x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x}{101568\sqrt{2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x
))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526
075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (62
5*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(819
2*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{3839123}{256}-\frac{1983543x}{128}-\frac{1464801x^2}{64}+\frac{430905x^3}{32}+\frac{639975x^4}{16}+250}{\left(3-x+2x^2\right)^{3/2}}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}+\frac{4\int \frac{-\frac{141812733}{256}-\frac{1880595x}{16}+\frac{15512925x^2}{64}+\frac{3372375}{16}}{\sqrt{3-x+2x^2}}}{1587}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}+\frac{625}{32}x^3\sqrt{3-x+2x^2}+\frac{\int \frac{-\frac{141812733}{32}-1}{}}{}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}+\frac{38375}{384}x^2\sqrt{3-x+2x^2}+\frac{625}{32}x^3\sqrt{3}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}+\frac{526075x\sqrt{3-x+2x^2}}{3072}+\frac{38375}{384}x^2$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}-\frac{1308645\sqrt{3-x+2x^2}}{4096}+\frac{526075x}{3}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}-\frac{1308645\sqrt{3-x+2x^2}}{4096}+\frac{526075x}{3}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}-\frac{1308645\sqrt{3-x+2x^2}}{4096}+\frac{526075x}{3}$$

Mathematica [A] time = 0.51, size = 75, normalized size = 0.51

$$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+498838}{6500352\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781
260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x
+ 2*x^2)^(3/2)) - (16955197*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8192*Sqrt[2])
```

fricas [A] time = 0.65, size = 132, normalized size = 0.90

$$\frac{26907897639 \sqrt{2} \left(4 x^4 - 4 x^3 + 13 x^2 - 6 x + 9\right) \log \left(4 \sqrt{2} \sqrt{2 x^2 - x + 3} (4 x - 1) - 32 x^2 + 16 x - 25\right) + 8 (50\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*sq
rt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*x^
7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 - 36
481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.25, size = 81, normalized size = 0.55

$$\frac{16955197}{16384} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2 x^2 - x + 3}\right) + 1\right) + \frac{((4 (2645 (20 (40 (60 x + 247) x + 16563) x - 479847) \cdots}{650\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1
) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x + 99
62225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x + 3
)^(3/2)
```

maple [A] time = 0.03, size = 214, normalized size = 1.46

$$\frac{625 x^7}{8 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} + \frac{30875 x^6}{96 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} + \frac{138025 x^5}{256 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} - \frac{799745 x^4}{1024 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} + \frac{16955197 x^3}{12288 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} - \frac{\cdots}{1638\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 16955197/32768/(2*x^2-x+3)^(1/2)-2149616639/524288/(2*x^2-x+3)^(3/2)+992926
033/13000704*(4*x-1)/(2*x^2-x+3)^(1/2)-16955197/16384*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))+5141612725/36175872*(4*x-1)/(2*x^2-x+3)^(3/2)+625/8*x^7/(
2*x^2-x+3)^(3/2)+30875/96*x^6/(2*x^2-x+3)^(3/2)+138025/256*x^5/(2*x^2-x+3)^
(3/2)-799745/1024*x^4/(2*x^2-x+3)^(3/2)+16955197/12288*x^3/(2*x^2-x+3)^(3/2
)-67488035/16384*x^2/(2*x^2-x+3)^(3/2)+55167267/131072*x/(2*x^2-x+3)^(3/2)+
16955197/8192/(2*x^2-x+3)^(1/2)*x
```

maxima [B] time = 1.00, size = 253, normalized size = 1.72

$$\frac{625 x^7}{8 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} + \frac{30875 x^6}{96 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} + \frac{138025 x^5}{256 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} - \frac{799745 x^4}{1024 \left(2 x^2 - x + 3\right)^{\frac{3}{2}}} - \frac{16955197}{13000704} x \left(\frac{284 x}{\sqrt{2 x^2 - x + 3}} \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 625/8*x^7/(2*x^2 - x + 3)^(3/2) + 30875/96*x^6/(2*x^2 - x + 3)^(3/2) + 1380
25/256*x^5/(2*x^2 - x + 3)^(3/2) - 799745/1024*x^4/(2*x^2 - x + 3)^(3/2) -
16955197/13000704*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(
3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 -
 x + 3)^(3/2)) - 16955197/16384*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) +
1203818987/6500352*sqrt(2*x^2 - x + 3) + 3536205583/3250176*x/sqrt(2*x^2 -
x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^(3/2) + 257773037/1083392/sqrt(2*x
^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^(3/2) - 374445479/70656/(2*x
^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^4}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^4}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

3.94 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] -1331/1104*(17-45*x)/(2*x^2-x+3)^(3/2)-7495/256*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)+121/8464*(10679-6744*x)/(2*x^2-x+3)^(1/2)+3175/64*(2*x^2-x+3)^(
1/2)+125/16*x*(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(
8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 -
 x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{91275}{64}-\frac{57201x}{32}+\frac{66585x^2}{16}+\frac{39675x^3}{8}+\frac{8625x^4}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{1452105}{64}+\frac{277725x}{8}+\frac{198375x^2}{16}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{125}{16}x\sqrt{3-x+2x^2}+\frac{\int \frac{\frac{214245}{4}+\frac{5038725x}{32}}{\sqrt{3-x+2x^2}}\,d}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2}$$

Mathematica [A] time = 0.34, size = 65, normalized size = 0.62

$$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568\left(2x^2-x+3\right)^{3/2}}+\frac{7495\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 3174
000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) + (7495*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/(128*Sqrt[2])
```

fricas [A] time = 0.60, size = 122, normalized size = 1.16

$$\frac{11894565\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(3174000\right.}{812544\left(4x^4-4x^3+13x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/812544*(11894565*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(3174000*x^5 + 16
```

```
980900*x^4 - 29423976*x^3 + 101546529*x^2 - 62463282*x + 89784565)*sqrt(2*x
^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.27, size = 72, normalized size = 0.69

$$-\frac{7495}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{3\left(\left(4\left(13225\left(20x+107\right)x-2451998\right)x+33848843\right)x-\right.}{101568\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 20821094)
*x + 89784565)/(2*x^2 - x + 3)^(3/2)
```

maple [B] time = 0.01, size = 180, normalized size = 1.71

$$\frac{125x^5}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{2675x^4}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{7495x^3}{192\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{222809x^2}{256\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{281177x}{2048\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{74}{128\sqrt{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -7495/512/(2*x^2-x+3)^(1/2)+20961031/24576/(2*x^2-x+3)^(3/2)-3391139/203136
*(4*x-1)/(2*x^2-x+3)^(1/2)+7495/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-
14081711/565248*(4*x-1)/(2*x^2-x+3)^(3/2)+125/4/(2*x^2-x+3)^(3/2)*x^5+2675/
16/(2*x^2-x+3)^(3/2)*x^4-7495/192/(2*x^2-x+3)^(3/2)*x^3+222809/256/(2*x^2-x
+3)^(3/2)*x^2-281177/2048/(2*x^2-x+3)^(3/2)*x-7495/128/(2*x^2-x+3)^(1/2)*x
```

maxima [B] time = 1.02, size = 219, normalized size = 2.09

$$\frac{125x^5}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{2675x^4}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{7495}{203136}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{80}{\left(2x^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 125/4*x^5/(2*x^2 - x + 3)^(3/2) + 2675/16*x^4/(2*x^2 - x + 3)^(3/2) + 7495/
203136*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 7495/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 532145/101568*sqr
t(2*x^2 - x + 3) - 4515389/50784*x/sqrt(2*x^2 - x + 3) + 7197/8*x^2/(2*x^2
- x + 3)^(3/2) + 396211/50784/sqrt(2*x^2 - x + 3) - 269783/1104*x/(2*x^2 -
x + 3)^(3/2) + 1002137/1104/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2 + 3x + 2\right)^3}{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)
```

3.95 $\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{121(19-7x)}{276\left(2x^2-x+3\right)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] 121/276*(19-7*x)/(2*x^2-x+3)^(3/2)-25/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)-11/6348*(7351+2336*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1660, 12, 619, 215}

$$\frac{121(19-7x)}{276\left(2x^2-x+3\right)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*S
qrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{131}{16}+\frac{5865x}{8}+\frac{1725x^2}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$
$$= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{4\int \frac{39675}{16\sqrt{3-x+2x^2}}\,dx}{1587}$$
$$= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{4\sqrt{46}}$$
$$= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} - \frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.24, size = 55, normalized size = 0.81

$$\frac{25\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}} - \frac{11\left(2336x^3+6183x^2+714x+8623\right)}{3174\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) + (
25*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

fricas [B] time = 0.68, size = 112, normalized size = 1.65

$$\frac{39675\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)-88\left(2336x^3+\right.}{25392\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x^2
 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.27, size = 61, normalized size = 0.90

$$-\frac{25}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{11\left(((2336x+6183)x+714)x+8623\right)}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/31
74*(((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)
```

maple [B] time = 0.01, size = 146, normalized size = 2.15

$$-\frac{25x^3}{6\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{145x^2}{8\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{319x}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{25x}{4\sqrt{2x^2-x+3}}+\frac{25\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}-\frac{}{768\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -25/6/(2*x^2-x+3)^(3/2)*x^3-145/8/(2*x^2-x+3)^(3/2)*x^2-319/64/(2*x^2-x+3)^
(3/2)*x-15775/768/(2*x^2-x+3)^(3/2)+8493/5888*(4*x-1)/(2*x^2-x+3)^(3/2)+226
7/2116*(4*x-1)/(2*x^2-x+3)^(1/2)-25/4/(2*x^2-x+3)^(1/2)*x-25/16/(2*x^2-x+3)
^(1/2)+25/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [B] time = 0.98, size = 185, normalized size = 2.72

$$\frac{25}{6348}x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{25}{8}\sqrt{2}\,\text{arsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/
sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3
/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*x^2
 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/2) -
 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/138/(
2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^2}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^2}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)
```

3.96 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

```
[Out] -11/69*(5+3*x)/(2*x^2-x+3)^(3/2)-71/529*(1-4*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1660, 12, 613}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x
 + 2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{5/2}}\,dx &= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{213}{4\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{71}{46}\int \frac{1}{\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}-\frac{71(1-4x)}{529\sqrt{3-x+2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 33, normalized size = 0.70

$$\frac{2\left(852x^3-639x^2+1005x-952\right)}{1587\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
[Out] (2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

fricas [A] time = 0.57, size = 51, normalized size = 1.09

$$\frac{2\left(852\,x^3-639\,x^2+1005\,x-952\right)\sqrt{2\,x^2-x+3}}{1587\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
[Out] 2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.23, size = 29, normalized size = 0.62

$$\frac{2\left(3\left(71\left(4\,x-3\right)x+335\right)x-952\right)}{1587\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
[Out] 2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)
```

maple [A] time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x)
[Out] 2/1587/(2*x^2-x+3)^(3/2)*(852*x^3-639*x^2+1005*x-952)
```

maxima [A] time = 0.43, size = 59, normalized size = 1.26

$$\frac{284\,x}{529\,\sqrt{2\,x^2-x+3}}-\frac{71}{529\,\sqrt{2\,x^2-x+3}}-\frac{11\,x}{23\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{55}{69\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
[Out] 284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^2
 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)
```

mupad [B] time = 0.09, size = 29, normalized size = 0.62

$$\frac{2\,\left(852\,x^3-639\,x^2+1005\,x-952\right)}{1587\left(2\,x^2-x+3\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] (2*(1005*x - 639*x^2 + 852*x^3 - 952))/(1587*(2*x^2 - x + 3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

3.97 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=199

$$\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left((24\right.}{\sqrt{2x}}\right.$$

```
[Out] 1/759*(13-6*x)/(2*x^2-x+3)^(3/2)+1/128018*(3603-658*x)/(2*x^2-x+3)^(1/2)+1/
330088*arctan(1/31*(443-98*2^(1/2)+x*(247+345*2^(1/2)))*341^(1/2)/(-15457+2
5000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-10541674+17050000*2^(1/2))^(1/2)-1
/330088*arctanh(1/31*(443+x*(247-345*2^(1/2))+98*2^(1/2))*341^(1/2)/(15457+
25000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(10541674+17050000*2^(1/2))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left((24\right.}{\sqrt{2x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x
+ 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457
 + 25000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(1
5457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]])/484
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
```

```
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx}{8349}$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{-\frac{5184729}{2}-\frac{12481755x}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,d}{23235267}$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}\left(11-54\sqrt{2}\right)-\frac{211}{}}{\sqrt{3-x+2x^2}\left(2+\right.}}{51117587}$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32}\left(17457\left(50000-1\right.\right.$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+\right.}$$

Mathematica [C] time = 0.87, size = 218, normalized size = 1.10

$$-\frac{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}\left(119\sqrt{31}+247i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680} + \frac{\sqrt{\frac{1}{682}\left(13-i\sqrt{31}\right)}\left(119\sqrt{31}-247i\right)\mathrm{t}}{9680}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) - (
Sqrt[(13 + I*Sqrt[31])/682]*(247*I + 119*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
 + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])])/9680 + (Sqrt[(13 - I*Sqrt[31])/682]*(-247*I + 119*Sqrt[31])*ArcTanh
[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]
]*Sqrt[3 - x + 2*x^2])])/9680
```

fricas [B] time = 2.04, size = 2133, normalized size = 10.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/370971467791584000*(1123856268*sqrt(341)*200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3
 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/788938
9562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(347404*x^7 - 90
7814*x^6 + 2112962*x^5 - 2166688*x^4 + 787344*x^3 + 304128*x^2 - sqrt(2)*(3
5898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 1081800*x^
2 - 518400*x + 1043712) - 2087424*x + 518400) + 5*200^(1/4)*(712757*x^7 - 1
0233303*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 -
sqrt(2)*(158647*x^7 - 2935272*x^6 + 19428740*x^5 - 55765712*x^4 + 78380640*
x^3 - 84096000*x^2 - 37407744*x + 53208576) - 106417152*x + 37407744))*sqrt
(-772850000*sqrt(2) + 2500000000) + 22395686500000*sqrt(31)*sqrt(2)*(28180*
```

```
x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98
496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 132071
0*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192)
- sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(1665224*x^7
- 2325796*x^6 + 7065036*x^5 - 196416*x^4 - 2176416*x^3 + 8895744*x^2 + sqrt
(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 - 4195584*x) - 8895744*x) + 5*200^(1/4)*(3246491*x^7 - 41888524*x^
6 + 159670660*x^5 - 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 2*sqrt(
2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 - 9524160*x^4 + 34843680*x^3 - 740
06784*x^2 + 74006784*x) - 376648704*x))*sqrt(-772850000*sqrt(2) + 250000000
0) + 314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 32
93072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 11
8051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^
2 - 1036800*x) + 3276288*x) + 14277500*sqrt(31)*(254591*x^8 - 4815126*x^7 +
 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*s
qrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sq
rt(2) + 2500000000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) +
 13366595500*x - 17704100000)/x^2) + 254496437500*sqrt(31)*(2828123*x^8 - 9
696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 +
 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))
/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44
249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 1123856268*sqrt(341)*
200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2
) + 2500000000)*arctan(-1/7889389562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3
)*(11*200^(3/4)*(347404*x^7 - 907814*x^6 + 2112962*x^5 - 2166688*x^4 + 7873
44*x^3 + 304128*x^2 - sqrt(2)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 211723
3*x^4 + 1272680*x^3 - 1081800*x^2 - 518400*x + 1043712) - 2087424*x + 51840
0) + 5*200^(1/4)*(712757*x^7 - 10233303*x^6 + 48529768*x^5 - 94500260*x^4 +
 113086944*x^3 - 22282848*x^2 - sqrt(2)*(158647*x^7 - 2935272*x^6 + 1942874
0*x^5 - 55765712*x^4 + 78380640*x^3 - 84096000*x^2 - 37407744*x + 53208576)
 - 106417152*x + 37407744))*sqrt(-772850000*sqrt(2) + 2500000000) - 2239568
6500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5
 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
- 539136) + 1154304*x - 456192) - sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x
+ 3)*(11*200^(3/4)*(1665224*x^7 - 2325796*x^6 + 7065036*x^5 - 196416*x^4 -
2176416*x^3 + 8895744*x^2 + sqrt(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 +
 1685680*x^4 - 3693960*x^3 + 4195584*x^2 - 4195584*x) - 8895744*x) + 5*200^
(1/4)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4 + 1804962
24*x^3 + 376648704*x^2 - 2*sqrt(2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 -
9524160*x^4 + 34843680*x^3 - 74006784*x^2 + 74006784*x) - 376648704*x))*sqr
t(-772850000*sqrt(2) + 2500000000) - 314105000*sqrt(31)*sqrt(2)*(123408*x^8
 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38223
36*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105396
0*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 14277500*sqrt
(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^
4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
- 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(
(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) +
163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 38949
02000*sqrt(2)*(2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 2544964
37500*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 269
2*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5
184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 +
 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 185
```

```
79456)) + 1587*sqrt(341)*200^(1/4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 6500
00*x^2 + 15457*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 4500
00)*sqrt(-772850000*sqrt(2) + 2500000000)*log(77500000/5711*(sqrt(341)*200^
(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sq
rt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 3894902000*sqrt(2)*(
2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 1587*sqrt(341)*200^(1/
4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 650000*x^2 + 15457*sqrt(2)*(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 450000)*sqrt(-772850000*sqrt(2) + 25
00000000)*log(-77500000/5711*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000
000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) + 13366595500*x
- 17704100000)/x^2) - 965935696000*(3948*x^3 - 23592*x^2 + 19767*x - 39005)
*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 16.02Done
```

maple [B] time = 0.04, size = 751, normalized size = 3.77

$$\frac{\sqrt{\frac{8\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+\frac{3\sqrt{2}\,\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+8-3\sqrt{2}}\;\sqrt{2}\left(-993674\sqrt{2}\;\operatorname{arctanh}\left(\frac{31\sqrt{\frac{8\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+\frac{3\sqrt{2}\,\left(x+\sqrt{2}-1\right)^{2}}{\left(-x+\sqrt{2}+1\right)^{2}}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)-42\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/10232728*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(10111*2^(1/2)*(-8866+6820*2^(1/2
))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2
^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2
^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2
^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2
^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/
2)+1)*(8+3*2^(1/2)))+13910*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/
2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2
))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2
)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+2
2379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-993674*ar
```

```
ctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-42685
698*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8
-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+
2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+13/484/(2*x^2-x+3)^(1/
2)-329/256036*(4*x-1)/(2*x^2-x+3)^(1/2)+1/66/(2*x^2-x+3)^(3/2)-1/506*(4*x-1
)/(2*x^2-x+3)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{5/2}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)),x)
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^{2}-x+3\right)^{\frac{5}{2}}\left(5x^{2}+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

3.98 $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{15101-8654x}{1035276\left(2x^2-x+3\right)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}+\frac{625\sqrt{\frac{1}{682}\left(30463\right.}}{}$$

```
[Out] 1/1035276*(-15101+8654*x)/(2*x^2-x+3)^(3/2)+1/682*(4+65*x)/(2*x^2-x+3)^(3/2
)/(5*x^2+3*x+2)+1/523849656*(-3133427-1352542*x)/(2*x^2-x+3)^(1/2)-625/4502
40032*arctanh(1/31*(203+x*(687-445*2^(1/2))-242*2^(1/2))*341^(1/2)/(-30463+
23600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-20775766+16095200*2^(1/2))^(1/2)+
625/450240032*arctan(1/31*(203+242*2^(1/2)+x*(687+445*2^(1/2)))*341^(1/2)/(
30463+23600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(20775766+16095200*2^(1/2))^(
1/2)
```

Rubi [A] time = 0.54, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{15101-8654x}{1035276\left(2x^2-x+3\right)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}+\frac{625\sqrt{\frac{1}{682}\left(30463\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(15101 - 8654*x)/(1035276*(3 - x + 2*x^2)^(3/2)) - (3133427 + 1352542*x)/(
523849656*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 +
 3*x + 5*x^2)) + (625*Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31
*(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]])/660176 - (625*Sqrt[(-30463 + 23600*Sqrt[2])/682]*ArcTanh
[(Sqrt[11/(31*(-30463 + 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/660176
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
```

```
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682(3-x+2x^2)^{3/2}(2+3x+5x^2)} - \frac{\int \frac{-1738+\frac{4411x}{2}-5720x^2}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx}{7502}$$

$$= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2}(2+3x+5x^2)} -$$

$$= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682(3-x}$$

$$= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682(3-x}$$

$$= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682(3-x}$$

$$= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682(3-x}$$

Mathematica [C] time = 1.15, size = 296, normalized size = 1.26

$$-\frac{198375i\sqrt{286+22i\sqrt{31}}\left(687\sqrt{31}+31i\right)\sqrt{2x^2-x+3}\left(10x^4+x^3+16x^2+7x+6\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i}{2\sqrt{286+22i\sqrt{31}}\sqrt{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -1/2858123723136*(5456*(31010342 - 5712309*x + 84671384*x^2 + 2879479*x^3 +
 32686812*x^4 + 13525420*x^5) + (198375*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(31*
I + 687*Sqrt[31])*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*Arc
Tanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt
[31]]*Sqrt[3 - x + 2*x^2])] + 198375*Sqrt[286 - (22*I)*Sqrt[31]]*(31 + (687
*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh
[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]
]*Sqrt[3 - x + 2*x^2])])/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2))
```

fricas [B] time = 1.99, size = 2253, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/25604335602537914112*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
```

```
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(
321084*x^7 - 1338894*x^6 + 2762802*x^5 - 4721048*x^4 + 2438224*x^3 - 131731
2*x^2 - sqrt(2)*(277258*x^7 - 994619*x^6 + 2123978*x^5 - 3198193*x^4 + 1552
680*x^3 - 621000*x^2 - 1900800*x + 1181952) - 2363904*x + 1900800) + 1829*6
962^(1/4)*sqrt(341)*(25187*x^7 - 392073*x^6 + 2114488*x^5 - 4948060*x^4 + 6
460704*x^3 - 4452768*x^2 - sqrt(2)*(20477*x^7 - 310452*x^6 + 1610140*x^5 -
3584192*x^4 + 4580640*x^3 - 2620800*x^2 - 3400704*x + 2198016) - 4396032*x
+ 3400704))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) + 3155854864122
4*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(118/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(3
41)*(1050904*x^7 - 1523916*x^6 + 5005956*x^5 - 2572736*x^4 + 3615264*x^3 +
877824*x^2 - sqrt(2)*(1065206*x^7 - 1518091*x^6 + 4815081*x^5 - 1448880*x^4
 + 1303560*x^3 + 3131136*x^2 - 3131136*x) - 877824*x) + 1829*6962^(1/4)*sqr
t(341)*(84981*x^7 - 1100084*x^6 + 4256060*x^5 - 5639616*x^4 + 7745184*x^3 +
 2571264*x^2 - 242*sqrt(2)*(319*x^7 - 4124*x^6 + 15860*x^5 - 20160*x^4 + 24
480*x^3 + 20736*x^2 - 20736*x) - 2571264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463
*sqrt(2) + 47200) + 187549318*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1
578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*
(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x
^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 8524969*sqrt(31)*(254591*x^8 -
 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*
x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6962^(1/4)*sqrt(
341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x
+ 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)*(2*x^
2 - x + 3) + 676362371*x - 895844200)/x^2) + 358619870923*sqrt(31)*(2828123
*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493000
96*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1007
0*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94
887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*
x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 301208632500*
6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*
x^2 + 15*x + 18)*sqrt(30463*sqrt(2) + 47200)*arctan(1/11117215998613*(16826
8*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(321084*x^7 - 1338894*x^6 + 2762802*x^
5 - 4721048*x^4 + 2438224*x^3 - 1317312*x^2 - sqrt(2)*(277258*x^7 - 994619*
x^6 + 2123978*x^5 - 3198193*x^4 + 1552680*x^3 - 621000*x^2 - 1900800*x + 11
81952) - 2363904*x + 1900800) + 1829*6962^(1/4)*sqrt(341)*(25187*x^7 - 3920
73*x^6 + 2114488*x^5 - 4948060*x^4 + 6460704*x^3 - 4452768*x^2 - sqrt(2)*(2
0477*x^7 - 310452*x^6 + 1610140*x^5 - 3584192*x^4 + 4580640*x^3 - 2620800*x
^2 - 3400704*x + 2198016) - 4396032*x + 3400704))*sqrt(2*x^2 - x + 3)*sqrt(
30463*sqrt(2) + 47200) - 31558548641224*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(11
8/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(341)*(1050904*x^7 - 1523916*x^6 + 5005
956*x^5 - 2572736*x^4 + 3615264*x^3 + 877824*x^2 - sqrt(2)*(1065206*x^7 - 1
518091*x^6 + 4815081*x^5 - 1448880*x^4 + 1303560*x^3 + 3131136*x^2 - 313113
6*x) - 877824*x) + 1829*6962^(1/4)*sqrt(341)*(84981*x^7 - 1100084*x^6 + 425
6060*x^5 - 5639616*x^4 + 7745184*x^3 + 2571264*x^2 - 242*sqrt(2)*(319*x^7 -
 4124*x^6 + 15860*x^5 - 20160*x^4 + 24480*x^3 + 20736*x^2 - 20736*x) - 2571
264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) - 187549318*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 8524969*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 9086680
8*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8
 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) +
```

```
 144820224*x))*sqrt((6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sqrt(30463*sqrt(2) + 47200) + 21
9481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x + 3) - 676362371*x + 895844200)/
x^2) - 358619870923*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14
2835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(
1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^
2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24
772608*x + 18579456)) + 991875*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*(944
000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 - 30463*sqrt(2
)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 708000*x + 849600)
*sqrt(30463*sqrt(2) + 47200)*log(7375000000000/79*(6962^(1/4)*sqrt(341)*sqr
t(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sq
rt(30463*sqrt(2) + 47200) + 219481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x +
3) - 676362371*x + 895844200)/x^2) - 991875*6962^(1/4)*sqrt(341)*sqrt(118)*
sqrt(31)*(944000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 -
 30463*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 70800
0*x + 849600)*sqrt(30463*sqrt(2) + 47200)*log(-7375000000000/79*(6962^(1/4)
*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) -
277*x + 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)
*(2*x^2 - x + 3) + 676362371*x - 895844200)/x^2) - 48877259552*(13525420*x^
5 + 32686812*x^4 + 2879479*x^3 + 84671384*x^2 - 5712309*x + 31010342)*sqrt(
2*x^2 - x + 3))/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 18.73Done
```

maple [B] time = 0.10, size = 5975, normalized size = 25.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2),x)
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

3.99 $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=269

$$-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{}{1364}$$

```
[Out] 1/2824232928*(-12280939+19536786*x)/(2*x^2-x+3)^(3/2)+1/1364*(4+65*x)/(2*x^
2-x+3)^(3/2)/(5*x^2+3*x+2)^2+1/1860496*(46386+86885*x)/(2*x^2-x+3)^(3/2)/(5
*x^2+3*x+2)+1/476353953856*(-1134826571+1504660754*x)/(2*x^2-x+3)^(1/2)-35/
1228254807296*arctanh(1/31*(1432939+x*(6290431-3861685*2^(1/2))-2428746*2^(
1/2))*341^(1/2)/(-2243059557247+2011748500000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1
/2))*(-1529766618042454+1372012477000000*2^(1/2))^(1/2)+35/1228254807296*ar
ctan(1/31*(1432939+2428746*2^(1/2)+x*(6290431+3861685*2^(1/2)))*341^(1/2)/(
2243059557247+2011748500000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(152976661804
2454+1372012477000000*2^(1/2))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{}{1364}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(12280939 - 19536786*x)/(2824232928*(3 - x + 2*x^2)^(3/2)) - (1134826571 -
 1504660754*x)/(476353953856*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x
 + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x +
2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (35*Sqrt[(2243059557247 + 2011748500000*S
qrt[2])/682]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*
(1432939 + 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/1800960128 - (35*Sqrt[(-2243059557247 + 2011748500000*Sqrt[2])/682]
*ArcTanh[(Sqrt[11/(31*(-2243059557247 + 2011748500000*Sqrt[2]))]*(1432939 -
 2428746*Sqrt[2] + (6290431 - 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/18
00960128
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
 - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
```

```
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5687+\frac{8635x}{2}-8580x^2}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} + \frac{46386+86885x}{1860496\left(3-x+2x^2\right)^{3/2}\left(2 \right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} + \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{136}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{136}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{136}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{136}$$

Mathematica [C] time = 1.96, size = 242, normalized size = 0.90

$$\frac{11109\sqrt{286+22i\sqrt{31}}\left(4541903-6290431i\sqrt{31}\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-11109i\sqrt{286-22i\sqrt{31}}\ (}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((5456*(9739335532 + 218659985088*x + 178650961091*x^2 + 519223213785*x^3 +
 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 225699113100*x^7))
/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + 11109*Sqrt[286 + (22*I)*Sqrt
[31]]*(4541903 - (6290431*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4
*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (11
109*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(-4541903*I + 6290431*Sqrt[31])*ArcTanh[
(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
*Sqrt[3 - x + 2*x^2])])/7796961516715008
```

fricas [B] time = 1.99, size = 2343, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/611377875290135815296770157063555072*(2164988593398757980*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/4525340115746282619252370338578594 39*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
 20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 128461267801829958223938254 77
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
 - 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 -
1036800*x) + 3276288*x) + 508857664084041859437692 9*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x
^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(129508224872072^(
1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*
x + 2195288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000
000) - 19211019462513817817 83*x^2 - 17250711354094040489 48*sqrt(2)*(2*x^2 -
 x + 3) + 59201304874277275316 17*x - 78412324336791093134 00)/x^2) + 1459787
13411170407072657107696083 69*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560
*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*
sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3
+ 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 46612
00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296
*x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^(1/4)*
sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390
*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 40234970
00000)*arctan(1/4525340115746282619252370338578594 39*(11475013444*sqrt(4023
497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 11768684222*x^6
+ 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - s
qrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4
 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206
82752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(2145837
31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x
^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 1468800342
0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x
 + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*sqrt(22
```

```
43059557247*sqrt(2) + 4023497000000) - 12846126780182995822393825477255364 7
2*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(4023497)*(11*
129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 + 3977730
3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt(2)*(10
387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242
91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12
9508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 34803726780
*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(2)*(367
903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 270540014
40*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) - 111948686098489209076292
438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 +
244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800
*x) + 3276288*x) - 50885766408404185943769 29*sqrt(31)*(254591*x^8 - 4815126
*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689569
28*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 36
18*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((129508224872072^(1/4)*sqr
t(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195
288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 1
921101946251381781783*x^2 + 1725071135409404048948*sqrt(2)*(2*x^2 - x + 3)
- 5920130487427727531617*x + 7841232433679109313400)/x^2) - 145978713411170
40707265710769608369*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 1
42835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*
(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x
^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 2
4772608*x + 18579456)) + 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(341
)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*x^6
 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96966
2777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484589
2000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(246439191250000000
0000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sq
rt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*sqrt
(2243059557247*sqrt(2) + 4023497000000) + 1921101946251381781783*x^2 + 1725
071135409404048948*sqrt(2)*(2*x^2 - x + 3) - 5920130487427727531617*x + 784
1232433679109313400)/x^2) - 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(
341)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*
x^6 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96
9662777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484
5892000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(-24643919125000
00000000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31
)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*
sqrt(2243059557247*sqrt(2) + 4023497000000) - 1921101946251381781783*x^2 -
1725071135409404048948*sqrt(2)*(2*x^2 - x + 3) + 5920130487427727531617*x -
 7841232433679109313400)/x^2) + 42781764158153220413910 4*(225699113100*x^7
- 12234606480*x^6 + 592923725931*x^5 + 174241614961*x^4 + 519223213785*x^3
+ 178650961091*x^2 + 218659985088*x + 9739335532)*sqrt(2*x^2 - x + 3))/(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 70.56Done
```

maple [B] time = 0.21, size = 19014, normalized size = 70.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3 \left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

3.100 $\int \sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)^2\,dx$

Optimal. Leaf size=436

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df\right.\right.\right.}{1024c^{11/2}}$$

```
[Out] 1/960*(640*c^3*d*e-105*b^3*f^2+28*b*c*f*(7*a*f+10*b*e)-8*c^2*(32*a*e*f+25*b
*(2*d*f+e^2)))*(c*x^2+b*x+a)^(3/2)/c^4+1/160*(21*b^2*f^2-4*c*f*(5*a*f+14*b*
e)+40*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(3/2)/c^3+1/20*f*(-3*b*f+8*c*e)*x^2*
(c*x^2+b*x+a)^(3/2)/c^2+1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)/c-1/1024*(-4*a*c+b^
2)*(128*c^4*d^2+21*b^4*f^2-56*b^2*c*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d*f+e^
2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d^2+21*b^4*f^2-56*b^2*c
*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d*f+e^2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b
^2*(2*d*f+e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A] time = 0.79, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\,\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bc\right.\right.}{512c^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] ((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(
e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b +
2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*
b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x +
c*x^2)^(3/2))/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e
^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*
(a + b*x + c*x^2)^(3/2))/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c
^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2
 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c
^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2 dx = \frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2}\left(6cd^2+12cdex-3\left(af^2-2c\left(\ldots\right.\right.\right.}{6c}$$

$$= \frac{f(8ce-3bf)x^2\left(a+bx+cx^2\right)^{3/2}}{20c^2} + \frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c} + \frac{\int \sqrt{a+bx\ldots}}{\ldots}$$

$$= \frac{\left(21b^2f^2-4cf(14be+5af)+40c^2\left(e^2+2df\right)\right)x\left(a+bx+cx^2\right)^{3/2}}{160c^3} + \frac{f(8\ldots}{\ldots}$$

$$= \frac{\left(640c^3de-105b^3f^2+28bcf(10be+7af)-8c^2\left(32aef+25b\left(e^2+2df\right)\ldots\right.\right.}{960c^4}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8c^2\ldots\right.}{512c^5}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8c^2\ldots\right.}{512c^5}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8c^2\ldots\right.}{512c^5}$$

Mathematica [A] time = 0.93, size = 657, normalized size = 1.51

$$\frac{-f^2\left(-15\left(16a^2c^2-56ab^2c+21b^4\right)\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+x(b+cx)}}\right)\right)+16c^{3/2}\left(\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] (3840*c^(9/2)*d^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + 10240*c^(9/2)*d*e*(a
+ x*(b + c*x))^(3/2) + 3840*c^(9/2)*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(3/2)
 + 6144*c^(9/2)*e*f*x^2*(a + x*(b + c*x))^(3/2) + 2560*c^(9/2)*f^2*x^3*(a +
 x*(b + c*x))^(3/2) - 1920*c^4*(b^2 - 4*a*c)*d^2*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])] - 1920*b*c^3*d*e*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
 + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]) + 8*c*e*f*(-16*c^(3/2)*(-35*b^2 + 32*a*c + 42*b*c*x)*(a + x*(b
 + c*x))^(3/2) - 15*b*(7*b^2 - 12*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b
 + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
```

```
)])])) - 40*c^2*(e^2 + 2*d*f)*(80*b*c^(3/2)*(a + x*(b + c*x))^(3/2) - 3*(5*
b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])) - f^2*(2304*b*c^(7/
2)*x^2*(a + x*(b + c*x))^(3/2) + 16*c^(3/2)*(105*b^3 - 196*a*b*c - 126*b^2*
c*x + 120*a*c^2*x)*(a + x*(b + c*x))^(3/2) - 15*(21*b^4 - 56*a*b^2*c + 16*a
^2*c^2)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/(15360*c^(11/2))
```

fricas [A] time = 1.32, size = 1269, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a*b*c^4)*d*e
+ 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140*a*b^4*c + 2
40*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*
c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(c)*log(-8*c^2*x
^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) -
 4*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*x^4 +
16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e)*f)*x
^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2 + (31
5*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*b*c^5*
e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16*a*c^5
)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*a*b^2*
c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2*c^4 -
 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*(10*(5
*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a
*b*c^4)*d*e + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140
*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c
^3 + 16*a^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a
*c)) + 2*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*
x^4 + 16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e
)*f)*x^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2
 + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*
b*c^5*e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16
*a*c^5)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*
a*b^2*c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2
*c^4 - 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*
(10*(5*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^6]
```

giac [A] time = 0.36, size = 638, normalized size = 1.46

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,f^2x+\frac{bc^4f^2+24\,c^5fe}{c^5}\right)x+\frac{240\,c^5df-9\,b^2c^3f^2+20\,ac^4f^2+24\,bc^4fe+1}{c^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (b*c^4*f^2 + 24*c^5*f*
e)/c^5)*x + (240*c^5*d*f - 9*b^2*c^3*f^2 + 20*a*c^4*f^2 + 24*b*c^4*f*e + 12
0*c^5*e^2)/c^5)*x + (80*b*c^4*d*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2 + 640*c
^5*d*e - 56*b^2*c^3*f*e + 128*a*c^4*f*e + 40*b*c^4*e^2)/c^5)*x + (1920*c^5*
d^2 - 400*b^2*c^3*d*f + 960*a*c^4*d*f - 105*b^4*c*f^2 + 448*a*b^2*c^2*f^2 -
```

```
 240*a^2*c^3*f^2 + 640*b*c^4*d*e + 280*b^3*c^2*f*e - 928*a*b*c^3*f*e - 200*
b^2*c^3*e^2 + 480*a*c^4*e^2)/c^5)*x + (1920*b*c^4*d^2 + 1200*b^3*c^2*d*f -
4160*a*b*c^3*d*f + 315*b^5*f^2 - 1680*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2 - 19
20*b^2*c^3*d*e + 5120*a*c^4*d*e - 840*b^4*c*f*e + 3680*a*b^2*c^2*f*e - 2048
*a^2*c^3*f*e + 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2)/c^5) + 1/1024*(128*b^2*c
^4*d^2 - 512*a*c^5*d^2 + 80*b^4*c^2*d*f - 384*a*b^2*c^3*d*f + 256*a^2*c^4*d
*f + 21*b^6*f^2 - 140*a*b^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*c^3*f^2 -
128*b^3*c^3*d*e + 512*a*b*c^4*d*e - 56*b^5*c*f*e + 320*a*b^3*c^2*f*e - 384*
a^2*b*c^3*f*e + 40*b^4*c^2*e^2 - 192*a*b^2*c^3*e^2 + 128*a^2*c^4*e^2)*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.02, size = 1429, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)/c-7/32*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(1/2)-
7/20*e*f*b/c^2*x*(c*x^2+b*x+a)^(3/2)-7/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^(1/2)
+2/3*d*e*(c*x^2+b*x+a)^(3/2)/c+1/4*d^2/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d^2/c^(1
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d^2/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e^2-5/24
*b/c^2*(c*x^2+b*x+a)^(3/2)*e^2+5/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*e^2-5/128*b
^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/8*a^2/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2+21/512*f^2*b^5/c^5*(c*x^2+b
*x+a)^(1/2)-21/1024*f^2*b^6/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))+1/16*f^2*a^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/64
*f^2*b^3/c^4*(c*x^2+b*x+a)^(3/2)+3/8*e*f*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)-5/16
*e*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*e*f*b^2
/c^3*a*(c*x^2+b*x+a)^(1/2)+3/8*e*f*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+5/16*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*d*f+3/8*b^2/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*f-1/4*a/c*x*(c*x^2+b*x+a)^(1/
2)*d*f-1/8*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*f-1/2*d*e*b/c*x*(c*x^2+b*x+a)^(1/2
)-1/2*d*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/2*d^2*x
*(c*x^2+b*x+a)^(1/2)+21/256*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(1/2)-5/12*b/c^2*(c
*x^2+b*x+a)^(3/2)*d*f+5/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2+5/32*b^3/c^3*(
c*x^2+b*x+a)^(1/2)*d*f+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*e^2-5/64*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*d*f-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*e^2-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e^2
-1/4*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-1/4*d*e*b^
2/c^2*(c*x^2+b*x+a)^(1/2)+35/256*f^2*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*a-7/64*f^2*b^3/c^4*a*(c*x^2+b*x+a)^(1/2)-15/64*f^2*b^2/c^
(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+49/240*f^2*b/c^3*a*(c
*x^2+b*x+a)^(3/2)-1/8*f^2*a/c^2*x*(c*x^2+b*x+a)^(3/2)+1/16*f^2*a^2/c^2*x*(c
*x^2+b*x+a)^(1/2)+1/32*f^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b+2/5*e*f*x^2*(c*x^2
+b*x+a)^(3/2)/c+7/24*e*f*b^2/c^3*(c*x^2+b*x+a)^(3/2)-7/64*e*f*b^4/c^4*(c*x^
2+b*x+a)^(1/2)+7/128*e*f*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))-4/15*e*f*a/c^2*(c*x^2+b*x+a)^(3/2)-3/20*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(
3/2)+21/160*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(3/2)+1/2*x*(c*x^2+b*x+a)^(3/2)/c*d
*f+1/8*d*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 5.31, size = 1299, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] d^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e^2*x*(a + b*x + c*x^2)^(3/2
))/(4*c) + (a*f^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/
2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
 + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c)
 + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) - (3*b
*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^
3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
+ c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*(
(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b
*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log
((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5
/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^
2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (f^2*x^3*(a + b*
x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) +
 (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3
/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (4*a*e*f*((log((b + 2*c
*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((
8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c
) - (5*b*d*f*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(4*c) + (d*e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(12*c^2) + (d*f*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7
*b*e*f*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c) + (2*e*f*x^2*(a +
 b*x + c*x^2)^(3/2))/(5*c) - (a*d*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2
) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c
^(3/2))))/(2*c) + (d*e*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))
*(b^3 - 4*a*b*c))/(8*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2}\, \left(d + ex + fx^2\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2, x)
```

3.101 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=175

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8\cdots\right.}{64c^3}$$

```
[Out] 1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-1
/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*
d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^{2}\cdots\right.}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\left(16c^2d-8bce+\right.}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(\right.}{2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(\right.}{2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(\right.}{2}$$

Mathematica [A] time = 0.27, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4bc\left(2c\left(6d+2ex+fx^2\right)-13af\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)+15b^3f\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(7/2)
)
```

fricas [A] time = 0.77, size = 465, normalized size = 2.66

$$\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
```

```
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

giac [A] time = 0.30, size = 212, normalized size = 1.21

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{bc^2f+8\,c^3e}{c^3}\right)x+\frac{48\,c^3d-5\,b^2cf+12\,ac^2f+8\,bc^2e}{c^3}\right)x+\frac{48\,bc^2d+15\,b^3f-52\,a}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 453, normalized size = 2.59

$$-\frac{a^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}+\frac{3a\,b^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{5}{2}}}-\frac{abe\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{4c^{\frac{3}{2}}}+\frac{ad\ln\left(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f*b/c^2*(c*x^2+b*x+a)^(3/2)+5/32*f*b^2/c
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*f*b^2/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f*b^4/c^(7/2)*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e*b/c*(c*x^2+b*x+a)^(1/2)
*x-1/8*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)-1/4*e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a+1/16*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/2*d*(c*x^2+b*x+a)^(1/2)*x+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 3.91, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}-\frac{a\,f\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+\ldots\right.}{2\,c\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.102 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/
4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/
2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)
^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e-2
*d*f+e^2)*c)^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e
+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x
+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
)*(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(
1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {989, 621, 206, 1032, 724}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{\cdots}{(e\cdots}}{f\sqrt{e^2-4df}\cdots}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4d\cdots}\right.\right.}}{\sqrt{2}f\sqrt{\cdots}}$$

Mathematica [A] time = 1.22, size = 417, normalized size = 0.97

$$\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}\cdots\right.}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-\cdots}\right.}}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*
f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a
*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]) - Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2
*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**B**] time = 0.05, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.103 $\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx$

Optimal. Leaf size=488

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4\right.}{}$$

```
[Out] -(2*f*x+e)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)-1/2*arctanh(1/4*(
4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/
(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1
/2))^(1/2))*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)
^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^
(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f
+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b
*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e+(-4*d*f+e^
2)^(1/2)))/(-4*d*f+e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*
e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.93, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {971, 1032, 724, 206}

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) -
((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b
*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c
*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{\frac{1}{2}(be-4af)+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{-e^2+4df}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{}{(e}}{\left(e^2-4df\right)^{3/2}}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(2\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\right)}{}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\tan}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{ce^2-2cdf-bef}}$$

Mathematica [A] time = 5.09, size = 555, normalized size = 1.14

$$\frac{4f(e+2fx)\sqrt{a+x(b+cx)}}{\left(e^2-4df\right)\left(\sqrt{e^2-4df}-e-2fx\right)\left(\sqrt{e^2-4df}+e+2fx\right)} + \frac{\left(ce\left(\sqrt{e^2-4df}-e\right)-f\left(4af+b\left(\sqrt{e^2-4df}-2e\right)\right)\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{f\left(2af+b\left(\sqrt{e\right.\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] (4*f*(e + 2*f*x)*Sqrt[a + x*(b + c*x)])/((e^2 - 4*d*f)*(-e + Sqrt[e^2 - 4*d
*f] - 2*f*x)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + ((c*e*(-e + Sqrt[e^2 - 4*d*
f]) - f*(4*a*f + b*(-2*e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(-4*a*f + 2*c*(e -
Sqrt[e^2 - 4*d*f])*x + b*(e - Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]
))]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) - ((c*
```

```
e*(e + Sqrt[e^2 - 4*d*f]) + f*(4*a*f - b*(2*e + Sqrt[e^2 - 4*d*f])))*ArcTan
h[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x
))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2
)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))])])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.04, size = 22287, normalized size = 45.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\left(f\,x^2 + e\,x + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2,x)
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
[Out] Timed out
```

3.104 $\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx$

Optimal. Leaf size=564

$$\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2d\cdots\right.\right.\right.}{32768c^{13/2}}$$

```
[Out] 1/6144*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-128*c^3*(6*b*d*e+a*
(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+e^2)))*(2*c*x+b)*(c
*x^2+b*x+a)^(3/2)/c^5+1/13440*(5376*c^3*d*e-693*b^3*f^2+36*b*c*f*(31*a*f+56
*b*e)-32*c^2*(48*a*e*f+49*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(5/2)/c^4+1/1344*(9
9*b^2*f^2-12*c*f*(7*a*f+24*b*e)+224*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(5/2)/
c^3+1/112*f*(-11*b*f+32*c*e)*x^2*(c*x^2+b*x+a)^(5/2)/c^2+1/8*f^2*x^3*(c*x^2
+b*x+a)^(5/2)/c+1/32768*(-4*a*c+b^2)^2*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(
3*a*f+4*b*e)-128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+1
4*b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1
3/2)-1/16384*(-4*a*c+b^2)*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-
128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+
e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

Rubi [A] time = 0.93, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)\cdots\right.\right.}{6144c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 12
8*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2
*(e^2 + 2*d*f)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4
*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2
+ 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^5) + ((5376*c^3*d*e - 693*b^3*f^2 + 3
6*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d*f)))*(a + b*
x + c*x^2)^(5/2))/(13440*c^4) + ((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 22
4*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(1344*c^3) + (f*(32*c*e - 1
1*b*f)*x^2*(a + b*x + c*x^2)^(5/2))/(112*c^2) + (f^2*x^3*(a + b*x + c*x^2)^
(5/2))/(8*c) + ((b^2 - 4*a*c)^2*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b
*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*
a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(32768*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx = \frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}+\frac{\int \left(a+bx+cx^2\right)^{3/2}\left(8cd^2+16cdex-\left(3af\right.\right.}{}$$

$$= \frac{f(32ce-11bf)x^2\left(a+bx+cx^2\right)^{5/2}}{112c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}+\frac{\int\left(a\right.}{}$$

$$= \frac{\left(99b^2f^2-12cf(24be+7af)+224c^2\left(e^2+2df\right)\right)x\left(a+bx+cx^2\right)^{5/2}}{1344c^3}$$

$$= \frac{\left(5376c^3de-693b^3f^2+36bcf(56be+31af)-32c^2\left(48aef+49b\left(e\right.\right.\right.}{13440c^4}$$

$$= \frac{\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+a\left(e^2+2d\right.\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde\right.\right.}{}$$

Mathematica [A] time = 1.76, size = 829, normalized size = 1.47

$$\frac{430080f^2(a+x(b+cx))^{5/2}x^3+983040ef(a+x(b+cx))^{5/2}x^2+573440\left(e^2+2df\right)(a+x(b+cx))^{5/2}x+1376\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] (430080*d^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 1376256*d*e*(a + x*(b + c
*x))^(5/2) + 573440*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(5/2) + 983040*e*f*x^
2*(a + x*(b + c*x))^(5/2) + 430080*f^2*x^3*(a + x*(b + c*x))^(5/2) + (80640
*(b^2 - 4*a*c)*d^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (2
6880*b*d*e*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c
)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (96*e*f*(-256*c^(5/
2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*
a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*S
qrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(e^2 + 2*d*f)*(1792
*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
 + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]))))/c^(7/2) - (3*f^2*(112640*b*c^(9/2)*x^2*(a + x*(b + c*x))^(5
/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a + x*
(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^(3/2)*(b + 2
*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))))/c^(11/2))/(3440640*c)
```

fricas [B] time = 3.79, size = 2179, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^
2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 -
1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144
*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 -
 64*a^3*b*c^4)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(215040*c^8*f^2*x^7 + 15360*(32
*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*a*c^7)*f
^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*b*c^7*e^2
 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6 + 64*a*c^
7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*c^6 + 140*a
*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 + 32*(14*(3*b^
2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 - 26880*(3*b^3*
c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d*e
 - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - 3*(3465*b^7*c -
 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^
7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*a*b*c^6)*e^2 -
3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(14*(7*b^3*c^5 -
 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e)*f)*x^2 - 3
2*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(315*b^6*c^2 - 2
520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(26880*(b^2*c^6 +
 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35*b^4*c^4 - 216*
a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2
*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2
*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*
x^2 + b*x + a))/c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c
^6)*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 -
60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c
+ 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2
 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 +
```

```
b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2*
x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^
6 + 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e +
 2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^
2*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*
b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2
 + 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3
- 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 1
28*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 -
3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2
+ 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*
a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(
14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6
)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(
315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(2
6880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35
*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*
c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^
2*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e
)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7]
```

giac [B] time = 0.60, size = 1150, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (17*b*c^7
*f^2 + 32*c^8*f*e)/c^7)*x + (448*c^8*d*f + 3*b^2*c^6*f^2 + 252*a*c^7*f^2 +
480*b*c^7*f*e + 224*c^8*e^2)/c^7)*x + (5824*b*c^7*d*f - 33*b^3*c^5*f^2 + 15
6*a*b*c^6*f^2 + 5376*c^8*d*e + 96*b^2*c^6*f*e + 6144*a*c^7*f*e + 2912*b*c^7
*e^2)/c^7)*x + (26880*c^8*d^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f + 297*b^
4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2 + 59136*b*c^7*d*e - 864*b
^3*c^5*f*e + 4224*a*b*c^6*f*e + 672*b^2*c^6*e^2 + 31360*a*c^7*e^2)/c^7)*x +
 (80640*b*c^7*d^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f - 693*b^5*c^3*f^2
+ 4680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2 + 5376*b^2*c^6*d*e + 172032*a*c^7
*d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^
3*c^5*e^2 + 8064*a*b*c^6*e^2)/c^7)*x + (26880*b^2*c^6*d^2 + 537600*a*c^7*d^
2 + 15680*b^4*c^4*d*f - 96768*a*b^2*c^5*d*f + 107520*a^2*c^6*d*f + 3465*b^6
*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*b^2*c^4*f^2 - 20160*a^3*c^5*f^2
- 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e - 10080*b^5*c^3*f*e + 69888*a*b^3*
c^4*f*e - 112128*a^2*b*c^5*f*e + 7840*b^4*c^4*e^2 - 48384*a*b^2*c^5*e^2 + 5
3760*a^2*c^6*e^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*c^6*d^2 + 47040*
b^5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f + 10395*b^7*c*f^2
 - 91980*a*b^5*c^2*f^2 + 244944*a^2*b^3*c^3*f^2 - 176448*a^3*b*c^4*f^2 - 80
640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e - 30240*b^6*c^2
*f*e + 241920*a*b^4*c^3*f*e - 526848*a^2*b^2*c^4*f*e + 196608*a^3*c^5*f*e +
 23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2)/c^7) - 1/
32768*(768*b^4*c^4*d^2 - 6144*a*b^2*c^5*d^2 + 12288*a^2*c^6*d^2 + 448*b^6*c
^2*d*f - 3840*a*b^4*c^3*d*f + 9216*a^2*b^2*c^4*d*f - 4096*a^3*c^5*d*f + 99*
b^8*f^2 - 1008*a*b^6*c*f^2 + 3360*a^2*b^4*c^2*f^2 - 3840*a^3*b^2*c^3*f^2 +
768*a^4*c^4*f^2 - 768*b^5*c^3*d*e + 6144*a*b^3*c^4*d*e - 12288*a^2*b*c^5*d*
e - 288*b^7*c*f*e + 2688*a*b^5*c^2*f*e - 7680*a^2*b^3*c^3*f*e + 6144*a^3*b*
c^4*f*e + 224*b^6*c^2*e^2 - 1920*a*b^4*c^3*e^2 + 4608*a^2*b^2*c^4*e^2 - 204
8*a^3*c^5*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
/c^(13/2)
```

maple [B] time = 0.02, size = 2458, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*f^2*x^3*(c*x^2+b*x+a)^(5/2)/c+1/8*d^2/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d^2*(
c*x^2+b*x+a)^(1/2)*x*a-3/64*d^2/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d^2/c^(1/2)
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d^2/c^(5/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4+2/5*d*e*(c*x^2+b*x+a)^(5/2)/c+1/6*x
*(c*x^2+b*x+a)^(5/2)/c*e^2-7/60*b/c^2*(c*x^2+b*x+a)^(5/2)*e^2+7/192*b^3/c^3
*(c*x^2+b*x+a)^(3/2)*e^2-7/512*b^5/c^4*(c*x^2+b*x+a)^(1/2)*e^2+7/1024*b^6/c
^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/16*a^3/c^(3/2)*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2+3/128*f^2*a^4/c^(5/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+99/32768*f^2*b^8/c^(13/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-33/640*f^2*b^3/c^4*(c*x^2+b*x+a)^(5/2)+33/20
48*f^2*b^5/c^5*(c*x^2+b*x+a)^(3/2)-99/16384*f^2*b^7/c^6*(c*x^2+b*x+a)^(1/2)
-3/16*e*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)*x*a+1/8*e*f*b/c^2*a*x*(c*x^2+b*x+a)^(
3/2)+3/16*e*f*b/c^2*a^2*(c*x^2+b*x+a)^(1/2)*x+1/4*b^2/c^2*(c*x^2+b*x+a)^(1/
2)*x*a*d*f-3/8*d*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+3/16*d*e*b^3/c^(5/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/16*e*f*b^2/c^3*a*(c*x^2+b*x+a)^(3/
2)+3/32*e*f*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)-3/14*e*f*b/c^2*x*(c*x^2+b*x+a)^
(5/2)-3/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^(3/2)+9/256*e*f*b^5/c^4*(c*x^2+b*x+a
)^(1/2)*x-3/32*e*f*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a-15/64*e*f*b^3/c^(7/2)*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+21/256*e*f*b^5/c^(9/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*e*f*b/c^(5/2)*a^3*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+7/48*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)*d*f+1/8*b^
2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*e^2-7/128*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*d*f-5
7/512*f^2*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)*x+153/2048*f^2*b^4/c^4*(c*x^2+b*x
+a)^(1/2)*x*a-9/128*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(3/2)+1/8*b^3/c^3*(c*x^2+
b*x+a)^(1/2)*a*d*f+9/32*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a^2*d*f-15/128*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a*d*f-1/12*a/c*x*(c*x^2+b*x+a)^(3/2)*d*f-1/24*a/c^2*(c*x^2+b*x+a)^(3/2)*b*
d*f-1/8*a^2/c*(c*x^2+b*x+a)^(1/2)*x*d*f-1/16*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*
d*f-1/4*d*e*b/c*x*(c*x^2+b*x+a)^(3/2)+3/32*d*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*
x-3/16*d*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a-3/8*d*e*b/c^(3/2)*ln((c*x+1/2*b)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+1/4*d^2*x*(c*x^2+b*x+a)^(3/2)+153/4096*f^2*
b^5/c^5*(c*x^2+b*x+a)^(1/2)*a+93/1120*f^2*b/c^3*a*(c*x^2+b*x+a)^(5/2)-11/11
2*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(5/2)+105/1024*f^2*b^4/c^(9/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-1/8*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*d*f-7/30*b/c^2*(c*x^2+b*x+a)^(5/2)*d*f+7/96*b^2/c^2*x*(c
*x^2+b*x+a)^(3/2)*e^2+7/96*b^3/c^3*(c*x^2+b*x+a)^(3/2)*d*f-7/256*b^4/c^3*(c
*x^2+b*x+a)^(1/2)*x*e^2+1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*e^2-9/1024*e*f*b
^7/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/35*e*f*a/c^2*(c*x
^2+b*x+a)^(5/2)+3/20*e*f*b^2/c^3*(c*x^2+b*x+a)^(5/2)-3/64*e*f*b^4/c^4*(c*x^
2+b*x+a)^(3/2)+9/512*e*f*b^6/c^5*(c*x^2+b*x+a)^(1/2)+2/7*e*f*x^2*(c*x^2+b*x
+a)^(5/2)/c-9/256*f^2*b^3/c^4*a*(c*x^2+b*x+a)^(3/2)-57/1024*f^2*b^3/c^4*a^2
*(c*x^2+b*x+a)^(1/2)+33/448*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(5/2)-63/2048*f^2*b
^6/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-15/128*f^2*b^2/c^
(7/2)*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/64*f^2*a^2/c^2*x*(c
*x^2+b*x+a)^(3/2)+1/128*f^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b+3/128*f^2*a^3/c^2
*(c*x^2+b*x+a)^(1/2)*x+3/256*f^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b-1/16*f^2*a/c
^2*x*(c*x^2+b*x+a)^(5/2)+33/1024*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(3/2)-99/8192*
f^2*b^6/c^5*(c*x^2+b*x+a)^(1/2)*x-15/256*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a*e^2+7/512*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*d*f-1/24*a/c*x*(c*x^2+b*x+a)^(3/2)*e^2-1/48*a/c^2*(c*x^2+b*x
+a)^(3/2)*b*e^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*e^2+1/3*x*(c*x^2+b*x+a)^(5
/2)/c*d*f-1/8*d*e*b^2/c^2*(c*x^2+b*x+a)^(3/2)+3/64*d*e*b^4/c^3*(c*x^2+b*x+a
)^(1/2)-3/128*d*e*b^5/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3
/32*d^2/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*d^2/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16
*d^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/32*a^2/c^2
*(c*x^2+b*x+a)^(1/2)*b*e^2-7/256*b^5/c^4*(c*x^2+b*x+a)^(1/2)*d*f+9/64*b^2/c
```

```
^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)**2, x)
```

3.105 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=236

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af\right.}{512c^4}$$

```
[Out] 1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2)/
c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b
^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A] time = 0.23, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right.}{6c}$$

$$= \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\left(2c(6cd-a\right.}{}$$

$$= \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} + \frac{(12ce-}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}$$

Mathematica [A] time = 0.61, size = 392, normalized size = 1.66

$$\frac{\frac{360d\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}} - 60be\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2c\right.}{c^{5/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

fricas [A] time = 1.27, size = 839, normalized size = 3.56

$$\left[-\frac{15\left(24\left(b^4c^2-8ab^2c^3+16a^2c^4\right)d-12\left(b^5c-8ab^3c^2+16a^2bc^3\right)e+\left(7b^6-60ab^4c+144a^2b^2c^2-64a^3c^3\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

giac [A] time = 0.61, size = 417, normalized size = 1.77

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{13\,bc^5f+12\,c^6e}{c^5}\right)x+\frac{120\,c^6d+3\,b^2c^4f+140\,ac^5f+132\,bc^5e}{c^5}\right)x+\frac{360}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [B] time = 0.01, size = 862, normalized size = 3.65

$$-\frac{a^3 f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{9a^2b^2 f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{64c^{\frac{5}{2}}}-\frac{3a^2be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/8*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/
4*d*(c*x^2+b*x+a)^(3/2)*x+1/5*e*(c*x^2+b*x+a)^(5/2)/c+1/6*f*x*(c*x^2+b*x+a)
^(5/2)/c-3/32*d/c*(c*x^2+b*x+a)^(1/2)*x*b^2-3/16*e*b/c^(3/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*b^4+1/8*d/c*(c*x^2+b*x+a)^(3/2)*b-1/16*e*b^2/c^2*(c*x^2+
```

```
b*x+a)^(3/2)+3/128*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-3/256*e*b^5/c^(7/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/60*f*b/c^2*(c*x^2+b*x+a)^(5/2)+7/19
2*f*b^3/c^3*(c*x^2+b*x+a)^(3/2)-7/512*f*b^5/c^4*(c*x^2+b*x+a)^(1/2)+7/1024*
f*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*f*a^3/c^(3/2
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*d/c^(1/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/8*d*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d/c^2*(
c*x^2+b*x+a)^(1/2)*b^3-1/24*f*a/c*(c*x^2+b*x+a)^(3/2)*x-1/48*f*a/c^2*(c*x^2
+b*x+a)^(3/2)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x+7/96*f*b^2/c^2*(c*x^2+b*
x+a)^(3/2)*x-7/256*f*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x+1/16*f*b^3/c^3*(c*x^2+b*
x+a)^(1/2)*a+9/64*f*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a^2-15/256*f*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/1
6*d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+3/16*d/c*(c*x
^2+b*x+a)^(1/2)*b*a+3/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x-3/32*e*b^2/c^2*(c*
x^2+b*x+a)^(1/2)*a-1/8*e*b/c*(c*x^2+b*x+a)^(3/2)*x-1/32*f*a^2/c^2*(c*x^2+b*
x+a)^(1/2)*b+3/32*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

3.106 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$

Optimal. Leaf size=679

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-2f\left(-f^2\left(b^2d-a^2f\right)+2cdf(be-af)+c^2(-d)\left(e^2-df\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf}$$

```
[Out] 1/8*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c^2*(-d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^
(1/2)/(c*x^2+b*x+a)^(1/2))/f^3/c^(1/2)-1/4*(-2*c*f*x-5*b*f+4*c*e)*(c*x^2+b*
x+a)^(1/2)/f^2+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*
(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c*d*f*(-a*f+b*e)-f^2*(
-a^2*f+b^2*d)-c^2*d*(-d*f+e^2))+(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))*
(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(
-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a
)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*
(-2*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2))+(-b*f+c*e)*(
f*(-2*a*f+b*e)-c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+
e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2
)
```

Rubi [A] time = 11.03, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {977, 1076, 621, 206, 1032, 724}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-2f\left(-f^2\left(b^2d-a^2f\right)+2cdf(be-af)+c^2(-d)\left(e^2-df\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((3*b^2*f^2 -
12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]
)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2
*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*(b^2*d - a^
2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}\left(-4bcde+5b^2df+4af(cd-2af)\right)-\frac{1}{4}\left(8c^2de-4acef-bf(5be-16\right.}{\sqrt{a+bx+cx^2}}}{2\ldots}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{1}{4}f\left(-4bcde+5b^2df+4af(cd-2af)\right)-\frac{1}{4}d\left(-3b^2f^2+12cf(be-af)\right.}{\ldots}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\text{Subst}\left(\ldots\right.}{4f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\tanh^{-1}\ldots}{8\sqrt{c}\,f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\tanh^{-1}\ldots}{8\sqrt{c}\,f^3}$$

Mathematica [A] time = 4.58, size = 1232, normalized size = 1.81

$$\frac{\sqrt{a+x(b+cx)}\left(-4\left(e+\sqrt{e^2-4df}\right)^2c^2+4f\left(e+\sqrt{e^2-4df}\right)xc^2-16af^2c+10bf\left(e+\sqrt{e^2-4df}\right)c-4bf^2xc\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*b^2*f^2 - 16*a*c*f^2 + 10*b*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*c^2*(e + S
qrt[e^2 - 4*d*f])^2 - 4*b*c*f^2*x + 4*c^2*f*(e + Sqrt[e^2 - 4*d*f])*x)*Sqrt
[a + x*(b + c*x)] + 2*Sqrt[a + x*(b + c*x)]*(b^2*f^2 - 2*c^2*(-2*e^2 + 4*d*
f + 2*e*Sqrt[e^2 - 4*d*f] + e*f*x - f*Sqrt[e^2 - 4*d*f]*x) + c*f*(8*a*f + b
*(-5*e + 5*Sqrt[e^2 - 4*d*f] + 2*f*x))) - ((b*f + c*(-e + Sqrt[e^2 - 4*d*f]
))*(b^2*f^2 + 4*c^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f + b
*(-e + Sqrt[e^2 - 4*d*f]))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
c*x)])])/(Sqrt[c]*f) + ((-(b*f) + c*(e + Sqrt[e^2 - 4*d*f]))*(-(b^2*f^2) +
4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(-3*a*f + b*(e + Sqrt[e^2
 - 4*d*f]))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(Sqrt[
c]*f) + (8*Sqrt[2]*c*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f^2*(2*a^2*f^2 - 2*a*b*f*(e + Sqrt[e^2 -
 4*d*f]) + b^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])) + 2*c*f*(a*f*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*
Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e
+ Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/
(f*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
- 4*d*f]))]) + (8*Sqrt[2]*c*(c^2*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*Sqrt[e
^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f^2*(-2*a^2*f^2 + 2*a*b*f*(e - S
qrt[e^2 - 4*d*f]) + b^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])) + 2*c*f*(a*f*
(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) + b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f
])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f -
```

```
 e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*
(b + c*x)])])/(f*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))])))/(16*c*f^2*Sqrt[e^2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.03, size = 22523, normalized size = 33.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.107 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}\,dx$

Optimal. Leaf size=704

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c\cdots}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-be\cdots}}$$

```
[Out] -(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)+c^(3/2)*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2-(-2*c*f*x-2*b*f+c*e)*(c*x^2+b
*x+a)^(1/2)/f/(-4*d*f+e^2)-1/4*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2
)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*
d*f+e^2)+f*(2*b^2*d*f+4*a*f*(a*f+c*d)-b*e*(3*a*f+c*d)))+(-b*f+c*e)*(f*(-2*a
*f+b*e)+2*c*(-5*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2/(-4*d*f+e^2)^(3/2)*2^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/4
*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/
2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(
-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*d*f+e^2)+f*(2*b^2*d*f+4*a*f*(a
*f+c*d)-b*e*(3*a*f+c*d)))+(-b*f+c*e)*(f*(-2*a*f+b*e)+2*c*(-5*d*f+e^2))*(e+(
-4*d*f+e^2)^(1/2)))/f^2/(-4*d*f+e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 11.95, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {971, 1066, 1076, 621, 206, 1032, 724}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c\cdots}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-be\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e
+ 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3
/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e -
b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2
*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f)
)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^
(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f
]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4
*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*
e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[2]*f^2
*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*S
qrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}(3be-4af)+(3ce+bf)x+4cfx^2\right)}{d+ex+fx^2}\,dx}{-e^2+4df}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{cf\left(2b^2df+4af(cd+af)\right.}{}}{}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^2\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} -$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\right.}{f^2}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^{3/2}\tanh^{-1}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{f^2}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^{3/2}\tanh^{-1}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{f^2}$$

Mathematica [B] time = 6.83, size = 2843, normalized size = 4.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] (-2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e - Sqrt[e^2 - 4*d*f] + 2*f*
x)) - (2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e + Sqrt[e^2 - 4*d*f] +
 2*f*x)) - (3*f*(a + x*(b + c*x))^(3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e +
Sqrt[e^2 - 4*d*f])) - 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqr
t[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*
(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2
 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^
2 - 4*d*f]) - 4*a*c*f*(e - Sqrt[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt
[e^2 - 4*d*f])) + 4*c*(-e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTan
h[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 -
4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b
*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))
)/((e^2 - 4*d*f)*(a + b*x + c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^(3/2)*(((-
4*c*f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2
- 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*
```

```
a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*f) + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(-4*(-e + Sqrt[e^2 - 4
*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*
f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*
(e - Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])))
)*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^
2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqr
t[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f
^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16
*c*f^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) - (f*(a + x*(b + c*
x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f -
c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*(e
+ Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])
)) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e^2 - 4*d
*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e
^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f]
)^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f*(a
 + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f]
)) + 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*Sqrt[c]*(b^2*f^2 +
4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 -
4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*
Sqrt[e^2 - 4*d*f]]*(4*c*(e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*
f*(3*b^2*f*(e + Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*
(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 -
4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)*(a +
 b*x + c*x^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,8,0,0,0]%%%}+%%%{%%{[%%%{16,[
1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,6,0,1,0]%%%}+%%%{%%{[%%%{-96,[2]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,4,0,2,0]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[8,4,2,0,3,0]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[8,4,0,0,4,0]%%%}+%%%{%%%{4,[1]%%%},[7,3,8,1,0,0]%%%}+%%%{
%%%{-64,[2]%%%},[7,3,6,1,1,0]%%%}+%%%{%%%{384,[3]%%%},[7,3,4,1,2,0]%%%}+%%%
{%%%{-1024,[4]%%%},[7,3,2,1,3,0]%%%}+%%%{%%%{1024,[5]%%%},[7,3,0,1,4,0]%%%}
+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,4,8,0,1,0]%%%}+%%%{%%{[%%%{-64,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,4,6,0,2,0]%%%}+%%%{%%{[%%%{384,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[6,4,4,0,3,0]%%%}+%%%{%%{[%%%{-1024,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[6,4,2,0,4,0]%%%}+%%%{%%{[%%%{1024,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[6,4,0,0,5,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[6
,3,9,1,0,0]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,8,0,
0,1]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,7,1,1,0]%%%
}+%%%{%%{[%%%{128,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,6,0,1,1]%%%}+%%%{
%%{[%%%{-192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,5,1,2,0]%%%}+%%%{%%{[%
%%{-768,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,4,0,2,1]%%%}+%%%{%%{[%%%{51
2,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,3,1,3,0]%%%}+%%%{%%{[%%%{2048,[4]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,2,0,3,1]%%%}+%%%{%%{[%%%{-512,[4]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,1,1,4,0]%%%}+%%%{%%{[%%%{-2048,[5]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[6,3,0,0,4,1]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[6,2,8,2,0,0]%%%}+%%%{%%{[%%%{64,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[6,2,6,2,1,0]%%%}+%%%{%%{[%%%{-384,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[6,2,4,2,2,0]%%%}+%%%{%%{[%%%{1024,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
6,2,2,2,3,0]%%%}+%%%{%%{[%%%{-1024,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,2,
0,2,4,0]%%%}+%%%{%%%{8,[1]%%%},[5,3,9,0,0,1]%%%}+%%%{%%%{-12,[1]%%%},[5,3,8
,1,1,0]%%%}+%%%{%%%{-128,[2]%%%},[5,3,7,0,1,1]%%%}+%%%{%%%{192,[2]%%%},[5,3
,6,1,2,0]%%%}+%%%{%%%{768,[3]%%%},[5,3,5,0,2,1]%%%}+%%%{%%%{-1152,[3]%%%},[
5,3,4,1,3,0]%%%}+%%%{%%%{-2048,[4]%%%},[5,3,3,0,3,1]%%%}+%%%{%%%{3072,[4]%%
%},[5,3,2,1,4,0]%%%}+%%%{%%%{2048,[5]%%%},[5,3,1,0,4,1]%%%}+%%%{%%%{-3072,[
5]%%%},[5,3,0,1,5,0]%%%}+%%%{%%%{4,[1]%%%},[5,2,9,2,0,0]%%%}+%%%{%%%{16,[2]
%%%},[5,2,8,1,0,1]%%%}+%%%{%%%{-64,[2]%%%},[5,2,7,2,1,0]%%%}+%%%{%%%{-256,[
3]%%%},[5,2,6,1,1,1]%%%}+%%%{%%%{384,[3]%%%},[5,2,5,2,2,0]%%%}+%%%{%%%{1536
,[4]%%%},[5,2,4,1,2,1]%%%}+%%%{%%%{-1024,[4]%%%},[5,2,3,2,3,0]%%%}+%%%{%%%{
-4096,[5]%%%},[5,2,2,1,3,1]%%%}+%%%{%%%{1024,[5]%%%},[5,2,1,2,4,0]%%%}+%%%{
%%%{4096,[6]%%%},[5,2,0,1,4,1]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[4
,4,8,0,2,0]%%%}+%%%{%%{[%%%{96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,6,0,
3,0]%%%}+%%%{%%{[%%%{-576,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,4,0,4,0]%
%%}+%%%{%%{[%%%{1536,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,2,0,5,0]%%%}+%
%%{%%{[%%%{-1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,0,0,6,0]%%%}+%%%{%
%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,10,0,0,1]%%%}+%%%{%%{[6,0]:[1,0,%%%{-
1,[1]%%%}]%%},[4,3,9,1,1,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[4,3,8,0,1,1]%%%}+%%%{%%{[%%%{-96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[4,3,7,1,2,0]%%%}+%%%{%%{[%%%{-448,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3
,6,0,2,1]%%%}+%%%{%%{[%%%{576,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,5,1,3
,0]%%%}+%%%{%%{[%%%{2048,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,4,0,3,1]%%
%}+%%%{%%{[%%%{-1536,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,3,1,4,0]%%%}+%
%%{%%{[%%%{-4608,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,2,0,4,1]%%%}+%%%{%
%{[%%%{1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,1,1,5,0]%%%}+%%%{%%{[%%
%{4096,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,0,0,5,1]%%%}+%%%{%%{[-1,0]:[
1,0,%%%{-1,[1]%%%}]%%},[4,2,10,2,0,0]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[4,2,9,1,0,1]%%%}+%%%{%%{[%%%{24,[1]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[4,2,8,2,1,0]%%%}+%%%{%%{[%%%{-16,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[4,2,8,0,0,2]%%%}+%%%{%%{[%%%{384,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
4,2,7,1,1,1]%%%}+%%%{%%{[%%%{-224,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,6
,2,2,0]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,6,0,1,2
]%%%}+%%%{%%{[%%%{-2304,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,5,1,2,1]%%%
```

}+%%%{%%{[%%%{1024,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,4,2,3,0]%%%}+%%%
{%%{[%%%{-1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,4,0,2,2]%%%}+%%%{%%{
[%%%{6144,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,3,1,3,1]%%%}+%%%{%%{[%%%{
-2304,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,2,2,4,0]%%%}+%%%{%%{[%%%{4096
,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,2,0,3,2]%%%}+%%%{%%{[%%%{-6144,[5]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,1,1,4,1]%%%}+%%%{%%{[%%%{2048,[5]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,0,2,5,0]%%%}+%%%{%%{[%%%{-4096,[6]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[4,2,0,0,4,2]%%%}+%%%{%%%{-16,[1]%%%},[3,3,9,0,1,1]%
%%}+%%%{%%%{12,[1]%%%},[3,3,8,1,2,0]%%%}+%%%{%%%{256,[2]%%%},[3,3,7,0,2,1]%
%%}+%%%{%%%{-192,[2]%%%},[3,3,6,1,3,0]%%%}+%%%{%%%{-1536,[3]%%%},[3,3,5,0,3
,1]%%%}+%%%{%%%{1152,[3]%%%},[3,3,4,1,4,0]%%%}+%%%{%%%{4096,[4]%%%},[3,3,3,
0,4,1]%%%}+%%%{%%%{-3072,[4]%%%},[3,3,2,1,5,0]%%%}+%%%{%%%{-4096,[5]%%%},[3
,3,1,0,5,1]%%%}+%%%{%%%{3072,[5]%%%},[3,3,0,1,6,0]%%%}+%%%{%%%{12,[1]%%%},[
3,2,10,1,0,1]%%%}+%%%{%%%{-8,[1]%%%},[3,2,9,2,1,0]%%%}+%%%{%%%{32,[2]%%%},[
3,2,9,0,0,2]%%%}+%%%{%%%{-208,[2]%%%},[3,2,8,1,1,1]%%%}+%%%{%%%{128,[2]%%%}
,[3,2,7,2,2,0]%%%}+%%%{%%%{-512,[3]%%%},[3,2,7,0,1,2]%%%}+%%%{%%%{1408,[3]%
%%},[3,2,6,1,2,1]%%%}+%%%{%%%{-768,[3]%%%},[3,2,5,2,3,0]%%%}+%%%{%%%{3072,[
4]%%%},[3,2,5,0,2,2]%%%}+%%%{%%%{-4608,[4]%%%},[3,2,4,1,3,1]%%%}+%%%{%%%{20
48,[4]%%%},[3,2,3,2,4,0]%%%}+%%%{%%%{-8192,[5]%%%},[3,2,3,0,3,2]%%%}+%%%{%%
%{7168,[5]%%%},[3,2,2,1,4,1]%%%}+%%%{%%%{-2048,[5]%%%},[3,2,1,2,5,0]%%%}+%%
%{%%%{8192,[6]%%%},[3,2,1,0,4,2]%%%}+%%%{%%%{-4096,[6]%%%},[3,2,0,1,5,1]%%%
}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,8,0,3,0]%%%}+%%%{%%{[%%%{-64,[1
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,6,0,4,0]%%%}+%%%{%%{[%%%{384,[2]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,4,0,5,0]%%%}+%%%{%%{[%%%{-1024,[3]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[2,4,2,0,6,0]%%%}+%%%{%%{[%%%{1024,[4]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[2,4,0,0,7,0]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2
,3,10,0,1,1]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,9,1,2,0]%%%}+%%
%{%%{[%%%{-72,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,8,0,2,1]%%%}+%%%{%%{[
%%%{96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,7,1,3,0]%%%}+%%%{%%{[%%%{512
,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,6,0,3,1]%%%}+%%%{%%{[%%%{-576,[2]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,5,1,4,0]%%%}+%%%{%%{[%%%{-1792,[3]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,4,0,4,1]%%%}+%%%{%%{[%%%{1536,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[2,3,3,1,5,0]%%%}+%%%{%%{[%%%{3072,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[2,3,2,0,5,1]%%%}+%%%{%%{[%%%{-1536,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[2,3,1,1,6,0]%%%}+%%%{%%{[%%%{-2048,[5]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[2,3,0,0,6,1]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,11,1,
0,1]%%%}+%%%{%%{[2,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,10,2,1,0]%%%}+%%%{%%{[%%
%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,10,0,0,2]%%%}+%%%{%%{[%%%{56,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,9,1,1,1]%%%}+%%%{%%{[%%%{-36,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,8,2,2,0]%%%}+%%%{%%{[%%%{384,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[2,2,8,0,1,2]%%%}+%%%{%%{[%%%{-576,[2]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[2,2,7,1,2,1]%%%}+%%%{%%{[%%%{256,[2]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[2,2,6,2,3,0]%%%}+%%%{%%{[%%%{-2304,[3]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,2,6,0,2,2]%%%}+%%%{%%{[%%%{2816,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[2,2,5,1,3,1]%%%}+%%%{%%{[%%%{-896,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2
,2,4,2,4,0]%%%}+%%%{%%{[%%%{6144,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,4,
0,3,2]%%%}+%%%{%%{[%%%{-6656,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,3,1,4,
1]%%%}+%%%{%%{[%%%{1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,2,2,5,0]%%%
}+%%%{%%{[%%%{-6144,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,2,0,4,2]%%%}+%%
%{%%{[%%%{6144,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,1,1,5,1]%%%}+%%%{%%{
[%%%{-1024,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0,2,6,0]%%%}+%%%{%%%{8,[
1]%%%},[1,3,9,0,2,1]%%%}+%%%{%%%{-4,[1]%%%},[1,3,8,1,3,0]%%%}+%%%{%%%{-128,
[2]%%%},[1,3,7,0,3,1]%%%}+%%%{%%%{64,[2]%%%},[1,3,6,1,4,0]%%%}+%%%{%%%{768,
[3]%%%},[1,3,5,0,4,1]%%%}+%%%{%%%{-384,[3]%%%},[1,3,4,1,5,0]%%%}+%%%{%%%{-2
048,[4]%%%},[1,3,3,0,5,1]%%%}+%%%{%%%{1024,[4]%%%},[1,3,2,1,6,0]%%%}+%%%{%%
%{2048,[5]%%%},[1,3,1,0,6,1]%%%}+%%%{%%%{-1024,[5]%%%},[1,3,0,1,7,0]%%%}+%%
%{%%%{8,[1]%%%},[1,2,11,0,0,2]%%%}+%%%{%%%{-12,[1]%%%},[1,2,10,1,1,1]%%%}+%
%%{%%%{4,[1]%%%},[1,2,9,2,2,0]%%%}+%%%{%%%{-128,[2]%%%},[1,2,9,0,1,2]%%%}+%
%%{%%%{192,[2]%%%},[1,2,8,1,2,1]%%%}+%%%{%%%{-64,[2]%%%},[1,2,7,2,3,0]%%%}+

%%%{%%%{768,[3]%%%},[1,2,7,0,2,2]%%%}+%%%{%%%{-1152,[3]%%%},[1,2,6,1,3,1]%%
%}+%%%{%%%{384,[3]%%%},[1,2,5,2,4,0]%%%}+%%%{%%%{-2048,[4]%%%},[1,2,5,0,3,2
]%%%}+%%%{%%%{3072,[4]%%%},[1,2,4,1,4,1]%%%}+%%%{%%%{-1024,[4]%%%},[1,2,3,2
,5,0]%%%}+%%%{%%%{2048,[5]%%%},[1,2,3,0,4,2]%%%}+%%%{%%%{-3072,[5]%%%},[1,2
,2,1,5,1]%%%}+%%%{%%%{1024,[5]%%%},[1,2,1,2,6,0]%%%}+%%%{%%{[-1,0]:[1,0,%%%
{-1,[1]%%%}]%%},[0,4,8,0,4,0]%%%}+%%%{%%{[%%%{16,[1]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[0,4,6,0,5,0]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[0,4,4,0,6,0]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,
4,2,0,7,0]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,4,0,0
,8,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,10,0,2,1]%%%}+%%%{%%{[
2,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,9,1,3,0]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[0,3,8,0,3,1]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[0,3,7,1,4,0]%%%}+%%%{%%{[%%%{-192,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[0,3,6,0,4,1]%%%}+%%%{%%{[%%%{192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[0,3,5,1,5,0]%%%}+%%%{%%{[%%%{512,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,3,4,0,5,1]%%%}+%%%{%%{[%%%{-512,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,3
,1,6,0]%%%}+%%%{%%{[%%%{-512,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,2,0,6,
1]%%%}+%%%{%%{[%%%{512,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,1,1,7,0]%%%}
+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,12,0,0,2]%%%}+%%%{%%{[2,0]:[1,0
,%%%{-1,[1]%%%}]%%},[0,2,11,1,1,1]%%%}+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%
},[0,2,10,2,2,0]%%%}+%%%{%%{[%%%{16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2
,10,0,1,2]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,9,1,
2,1]%%%}+%%%{%%{[%%%{16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,8,2,3,0]%%%
}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,8,0,2,2]%%%}+%%%{
%%{[%%%{192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,7,1,3,1]%%%}+%%%{%%{[%%
%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,6,2,4,0]%%%}+%%%{%%{[%%%{256,
[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,6,0,3,2]%%%}+%%%{%%{[%%%{-512,[3]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,5,1,4,1]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[0,2,4,2,5,0]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[0,2,4,0,4,2]%%%}+%%%{%%{[%%%{512,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[0,2,3,1,5,1]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[0,2,2,2,6,0]%%%} / %%%{%%%{1,[1]%%%},[8,2,0,0,0,0]%%%}+%%%{%%{poly
1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,1,0,1,0,0]%%%}+%%%{%%%{-4,[1
]%%%},[6,2,0,0,1,0]%%%}+%%%{%%%{2,[1]%%%},[6,1,1,1,0,0]%%%}+%%%{%%%{8,[2]%%
%},[6,1,0,0,0,1]%%%}+%%%{%%%{4,[2]%%%},[6,0,0,2,0,0]%%%}+%%%{%%{poly1[%%%{-
8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,1,0,0,1]%%%}+%%%{%%{poly1[%%%{12,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,0,1,1,0]%%%}+%%%{%%{poly1[%%%{-4,[1
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{-16,[2]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,0,0,1,0,1]%%%}+%%%{%%%{6,[1]%%%},[4,2,0,
0,2,0]%%%}+%%%{%%%{2,[1]%%%},[4,1,2,0,0,1]%%%}+%%%{%%%{-6,[1]%%%},[4,1,1,1,
1,0]%%%}+%%%{%%%{-16,[2]%%%},[4,1,0,0,1,1]%%%}+%%%{%%%{1,[1]%%%},[4,0,2,2,0
,0]%%%}+%%%{%%%{24,[2]%%%},[4,0,1,1,0,1]%%%}+%%%{%%%{-8,[2]%%%},[4,0,0,2,1,
0]%%%}+%%%{%%%{16,[3]%%%},[4,0,0,0,0,2]%%%}+%%%{%%{poly1[%%%{16,[1]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,1,1,0,1,1]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1,2,0]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[3,0,2,1,0,1]%%%}+%%%{%%{poly1[%%%{8,[1]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[3,0,1,2,1,0]%%%}+%%%{%%{poly1[%%%{-32,[2]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[3,0,1,0,0,2]%%%}+%%%{%%{poly1[%%%{16,[2]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[3,0,0,1,1,1]%%%}+%%%{%%%{-4,[1]%%%},[2,2,0,0,3,0]%%%}+%%%{
%%%{-4,[1]%%%},[2,1,2,0,1,1]%%%}+%%%{%%%{6,[1]%%%},[2,1,1,1,2,0]%%%}+%%%{%%
%{8,[2]%%%},[2,1,0,0,2,1]%%%}+%%%{%%%{2,[1]%%%},[2,0,3,1,0,1]%%%}+%%%{%%%{-
2,[1]%%%},[2,0,2,2,1,0]%%%}+%%%{%%%{24,[2]%%%},[2,0,2,0,0,2]%%%}+%%%{%%%{-2
4,[2]%%%},[2,0,1,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2,2,0]%%%}+%%%{%%{poly
1[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,1,0,2,1]%%%}+%%%{%%{poly1[
%%%{4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,0,1,3,0]%%%}+%%%{%%{poly1[%%%
{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,3,0,0,2]%%%}+%%%{%%{poly1[%%%{1
2,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2,1,1,1]%%%}+%%%{%%{poly1[%%%{-4,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2,2,0]%%%}+%%%{%%%{1,[1]%%%},[0,2
,0,0,4,0]%%%}+%%%{%%%{2,[1]%%%},[0,1,2,0,2,1]%%%}+%%%{%%%{-2,[1]%%%},[0,1,1

```
,1,3,0]%%%}+%%%{%%%{1,[1]%%%},[0,0,4,0,0,2]%%%}+%%%{%%%{-2,[1]%%%},[0,0,3,1
,1,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2,2,0]%%%} Error: Bad Argument Value
```

maple [**B**] time = 0.04, size = 72576, normalized size = 103.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + b\,x + a\right)^{3/2}}{\left(f\,x^2 + e\,x + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x)
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
[Out] Timed out
```

3.108 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx$

Optimal. Leaf size=671

$$-\frac{3\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2\left(4df+e^2\right)\right)\right)\right)\tanh^{-1}}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+c}}$$

```
[Out] -1/2*(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)^2+3/4*(4*c*d*
e+4*a*e*f-b*(4*d*f+e^2)+2*(4*a*f^2-2*b*e*f+c*e^2)*x)*(c*x^2+b*x+a)^(1/2)/(-
4*d*f+e^2)^2/(f*x^2+e*x+d)-3/8*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2
)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(4*b*e*(3*a*f+c
*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-
(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(5/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^
2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+3/8*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+
e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)
/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(4*
b*e*(3*a*f+c*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-
b*f+c*e)*(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(5/2)*2^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 11.60, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {971, 1013, 1032, 724, 206}

$$\frac{3\left(-2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)+4bef(3af+cd)-4af\left(4af^2+ce^2\right)+b^2(-f)\left(4df+e^2\right)\right)\tanh}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2
) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)
*x)*Sqrt[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*b*
e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*
d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*a*
f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f
)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4
*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^
2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{\int \frac{\left(\frac{3}{2}(be-4af)+3(ce-bf)x\right)\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx}{2\left(e^2-4df\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

Mathematica [B] time = 7.23, size = 4727, normalized size = 7.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] (-2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/2)*(e - Sqrt[e^2 - 4*d*f
] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^2*(e - Sqrt[
e^2 - 4*d*f] + 2*f*x)) + (2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/
2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e
^2 - 4*d*f)^2*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (9*f^2*(a + x*(b + c*x))^(
3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c^2*f*x)*Sq
rt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*
(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^2 - 4*d*f]) - 4*a*c*f*(e - Sqrt[e^2
 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + 4*c*(-e + Sqrt[e^
2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a
*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*
f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(
-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2
)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((-4*c*f*(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b*f + 2*c*(-e + Sqrt[e^
2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*
x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f
])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) +
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] +
 b*f*Sqrt[e^2 - 4*d*f]]*(-4*(-e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3
```

```
*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e - Sqrt[e^2 -
4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2
*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + S
qrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sq
rt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4
*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(5/2
)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((-2*b*f - 2*c
*(-e + Sqrt[e^2 - 4*d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 - 2*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) - c*(-e + Sqrt[e^2 - 4*d*f])^2)*(-e + Sqrt[e^2 - 4*d*f
] - 2*f*x)) + (((-4*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])) -
 4*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))
 - 8*c^2*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c
*f^2) - ((16*c^(3/2)*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(32*c^
2*(-e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 16*c*f*
(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b
*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt
[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 -
2*b*f*(-e + Sqrt[e^2 - 4*d*f]) - c*(-e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*
d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((4*c
*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e + Sqrt[e^2 -
4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e + Sqrt[e^2
- 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*
a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*(e + Sqrt[e^2 - 4*d
*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*
(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e^2 - 4*d*f]))))*
ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 -
4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^
2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 -
 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2
)))/((e^2 - 4*d*f)^(5/2)*(a + b*x + c*x^2)^(3/2)) - (9*f^2*(a + x*(b + c*x)
)^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) + 4*c^2*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d
*f]]*(4*c*(e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*f*(3*b^2*f*(e
+ Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*(2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*
(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2
)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((2*b*f - 2*c*(e + Sqrt[e^2 - 4*
d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4*d*f])
- c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (((4*c*(b
```

```
*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) + 4*
c*f*(-(b^2*f) - 4*a*c*f + 2*b*c*(e + Sqrt[e^2 - 4*d*f])) - 8*c^2*f*(b*f - c
*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((16*c^(3/2
)*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/f - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-32*c^2*(e + Sqrt[e^2 -
4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 16*c*f*(b^2*f + 4*a*c*f -
2*b*c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2]])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(
e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4
*d*f]) - c*(e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x
^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 178044, normalized size = 265.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{{\left(f\,x^2+e\,x+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x)
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
[Out] Timed out
```

3.109 $\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=717

$$\frac{x\sqrt{a+bx+cx^2}\left(24c^2f\left(50a^2f^2+322abef+175b^2\left(df+e^2\right)\right)-252b^2cf^2(14af+15be)-160c^3\left(27af\left(df+e^2\right)\right.\right.}{3840c^5}$$

```
[Out] 1/1024*(1024*c^6*d^3+231*b^6*f^3-252*b^4*c*f^2*(5*a*f+3*b*e)-1536*c^5*d*(b*
d*e+a*(d*f+e^2))+840*b^2*c^2*f*(4*a*b*e*f+2*a^2*f^2+b^2*(d*f+e^2))+384*c^4*
(3*b^2*d*(d*f+e^2)+3*a^2*f*(d*f+e^2)+2*a*b*e*(6*d*f+e^2))-320*c^3*(9*a^2*b*
e*f^2+a^3*f^3+9*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)+1/7680*(23040*c^5*d^2*e-3465*b^5*f^3
+420*b^3*c*f^2*(34*a*f+27*b*e)-504*b*c^2*f*(70*a*b*e*f+22*a^2*f^2+25*b^2*(d
*f+e^2))-640*c^4*(27*b*d*(d*f+e^2)+8*a*e*(6*d*f+e^2))+96*c^3*(128*a^2*e*f^2
+275*a*b*f*(d*f+e^2)+50*b^2*(6*d*e*f+e^3)))*(c*x^2+b*x+a)^(1/2)/c^6+1/3840*
(1155*b^4*f^3-252*b^2*c*f^2*(14*a*f+15*b*e)+5760*c^4*d*(d*f+e^2)+24*c^2*f*(
322*a*b*e*f+50*a^2*f^2+175*b^2*(d*f+e^2))-160*c^3*(27*a*f*(d*f+e^2)+10*b*(6
*d*e*f+e^3)))*x*(c*x^2+b*x+a)^(1/2)/c^5-1/960*(231*b^3*f^3-36*b*c*f^2*(13*a
*f+21*b*e)-320*c^3*(6*d*e*f+e^3)+24*c^2*f*(32*a*e*f+35*b*(d*f+e^2)))*x^2*(c
*x^2+b*x+a)^(1/2)/c^4+1/480*f*(99*b^2*f^2-4*c*f*(25*a*f+81*b*e)+360*c^2*(d*
f+e^2))*x^3*(c*x^2+b*x+a)^(1/2)/c^3+1/60*f^2*(-11*b*f+36*c*e)*x^4*(c*x^2+b*
x+a)^(1/2)/c^2+1/6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A] time = 2.71, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = $\{1661, 640, 621, 206\}$

$$\frac{\sqrt{a+bx+cx^2}\left(-504bc^2f\left(22a^2f^2+70abef+25b^2\left(df+e^2\right)\right)+96c^3\left(128a^2ef^2+275abf\left(df+e^2\right)+50b^2\left(6de\right.\right.\right.}{7680c^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*
c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2
 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d
*f) + 50*b^2*(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b
^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*
f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 +
 d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*
b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f
*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(
99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*
x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(
60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*
f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*
(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2
*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{6cd^3+18cd^2ex+18cd\left(e^2+df\right)x^2+6ce\left(e^2+6df\right)x^3-f\left(5af^2-18c\left(e^2+df\right)\right)x^4+\frac{1}{2}f^2(\cdots}{\sqrt{a+bx+cx^2}}}{6c}$$

$$= \frac{f^2(36ce-11bf)x^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{30c^2d^3+90c^2d^2ex+90c^2d\left(e^2+d\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{f\left(99b^2f^2-4cf(81be+25af)+360c^2\left(e^2+df\right)\right)x^3\sqrt{a+bx+cx^2}}{480c^3} + \frac{f^2(36ce-11b\cdots}{\cdots}$$

$$= -\frac{\left(231b^3f^3-36bcf^2(21be+13af)-320c^3\left(e^3+6def\right)+24c^2f\left(32aef+35b\left(e^2+d\cdots\right.\right.\right.}{960c^4}$$

$$= \frac{\left(1155b^4f^3-252b^2cf^2(15be+14af)+5760c^4d\left(e^2+df\right)+24c^2f\left(322abef+50a^2f\cdots\right.\right.}{3840\cdots}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+2\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+2\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+2\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 1.35, size = 615, normalized size = 0.86

$$\frac{\sqrt{a+x(b+cx)}\left(48c^3\left(2a^2f^2(128e+25fx)+2abf\left(f\left(275d+39fx^2\right)+275e^2+161efx\right)+b^2\left(6ef\left(100d+2\cdots\right.\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 68*a*f + 11
*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(75*e^2 + 75
*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 15*d*x*(6*e^
2 + 8*e*f*x + 3*f^2*x^2) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3
*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 175*e^2*f*x +
6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*f*(275*e^2 + 1
61*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*e^2*f*x + 96*e*
f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f + 15*d*(18*e^2 +
 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^2*x^2 + 22*f^3*x^
3)))))/(7680*c^6) + ((1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5
*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a
^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d
*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f
*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(1024*c^(13/2))
```

fricas [A] time = 3.20, size = 1583, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*a*c^5)*
d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c + 1680*a^
2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^2*c
^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(3*b^2*c
^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 - 120*a*b
^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f^3*x^5 + 230
40*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*b
*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*d
 - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
- 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(1
6*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16*
a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600*
b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(35
*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*d
^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a
))/c^7, -1/15360*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*
a*c^5)*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c +
1680*a^2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 4
8*a^2*c^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(
3*b^2*c^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 -
120*a*b^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f^3*x^5 + 23
040*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
 320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*
b*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*
d - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
 - 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
 156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(
16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16
*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600
*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(3
5*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*
```

```
d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x +
a))/c^7]
```

giac [A] time = 0.53, size = 824, normalized size = 1.15

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(\frac{10\,f^3x}{c}-\frac{11\,bc^4f^3-36\,c^5f^2e}{c^6}\right)x+\frac{360\,c^5df^2+99\,b^2c^3f^3-100\,ac^4f^3-324\,b\right.\right.\right.\right.\right.}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^3*x/c - (11*b*c^4*f^3 - 36*c
^5*f^2*e)/c^6)*x + (360*c^5*d*f^2 + 99*b^2*c^3*f^3 - 100*a*c^4*f^3 - 324*b*
c^4*f^2*e + 360*c^5*f*e^2)/c^6)*x - (840*b*c^4*d*f^2 + 231*b^3*c^2*f^3 - 46
8*a*b*c^3*f^3 - 1920*c^5*d*f*e - 756*b^2*c^3*f^2*e + 768*a*c^4*f^2*e + 840*
b*c^4*f*e^2 - 320*c^5*e^3)/c^6)*x + (5760*c^5*d^2*f + 4200*b^2*c^3*d*f^2 -
4320*a*c^4*d*f^2 + 1155*b^4*c*f^3 - 3528*a*b^2*c^2*f^3 + 1200*a^2*c^3*f^3 -
 9600*b*c^4*d*f*e - 3780*b^3*c^2*f^2*e + 7728*a*b*c^3*f^2*e + 5760*c^5*d*e^
2 + 4200*b^2*c^3*f*e^2 - 4320*a*c^4*f*e^2 - 1600*b*c^4*e^3)/c^6)*x - (17280
*b*c^4*d^2*f + 12600*b^3*c^2*d*f^2 - 26400*a*b*c^3*d*f^2 + 3465*b^5*f^3 - 1
4280*a*b^3*c*f^3 + 11088*a^2*b*c^2*f^3 - 23040*c^5*d^2*e - 28800*b^2*c^3*d*
f*e + 30720*a*c^4*d*f*e - 11340*b^4*c*f^2*e + 35280*a*b^2*c^2*f^2*e - 12288
*a^2*c^3*f^2*e + 17280*b*c^4*d*e^2 + 12600*b^3*c^2*f*e^2 - 26400*a*b*c^3*f*
e^2 - 4800*b^2*c^3*e^3 + 5120*a*c^4*e^3)/c^6) - 1/1024*(1024*c^6*d^3 + 1152
*b^2*c^4*d^2*f - 1536*a*c^5*d^2*f + 840*b^4*c^2*d*f^2 - 2880*a*b^2*c^3*d*f^
2 + 1152*a^2*c^4*d*f^2 + 231*b^6*f^3 - 1260*a*b^4*c*f^3 + 1680*a^2*b^2*c^2*
f^3 - 320*a^3*c^3*f^3 - 1536*b*c^5*d^2*e - 1920*b^3*c^3*d*f*e + 4608*a*b*c^
4*d*f*e - 756*b^5*c*f^2*e + 3360*a*b^3*c^2*f^2*e - 2880*a^2*b*c^3*f^2*e + 1
152*b^2*c^4*d*e^2 - 1536*a*c^5*d*e^2 + 840*b^4*c^2*f*e^2 - 2880*a*b^2*c^3*f
*e^2 + 1152*a^2*c^4*f*e^2 - 320*b^3*c^3*e^3 + 768*a*b*c^4*e^3)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

maple [B] time = 0.03, size = 1930, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 3*d^2*e/c*(c*x^2+b*x+a)^(1/2)+1/3*x^2/c*(c*x^2+b*x+a)^(1/2)*e^3+5/8*b^2/c^3
*(c*x^2+b*x+a)^(1/2)*e^3-5/16*b^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*e^3-2/3*a/c^2*(c*x^2+b*x+a)^(1/2)*e^3-231/512*f^3*b^5/c^6*(c*x^2
+b*x+a)^(1/2)+231/1024*f^3*b^6/c^(13/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-5/16*f^3*a^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d
^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+161/80*e*f^2*b/c^3*a
*x*(c*x^2+b*x+a)^(1/2)-5/2*b/c^2*x*(c*x^2+b*x+a)^(1/2)*d*e*f+9/2*b/c^(5/2)*
a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f-45/16*b^2/c^(7/2)*a*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f+55/16*b/c^3*a*(c*x^2+b*x+a)^
(1/2)*d*f^2+55/16*b/c^3*a*(c*x^2+b*x+a)^(1/2)*e^2*f-9/8*a/c^2*x*(c*x^2+b*x+
a)^(1/2)*d*f^2-147/160*f^3*b^2/c^4*a*x*(c*x^2+b*x+a)^(1/2)+39/80*f^3*b/c^3*
a*x^2*(c*x^2+b*x+a)^(1/2)-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2*f+2*x^2/c*(c*
x^2+b*x+a)^(1/2)*d*e*f+15/4*b^2/c^3*(c*x^2+b*x+a)^(1/2)*d*e*f-15/8*b^3/c^(7
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f-4*a/c^2*(c*x^2+b*x+a)
^(1/2)*d*e*f-27/40*e*f^2*b/c^2*x^3*(c*x^2+b*x+a)^(1/2)+63/80*e*f^2*b^2/c^3*
x^2*(c*x^2+b*x+a)^(1/2)-63/64*e*f^2*b^3/c^4*x*(c*x^2+b*x+a)^(1/2)+105/32*e*
f^2*b^3/c^(9/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-147/32*e*f^2*
b^2/c^4*a*(c*x^2+b*x+a)^(1/2)-45/16*e*f^2*b/c^(7/2)*a^2*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))-4/5*e*f^2*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)-7/8*b/c^2*
```

```
x^2*(c*x^2+b*x+a)^(1/2)*d*f^2-7/8*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)*e^2*f+35/32
*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*d*f^2+35/32*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*e^
2*f-45/16*b^2/c^(7/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+1
/6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*e^2*f-5/12*b/c^2*x*(c*x^2+b*x+a)^(1/2)*e^3+3/4*b/c^(5/2)*
a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3-105/64*b^3/c^4*(c*x^2+b*x
+a)^(1/2)*e^2*f+105/128*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*d*f^2+105/128*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e^2*f+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+3/4
*x^3/c*(c*x^2+b*x+a)^(1/2)*d*f^2+3/4*x^3/c*(c*x^2+b*x+a)^(1/2)*e^2*f-105/64
*b^3/c^4*(c*x^2+b*x+a)^(1/2)*d*f^2+3/5*e*f^2*x^4/c*(c*x^2+b*x+a)^(1/2)+189/
128*e*f^2*b^4/c^5*(c*x^2+b*x+a)^(1/2)-189/256*e*f^2*b^5/c^(11/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/5*e*f^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)-315
/256*f^3*b^4/c^(11/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+119/64*
f^3*b^3/c^5*a*(c*x^2+b*x+a)^(1/2)+105/64*f^3*b^2/c^(9/2)*a^2*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))-231/160*f^3*b/c^4*a^2*(c*x^2+b*x+a)^(1/2)-5/2
4*f^3*a/c^2*x^3*(c*x^2+b*x+a)^(1/2)+5/16*f^3*a^2/c^3*x*(c*x^2+b*x+a)^(1/2)-
11/60*f^3*b/c^2*x^4*(c*x^2+b*x+a)^(1/2)+33/160*f^3*b^2/c^3*x^3*(c*x^2+b*x+a
)^(1/2)-77/320*f^3*b^3/c^4*x^2*(c*x^2+b*x+a)^(1/2)+77/256*f^3*b^4/c^5*x*(c*
x^2+b*x+a)^(1/2)-9/4*b/c^2*(c*x^2+b*x+a)^(1/2)*f*d^2-9/4*b/c^2*(c*x^2+b*x+a
)^(1/2)*e^2*d+9/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f
*d^2+9/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3/2*
a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2-3/2*a/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3/2*d^2*e*b/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/2*x/c*(c*x^2+b*x+a)^(1/2)*f*d^2+3/2
*x/c*(c*x^2+b*x+a)^(1/2)*e^2*d
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/sqrt(a + b*x + c*x**2), x)
```

3.110 $\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bd\right.}{128c^{9/2}}$$

```
[Out] 1/128*(128*c^4*d^2+35*b^4*f^2-40*b^2*c*f*(3*a*f+2*b*e)-64*c^3*(2*b*d*e+a*(2
*d*f+e^2))+48*c^2*(4*a*b*e*f+a^2*f^2+b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/192*(384*c^3*d*e-105*b^3*f^2+20*b*
c*f*(11*a*f+12*b*e)-16*c^2*(16*a*e*f+9*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(1/2)/
c^4+1/96*(35*b^2*f^2-4*c*f*(9*a*f+20*b*e)+48*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+
a)^(1/2)/c^3+1/24*f*(-7*b*f+16*c*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*f^2*x^3
*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A] time = 0.63, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bd\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f
 + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*
c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c
^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sq
rt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e
 + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^
2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{4cd^2+8cdex-\left(3af^2-4c\left(e^2+2df\right)\right)x^2+\frac{1}{2}f(16ce-7bf)x^3}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{f(16ce-7bf)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{12c^2d^2+\left(24c^2de-16acef+7abf^2\right)x+\frac{1}{4}}{\sqrt{a}}}{}$$

$$= \frac{\left(35b^2f^2-4cf(20be+9af)+48c^2\left(e^2+2df\right)\right)x\sqrt{a+bx+cx^2}}{96c^3} + \frac{f(16ce-7bf)x^2\sqrt{a+}}{24c^2}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+c}}{192c^4}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+c}}{192c^4}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+c}}{192c^4}$$

Mathematica [A] time = 0.50, size = 251, normalized size = 0.79

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f + 7*b*f*x)
 + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) - 8*c^2*(a*f
*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2))))/(192*c^4) +
 ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e
+ a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

fricas [A] time = 1.44, size = 637, normalized size = 2.02

$$\left[\frac{3\left(128c^4d^2-128bc^3de+16\left(3b^2c^2-4ac^3\right)e^2+\left(35b^4-120ab^2c+48a^2c^2\right)f^2+16\left(2\left(3b^2c^2-4ac^3\right)d-\left(5b^3\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35
*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b
^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*
```

```
x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c^4*d
*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^
3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2
+ (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 +
 b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*
a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4
*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f^2*x^3 +
 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*
f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(4
8*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sq
rt(c*x^2 + b*x + a))/c^5]
```

giac [A] time = 0.63, size = 304, normalized size = 0.96

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6f^2x}{c}-\frac{7bc^2f^2-16c^3fe}{c^4}\right)x+\frac{96c^3df+35b^2cf^2-36ac^2f^2-80bc^2fe+48c^3e^2}{c^4}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c - (7*b*c^2*f^2 - 16*c^3*f*e)/c
^4)*x + (96*c^3*d*f + 35*b^2*c*f^2 - 36*a*c^2*f^2 - 80*b*c^2*f*e + 48*c^3*e
^2)/c^4)*x - (288*b*c^2*d*f + 105*b^3*f^2 - 220*a*b*c*f^2 - 384*c^3*d*e - 2
40*b^2*c*f*e + 256*a*c^2*f*e + 144*b*c^2*e^2)/c^4) - 1/128*(128*c^4*d^2 + 9
6*b^2*c^2*d*f - 128*a*c^3*d*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 + 48*a^2*c^2*f
^2 - 128*b*c^3*d*e - 80*b^3*c*f*e + 192*a*b*c^2*f*e + 48*b^2*c^2*e^2 - 64*a
*c^3*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9
/2)
```

maple [B] time = 0.01, size = 706, normalized size = 2.23

$$\frac{\sqrt{c\,x^2+bx+a}\;f^2x^3}{4c}-\frac{7\sqrt{c\,x^2+bx+a}\;b\,f^2x^2}{24c^2}+\frac{2\sqrt{c\,x^2+bx+a}\;ef\,x^2}{3c}+\frac{3a^2f^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*d*e/c*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e^2-3/4*b/c^2*(c*x^
2+b*x+a)^(1/2)*e^2+3/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e^2-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-35/64
*f^2*b^3/c^4*(c*x^2+b*x+a)^(1/2)+35/128*f^2*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))+3/8*f^2*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-5/6*e*f*b/c^2*x*(c*x^2+b*x+a)^(1/2)+1/4*f^2*x^3*(c*x^2+b*x+a)
^(1/2)/c+3/2*e*f*b/c^(5/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d^
2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-5/8*e*f*b^3/c^(7/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3*e*f*a/c^2*(c*x^2+b*x+a)^(1/2
)+x/c*(c*x^2+b*x+a)^(1/2)*d*f-3/2*b/c^2*(c*x^2+b*x+a)^(1/2)*d*f+3/4*b^2/c^(
5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-a/c^(3/2)*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-d*e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))-3/8*f^2*a/c^2*x*(c*x^2+b*x+a)^(1/2)+55/48*f^2*b/c^3*a*(c
*x^2+b*x+a)^(1/2)-7/24*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)+35/96*f^2*b^2/c^3*
x*(c*x^2+b*x+a)^(1/2)-15/16*f^2*b^2/c^(7/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))+2/3*e*f*x^2/c*(c*x^2+b*x+a)^(1/2)+5/4*e*f*b^2/c^3*(c*x^2+b*x
+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/sqrt(a + b*x + c*x**2), x)
```

3.111 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} \, dx$

Optimal. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] 1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x
^2+b*x+a)^(1/2)/c
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{}{\sqrt{a+b}}}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\text{Subst}}{2c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}\left(\frac{}{2}\right.}{8c^{5/2}}$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\,(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d + 3*b
^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(8*c^(5/2))
```

fricas [A] time = 1.45, size = 227, normalized size = 1.96

$$\left[-\frac{\left(8\,c^2d-4\,bce+\left(3\,b^2-4\,ac\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)-4\left(2\,c^2fx\right.}{16\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

giac [A] time = 0.50, size = 98, normalized size = 0.84

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}-\frac{3\,bf-4\,ce}{c^2}\right)-\frac{\left(8\,c^2d+3\,b^2f-4\,acf-4\,bce\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}\,-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 185, normalized size = 1.59

$$-\frac{af\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3b^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{d\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*f*b^2/c^(
5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*f*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.112 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^
(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+f*arctanh(1/4*(4*a*f-b*(e+(-4
*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*2^
(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2)
```

Rubi [**A**] time = 0.58, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {983, 724, 206}

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 983

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
```

```
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,d}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\,\mathrm{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-\right.}}{}\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}\,f}{\sqrt{e^2-}}$$

Mathematica [A] time = 1.45, size = 376, normalized size = 1.01

$$\frac{\sqrt{2}\,f\left(\frac{\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2d\right.}}\right.}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right.}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^
2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] -
 ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f
] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])]/Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sq
rt[e^2 - 4*d*f]
```

fricas [B] time = 14.92, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
```

2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
e^3 + 2(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
 a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
 a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c

```
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d -
a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*
b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d
^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3
*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^
4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3
*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e
^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
 + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^
2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*
x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c
*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2
)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d
*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d
*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 -
 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)
*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
 + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
 (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^
2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 -
4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 -
 (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)
*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 -
(b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f +
 b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c
^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*
c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 -
 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
 (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4
*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3
 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^
2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a
^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*
f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*
b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^
3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4
)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
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e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2
- 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b
*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 +
 a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 -
 a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b
^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3
*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d
^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^
2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e
^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^
3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*
b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2
)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d
*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^
2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 -
(2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
- a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c
^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^
2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2
*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^
4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b
^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)
*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*
e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c
*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
 + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
 a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
(b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 -
 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 +
 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)
*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*
d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2
*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e
^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b
*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e
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^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*
e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^
3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 -
 a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*
b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4
- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b
^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2
 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 -
 (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(
a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a
*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f
+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 +
(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*
c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^
2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^
2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2
)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b
^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 761, normalized size = 2.03

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}\;\cdots}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\;\cdots}{\cdots}\right)}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,b\cdots}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^
(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*
f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2
)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.113 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx$

Optimal. Leaf size=789

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f\left(f\left(-4a^2f^2+3abef+b^2\left(e^2-6df\right)\right)-c\left(4af\left(e^2-3df\right)+b\left(\right.\right.\right.\right.}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{\right.}}}$$

```
[Out] (f*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)+f*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2)
)*x)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/
(f*x^2+e*x+d)+1/4*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(
e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*d*f+e^2)+f*(3
*a*b*e*f-4*a^2*f^2+b^2*(-6*d*f+e^2))-c*(4*a*f*(-3*d*f+e^2)+b*(-5*d*e*f+e^3)
))+f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(3/2
)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/4*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e
^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/
(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2
*c^2*d*(-4*d*f+e^2)+f*(3*a*b*e*f-4*a^2*f^2+b^2*(-6*d*f+e^2))-c*(4*a*f*(-3*d
*f+e^2)+b*(-5*d*e*f+e^3)))+f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e+(-4*d*f+e^2)^(
1/2)))/(-4*d*f+e^2)^(3/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(c*e
^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 8.21, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3\right.\right.\right.}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*
(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e
- b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2
- 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e
*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[2]*(e^2 - 4*d*f)^(3
/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*
e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^
2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d
*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[2]*(e^2 - 4*d*f)^
(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx = \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right.\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+f\right.}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right.\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+f\right.}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right.\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+f\right.}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right.\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+f\right.}$$

Mathematica [A] time = 6.69, size = 1377, normalized size = 1.75

$$-\frac{8\left(cx^2+bx+a\right)f^3}{\left(e^2-4df\right)\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{a+x(b+cx)}}-\frac{\quad}{\left(e^2-4\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] (-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 -
4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt
[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*
b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 -
4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^
2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)^(3/
2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2
- 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(2*b*f + 2
*c*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(-4*a*f - b*(-e
+ Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]
*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)*(4*a*f^2 + 2*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) + c*(-e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 + 8*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)
]) - (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt
[a + b*x + c*x^2])])/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) -
 (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d
*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[a
+ b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(
e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2
- 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2
- 4*d*f])^2)*Sqrt[a + x*(b + c*x)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 3858, normalized size = 4.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] 2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*
f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f
^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)
*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))+2/(4*d*f-e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2/(x-1/2/f*(-4*d*f+e^2)^(1/2)+1/2*e/f)*((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c
*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2
-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+
(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)
/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b
*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2
-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*(-4*d*f+e^2)^(1/2)-1/(4*d*f-e^2)*
f^2/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(
-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*b+1/(4*d*f-e^
2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e+2/(4*d*f
-e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e)*f^2/(x+1/2/f*(-4*d*f+e^2)^(1/2)+1/2*e/f)*((x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/
f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)
^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*
f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f
^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)
*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d
```

```
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))*c*(-4*d*f+e^2)^(1/2)-1/(4*d*f-e^2)*f^2/(2*a*f^2-b*e*f-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*
f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*l
n(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f
^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1
/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-
(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f))*b+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/
((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
[Out] Timed out
```

3.114 $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=649

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+\right.\right.}{128c^{11/2}}$$

```
[Out] 3/128*(105*b^4*f^3-280*b^2*c*f^2*(a*f+b*e)+128*c^4*d*(d*f+e^2)+80*c^2*f*(6*
a*b*e*f+a^2*f^2+3*b^2*(d*f+e^2))-64*c^3*(3*a*f*(d*f+e^2)+b*(6*d*e*f+e^3)))*
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+2*(3*a*b^4*c*e*
f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3*d^3+5*a^3*f^3+3*
a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f^2-c*(6*d*f+e^2))
+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f+b^2*f-b*c*e+2*c^
2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-3*a*c^3*e^2+b^4*f
^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*x)/c^5/(-4*a*c+b^
2)/(c*x^2+b*x+a)^(1/2)-1/64*(187*b^3*f^3-4*b*c*f^2*(73*a*f+114*b*e)-64*c^3*
(6*d*e*f+e^3)+16*c^2*f*(20*a*e*f+21*b*(d*f+e^2)))*(c*x^2+b*x+a)^(1/2)/c^5+1
/32*f*(41*b^2*f^2-4*c*f*(7*a*f+22*b*e)+48*c^2*(d*f+e^2))*x*(c*x^2+b*x+a)^(1
/2)/c^4+1/8*f^2*(-5*b*f+8*c*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/4*f^3*x^3*(c*x
^2+b*x+a)^(1/2)/c^2
```

Rubi [A] time = 2.11, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 621, 206}

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^2c^2df+b^2c^2e^2-2b^3cef\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*(b^2 -
 4*a*c)*Sqrt[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*
f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[
a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c
^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^
2*Sqrt[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*Sqrt[a + b*x + c*x^2])/(4*c^2)
+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*
c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f)
 + b*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(128*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2\right.\right.\right.}{}$$

Mathematica [A] time = 1.66, size = 745, normalized size = 1.15

$$\frac{3\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3a\right.\right.}{128c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c*f*(105*a*
f*(4*e + 9*f*x) + c*x*(-360*e^2 - 360*d*f + 140*e*f*x + 21*f^2*x^2)) - 8*b^
3*c*(210*a^2*f^3 - c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2
*e*f*(-72*d + 7*f*x^2)) + a*c*f*(-90*e^2 - 530*e*f*x + f*(-90*d + 77*f*x^2)
)) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 186*e^2*f*x +
 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-24*d^2*f + 6
*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*
x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*(16*e^3 + 36
*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*a*c^2*(-12*d
^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3 + 6*e^2*f*x
+ 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*(d - 3*e*x)
+ a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*c^2*(12*d^2*
f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*x + 14*e*f^2
*x^2 + 3*f^3*x^3))))/(64*c^5*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (3*(10
5*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6
*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^
3 + 6*d*e*f)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(128*c^(11
/2))
```

fricas [B] time = 5.60, size = 3143, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4)
*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8*
(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2*b^
3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4
 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*
c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*c^2 -
40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2 - 24*(b
^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e^2)*f)
*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e
 + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b^3*c^4 - 4
*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 140*a*b^5*c
+ 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*b^3*c^3 + 16
*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)*f^2 + 16*(8*
(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e + 3*(5*b^5*c^2 -
 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
 b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*b*c^6*
d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5
- 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5)*f^3)*x^4 - 64*(3
*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*
c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3 - 104*a*b^2*c^4 +
80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4 - 4*a*b*c^5)*e)*f
^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b^4*c^2 - 460*a^2*b
^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^3 - 7*(15*b^5*c^2
- 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*a*b*c^5)*d - (35*b
^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2*c^5 - 4*a*c^6)*d*
e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d^2 - 8*(3*a*b^2*c^
4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f + (256*c^7*d^3 -
384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b^3*c^4 - 10*a*b*c^
5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3
+ 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*b^5*c^2 - 530*a*b^
3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)*d^2 - 8*(3*b^3*c^
4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*e^2)*f)*x)*s
qrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b
^3*c^6 - 4*a*b*c^7)*x), -1/128*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(
a*b^3*c^3 - 4*a^2*b*c^4)*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^
2 - 64*a^4*c^3)*f^3 + 8*(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d -
5*(7*a*b^5*c - 40*a^2*b^3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*
c^6)*d*e^2 - 64*(b^3*c^4 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 2
40*a^2*b^2*c^3 - 64*a^3*c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*
c^5)*d - 5*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^
5 - 4*a*c^6)*d^2 - 24*(b^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c
^4 + 16*a^2*c^5)*e^2)*f)*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^
3*c^3 - 4*a^2*b*c^4)*d*e + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^
2)*f + (128*(b^3*c^4 - 4*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 +
5*(21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5
*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*
b^2*c^3)*e)*f^2 + 16*(8*(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c
^4)*d*e + 3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(-c)*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c
)) + 2*(128*b*c^6*d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 -
 4*a*c^6)*f^3*x^5 - 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5
)*f^3)*x^4 - 64*(3*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3
```

```
*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3
- 104*a*b^2*c^4 + 80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4
 - 4*a*b*c^5)*e)*f^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b
^4*c^2 - 460*a^2*b^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^
3 - 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*
a*b*c^5)*d - (35*b^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2
*c^5 - 4*a*c^6)*d*e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d
^2 - 8*(3*a*b^2*c^4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f
 + (256*c^7*d^3 - 384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b
^3*c^4 - 10*a*b*c^5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 -
 480*a^3*c^4)*f^3 + 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*
b^5*c^2 - 530*a*b^3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)
*d^2 - 8*(3*b^3*c^4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2
*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 -
 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*x)]
```

giac [A] time = 0.42, size = 1099, normalized size = 1.69

$$\left(\left(2\left(4\left(\frac{2\left(b^{2}c^{4}f^{3}-4\,ac^{5}f^{3}\right)x}{b^{2}c^{5}-4\,ac^{6}}-\frac{3\,b^{3}c^{3}f^{3}-12\,abc^{4}f^{3}-8\,b^{2}c^{4}f^{2}e+32\,ac^{5}f^{2}e}{b^{2}c^{5}-4\,ac^{6}}\right)x+\frac{48\,b^{2}c^{4}df^{2}-192\,ac^{5}df^{2}+21\,b^{4}c^{2}f^{3}-104\,ab^{2}c^{3}f^{3}+80\,a^{2}c^{4}f^{3}-5}{b^{2}c^{5}-4\,ac^{6}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) - (3*b^3*
c^3*f^3 - 12*a*b*c^4*f^3 - 8*b^2*c^4*f^2*e + 32*a*c^5*f^2*e)/(b^2*c^5 - 4*a
*c^6))*x + (48*b^2*c^4*d*f^2 - 192*a*c^5*d*f^2 + 21*b^4*c^2*f^3 - 104*a*b^2
*c^3*f^3 + 80*a^2*c^4*f^3 - 56*b^3*c^3*f^2*e + 224*a*b*c^4*f^2*e + 48*b^2*c
^4*f*e^2 - 192*a*c^5*f*e^2)/(b^2*c^5 - 4*a*c^6))*x - (240*b^3*c^3*d*f^2 - 9
60*a*b*c^4*d*f^2 + 105*b^5*c*f^3 - 616*a*b^3*c^2*f^3 + 784*a^2*b*c^3*f^3 -
384*b^2*c^4*d*f*e + 1536*a*c^5*d*f*e - 280*b^4*c^2*f^2*e + 1376*a*b^2*c^3*f
^2*e - 1024*a^2*c^4*f^2*e + 240*b^3*c^3*f*e^2 - 960*a*b*c^4*f*e^2 - 64*b^2*
c^4*e^3 + 256*a*c^5*e^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 + 384*b^2*c^
4*d^2*f - 768*a*c^5*d^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152
*a^2*c^4*d*f^2 + 315*b^6*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 48
0*a^3*c^3*f^3 - 384*b*c^5*d^2*e - 1152*b^3*c^3*d*f*e + 3840*a*b*c^4*d*f*e -
 840*b^5*c*f^2*e + 4240*a*b^3*c^2*f^2*e - 3904*a^2*b*c^3*f^2*e + 384*b^2*c^
4*d*e^2 - 768*a*c^5*d*e^2 + 720*b^4*c^2*f*e^2 - 2976*a*b^2*c^3*f*e^2 + 1152
*a^2*c^4*f*e^2 - 192*b^3*c^3*e^3 + 640*a*b*c^4*e^3)/(b^2*c^5 - 4*a*c^6))*x
- (128*b*c^5*d^3 + 384*a*b*c^4*d^2*f + 720*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3
*d*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + 1808*a^3*b*c^2*f^3 - 768*a*c^
5*d^2*e - 1152*a*b^2*c^3*d*f*e + 3072*a^2*c^4*d*f*e - 840*a*b^4*c*f^2*e + 3
680*a^2*b^2*c^2*f^2*e - 2048*a^3*c^3*f^2*e + 384*a*b*c^4*d*e^2 + 720*a*b^3*
c^2*f*e^2 - 2496*a^2*b*c^3*f*e^2 - 192*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3)/(b^
2*c^5 - 4*a*c^6))/sqrt(c*x^2 + b*x + a) - 3/128*(128*c^4*d^2*f + 240*b^2*c^
2*d*f^2 - 192*a*c^3*d*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^2*c^2*f^3
- 384*b*c^3*d*f*e - 280*b^3*c*f^2*e + 480*a*b*c^2*f^2*e + 128*c^4*d*e^2 + 2
40*b^2*c^2*f*e^2 - 192*a*c^3*f*e^2 - 64*b*c^3*e^3)*log(abs(-2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.03, size = 2827, normalized size = 4.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*a/c^2/(c*x^2+b*x+a)^(1/2)*e^3+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+315/128*f^3*b^4/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1
```

```
5/8*f^3*a^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+315/256*f^3
*b^5/c^6/(c*x^2+b*x+a)^(1/2)+1/4*f^3*x^5/c/(c*x^2+b*x+a)^(1/2)+3/c^(3/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2+3/c^(3/2)*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3*d^2*e/c/(c*x^2+b*x+a)^(1/2)+x^2/c/(c*x^2
+b*x+a)^(1/2)*e^3-3/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*e^3-3/2*b/c^(5/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3+115/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*x-9*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e*f+12*
a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e*f-39/2*b^2/c^2*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*x*d*f^2-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x*e^2*f-16*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+21/32*f^3*b^2/
c^3*x^3/(c*x^2+b*x+a)^(1/2)-105/64*f^3*b^3/c^4*x^2/(c*x^2+b*x+a)^(1/2)-315/
128*f^3*b^4/c^5*x/(c*x^2+b*x+a)^(1/2)+315/256*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)-105/16*f^3*b^2/c^(9/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-5/8*f^3*a/c^2*x^3/(c*x^2+b*x+a)^(1/2)-15/8*f^3*a^2/c^3*x/(c*x^2+b*
x+a)^(1/2)-3*x/c/(c*x^2+b*x+a)^(1/2)*f*d^2-3*x/c/(c*x^2+b*x+a)^(1/2)*e^2*d+
3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*d*f^2+3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*e^2*f+45/
16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*d*f^2+45/16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*e^2*
f+45/8*b^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+45/8*b
^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f-9/2*a/c^(5/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2-9/2*a/c^(5/2)*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f+3/2*b/c^2*x/(c*x^2+b*x+a)^(1/2)*e^3-
3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^3+e*f^2*x^4/c/(c*x^2+b*x+a)^(
1/2)-105/32*e*f^2*b^4/c^5/(c*x^2+b*x+a)^(1/2)-105/16*e*f^2*b^3/c^(9/2)*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^(1/2)
+3/2*b/c^2/(c*x^2+b*x+a)^(1/2)*f*d^2+3/2*b/c^2/(c*x^2+b*x+a)^(1/2)*e^2*d-10
5/16*f^3*b^3/c^5*a/(c*x^2+b*x+a)^(1/2)+113/16*f^3*b/c^4*a^2/(c*x^2+b*x+a)^(
1/2)+24*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e*f+9*b/c^2*x/(c*x^2+b*x+
a)^(1/2)*d*e*f-9/2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e*f+4*a/c*b/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^3+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
*x*f*d^2+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*d-8*e*f^2*a^2/c^3*b^
2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)*x*d*f^2+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*f-39/4*b^3/c
^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f^2-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*e^2*f-105/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x
+115/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-45/4*e*f^2*b/c^3*a*x
/(c*x^2+b*x+a)^(1/2)+113/8*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x-105/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+105/16*e*f^2*b^3/c^
4*x/(c*x^2+b*x+a)^(1/2)-105/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)+115/8*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^(1/2)+45/4*e*f^2*b/c^(7/2)*a*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4*e*f^2*a/c^2*x^2/(c*x^2+b*x+a)^(1/2)+3
15/128*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-105/16*f^3*b^5/c^5*a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+105/16*f^3*b^2/c^4*a*x/(c*x^2+b*x+a)^(1/2)+4
9/16*f^3*b/c^3*a*x^2/(c*x^2+b*x+a)^(1/2)+113/16*f^3*b^3/c^4*a^2/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)+6*x^2/c/(c*x^2+b*x+a)^(1/2)*d*e*f-9/2*b^2/c^3/(c*x^2+b
*x+a)^(1/2)*d*e*f-3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^3-9*b/c^(
5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f+12*a/c^2/(c*x^2+b*x+
a)^(1/2)*d*e*f+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^3+3/2*b^3/c^2/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*d^2+3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)*e^2*d-6*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-3*d^2*e*b^2/c/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)-15/4*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)*d*f^2-15/4*b
/c^2*x^2/(c*x^2+b*x+a)^(1/2)*e^2*f-45/8*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)*d*f^2
-45/8*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)*e^2*f-7/4*e*f^2*b/c^2*x^3/(c*x^2+b*x+a)
^(1/2)+35/8*e*f^2*b^2/c^3*x^2/(c*x^2+b*x+a)^(1/2)+45/16*b^5/c^4/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*e^2*f-39/4*b/c^3*a/(c*x^2+b*x+a)^(1/2)*d*f^2-39/4*b/c^
3*a/(c*x^2+b*x+a)^(1/2)*e^2*f+9/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*d*f^2+9/2*a/c
^2*x/(c*x^2+b*x+a)^(1/2)*e^2*f-3/8*f^3*b/c^2*x^4/(c*x^2+b*x+a)^(1/2)+45/16*
b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f\,x^2+e\,x+d\right)}^3}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + fx^{2}\right)^{3}}{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/(a + b*x + c*x**2)**(3/2), x)
```

3.115 $\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=309

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] 1/8*(15*b^2*f^2-12*c*f*(a*f+2*b*e)+8*c^2*(2*d*f+e^2))*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*
(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a*c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2
*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2*d*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(
2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+1/4*f*(-7*b*f+8*c*e)*(
c*x^2+b*x+a)^(1/2)/c^3+1/2*f^2*x*(c*x^2+b*x+a)^(1/2)/c^2
```

Rubi [A] time = 0.45, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) +
 ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{c^3\left(b^2-4\right.}$$

Mathematica [A] time = 0.73, size = 288, normalized size = 0.93

$$\frac{4bc\left(-13a^2f^2+ac\left(4df+2e^2+20efx-5f^2x^2\right)+2c^2d(d-2ex)\right)+8c^2\left(a^2f(8e+3fx)+ac\left(x\left(-2e^2+4efx\right.\right.\right.}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(-13*a^2*f^2 +
 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*x^2)) - 2*b^2*
c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2*x^2)) + 8*c^2*
(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) + x*(-2*e^2 + 4*e*
f*x + f^2*x^2))))/(4*c^3*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((15*b^2*f
^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*Log[b + 2*c*x + 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/(8*c^(7/2))
```

fricas [B] time = 3.26, size = 1305, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3
*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2
*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2
+ 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3
*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b
^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(c)*log(-8*c^2
*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
 + 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^
2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b
^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e
)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c -
62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 -
10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c
^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c
^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c
^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a
*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*
d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^
5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*
c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*
a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f
^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a
*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*
(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2
*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x
 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*
c^5)*x)]
```

giac [A] time = 0.35, size = 407, normalized size = 1.32

$$\frac{\left(\left(\frac{2\left(b^2c^2f^2-4\,ac^3f^2\right)x}{b^2c^3-4\,ac^4}-\frac{5\,b^3cf^2-20\,abc^2f^2-8\,b^2c^2fe+32\,ac^3fe}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4d^2+16\,b^2c^2df-32\,ac^3df+15\,b^4f^2-62\,ab^2cf^2+24\,a^2c^2f^2-16\,bc^3de-24\,b^3 \cdots}{b^2c^3-4\,ac^4} \cdots}{4\,\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) - (5*b^3*c*f^2 -
 20*a*b*c^2*f^2 - 8*b^2*c^2*f*e + 32*a*c^3*f*e)/(b^2*c^3 - 4*a*c^4))*x - (1
6*c^4*d^2 + 16*b^2*c^2*d*f - 32*a*c^3*d*f + 15*b^4*f^2 - 62*a*b^2*c*f^2 + 2
4*a^2*c^2*f^2 - 16*b*c^3*d*e - 24*b^3*c*f*e + 80*a*b*c^2*f*e + 8*b^2*c^2*e^
2 - 16*a*c^3*e^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 + 16*a*b*c^2*d*f +
15*a*b^3*f^2 - 52*a^2*b*c*f^2 - 32*a*c^3*d*e - 24*a*b^2*c*f*e + 64*a^2*c^2*
f*e + 8*a*b*c^2*e^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/8*(16*c
^2*d*f + 15*b^2*f^2 - 12*a*c*f^2 - 24*b*c*f*e + 8*c^2*e^2)*log(abs(-2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.02, size = 1011, normalized size = 3.27

$$-\frac{13 a\,b^2 f^2 x}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2}+\frac{8abefx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}+\frac{15 b^4 f^2 x}{8\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^3}-\frac{3b^3efx}{\left(4ac-b^2\right)\sqrt{c\,x^2+ \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*d*e/c/(c*x^2+b*x+a)^(1/2)-x/c/(c*x^2+b*x+a)^(1/2)*e^2+1/2*b/c^2/(c*x^2+b
*x+a)^(1/2)*e^2+2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f+1
/2*f^2*x^3/c/(c*x^2+b*x+a)^(1/2)+15/16*f^2*b^3/c^4/(c*x^2+b*x+a)^(1/2)+15/8
*f^2*b^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*f^2*a/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*d^2*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)+4*e*f*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*b^2/
c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f-13/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*x-3*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+8*e*f*a/c
*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-13/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)-4*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+15/8*f^2*b^4/c^3/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-2*d*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+
3*e*f*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/2*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2+b^3/c^2/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*d*f+1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e
^2+1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2+2*e*f*x^2/c/(c*x^2+b*x+a
)^(1/2)-3/2*e*f*b^2/c^3/(c*x^2+b*x+a)^(1/2)-3*e*f*b/c^(5/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))+4*e*f*a/c^2/(c*x^2+b*x+a)^(1/2)-5/4*f^2*b/c^2*
x^2/(c*x^2+b*x+a)^(1/2)-15/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)+15/16*f^2*b^
5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-13/4*f^2*b/c^3*a/(c*x^2+b*x+a)^(1/2)+
3/2*f^2*a/c^2*x/(c*x^2+b*x+a)^(1/2)-2*x/c/(c*x^2+b*x+a)^(1/2)*d*f+b/c^2/(c*
x^2+b*x+a)^(1/2)*d*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + fx^{2}\right)^{2}}{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)
```

3.116 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b*
(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$
$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$
$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(2f)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \cdots\right.}{c}$$
$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.31, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(abf-2ac(e+fx)+b^2fx+bc(d-ex)+2c^2dx\right)}{\sqrt{a+x(b+cx)}}-f\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(c^(3/2)*(-b^2 + 4*a*c))
```

fricas [B] time = 1.74, size = 429, normalized size = 3.86

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)fx^2+\left(b^3-4\,abc\right)fx+\left(ab^2-4\,a^2c\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b\cdots\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

giac [A] time = 0.31, size = 122, normalized size = 1.10

$$-\frac{2\left(\frac{\left(2\,c^2d+b^2f-2\,acf-bce\right)x}{b^2c-4\,ac^2}+\frac{bcd+abf-2\,ace}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

maple [B] time = 0.01, size = 249, normalized size = 2.24

$$\frac{b^2 f x}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}-\frac{2bex}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{b^3 f}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2}-\frac{b^2 e}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f*b/c^2/(c*x^2+b*x+a)^(1/2)+f*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 3.73, size = 143, normalized size = 1.29

$$\frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,\left(4\,a+2\,b\,x\right)}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right.\right.}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.117 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)\cdots\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-\cdots\right.}$$

```
[Out] 2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d
)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-
1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+f*(2*a*f
-b*(e+(-4*d*f+e^2)^(1/2))))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-
4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e
-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*
(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+
e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)/((-a*f+c*d)^2-(-a*e+b*d)*
(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+
f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Rubi [A] time = 1.83, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 1032, 724, 206}

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)\cdots\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}+$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}+$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}-$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}-$$

Mathematica [A] time = 5.34, size = 700, normalized size = 1.05

$$2f\left(-\frac{2\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
 + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
 c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sq
rt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2))
/Sqrt[e^2 - 4*d*f]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.03, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
```

```
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*f
/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2+4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*x*b*c-4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*
d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f
^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2
*e-2*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*
e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c
^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f
+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*b^2-2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1
/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c
*e+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)
^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(
-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e
*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)
*b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*
f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^
2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)-4*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*
f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a
*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2
/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*c^2*e-2*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b
^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(
```

```
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f
+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)
*f^2/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f
+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*b*c*e-2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((
b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-
4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=891

$$\frac{x\sqrt{cx^2+bx+a}\,f^3}{2c^3}+\frac{(12ce-11bf)\sqrt{cx^2+bx+a}\,f^2}{4c^4}+\frac{\left(24\left(e^2+df\right)c^2-20f(3be+af)c+35b^2f^2\right)\tanh^{-1}\left(\frac{b\cdots}{2\sqrt{c}\sqrt{\cdots}}\right.}{8c^{9/2}}$$

```
[Out] 2/3*(3*a*b^4*c*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3
*d^3+5*a^3*f^3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f
^2-c*(6*d*f+e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f
+b^2*f-b*c*e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-
3*a*c^3*e^2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*
x)/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+1/8*f*(35*b^2*f^2-20*c*f*(a*f+3*b*e
)+24*c^2*(d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9
/2)-2/3*(3*b^6*c*e*f^2-b^7*f^3+3*b^5*c*f*(6*a*f^2-c*(d*f+e^2))-3*b^3*c^2*(2
9*a^2*f^3+c^2*d*(d*f+e^2)-10*a*c*f*(d*f+e^2))-4*b*c^3*(2*c^3*d^3-29*a^3*f^3
+3*a*c^2*d*(d*f+e^2)+24*a^2*c*f*(d*f+e^2))-24*a^2*c^4*e*(6*a*f^2-c*(6*d*f+e
^2))-b^4*c^2*e*(42*a*f^2-c*(6*d*f+e^2))+6*b^2*c^3*e*(2*c^2*d^2+28*a^2*f^2-a
*c*(6*d*f+e^2))-c*(16*c^6*d^3-10*b^6*f^3+3*b^4*c*f^2*(26*a*f+7*b*e)-24*c^5*
d*(b*d*e-a*(d*f+e^2))-6*b^2*c^2*f*(25*a*b*e*f+27*a^2*f^2+2*b^2*(d*f+e^2))+6
*c^4*(b^2*d*(d*f+e^2)-16*a^2*f*(d*f+e^2)-2*a*b*e*(6*d*f+e^2))+c^3*(240*a^2*
b*e*f^2+56*a^3*f^3+84*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*x)/c^5/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)^(1/2)+1/4*f^2*(-11*b*f+12*c*e)*(c*x^2+b*x+a)^(1/2)/c^4
+1/2*f^3*x*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A] time = 1.77, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{x\sqrt{cx^2+bx+a}\,f^3}{2c^3}+\frac{(12ce-11bf)\sqrt{cx^2+bx+a}\,f^2}{4c^4}+\frac{\left(24\left(e^2+df\right)c^2-20f(3be+af)c+35b^2f^2\right)\tanh^{-1}\left(\frac{b\cdots}{2\sqrt{c}\sqrt{\cdots}}\right.}{8c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(3*c^5*(b^2
 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c*
f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1
0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d
*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) -
b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^
2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e +
26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a
^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d*
f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2*
f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x))/(3*c^5*(b^2 - 4*a*c)^2*Sqrt[a + b
*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3
*x*Sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) +
```

```
 24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + d\right.\right.\right.}{}$$

Mathematica [A] time = 2.19, size = 872, normalized size = 0.98

$$\frac{-105f^3x^2b^7 - 10f^2x(21af + 2cx(7fx - 9e))b^6 - 3f\left(35a^2f^2 - 10acx(12e + 23fx)f + c^2x^2\left(24e^2 - 80fxe + 7f^2x^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-105*b^7*f^3*x^2 - 10*b^6*f^2*x*(21*a*f + 2*c*x*(-9*e + 7*f*x)) + 6*b^4*c*
f*(5*a^2*f*(6*e + 53*f*x) - 6*a*c*x*(4*e^2 + 4*d*f + 30*e*f*x - 31*f^2*x^2)
 + c^2*x^3*(-16*e^2 - 16*d*f + 6*e*f*x + f^2*x^2)) - 3*b^5*f*(35*a^2*f^2 -
10*a*c*f*x*(12*e + 23*f*x) + c^2*x^2*(24*e^2 + 24*d*f - 80*e*f*x + 7*f^2*x^
2)) - 48*b*c^2*(27*a^4*f^3 - 4*c^4*d^2*x^2*(d - e*x) + a^2*c^2*(-4*d^2*f +
4*e^3*x - 64*e*f^2*x^3 + 7*f^3*x^4 - 4*d*e*(e - 6*f*x)) - 2*a*c^3*(d^3 - e^
3*x^3 + 3*d*e*x^2*(e - 2*f*x) + 3*d^2*x*(-e + f*x)) - 2*a^3*c*f*(5*e^2 + 39
*e*f*x + f*(5*d - 14*f*x^2))) - 8*b^3*c*(-95*a^3*f^3 + c^3*(d^3 - e^3*x^3 +
 9*d^2*x*(e - f*x) - 3*d*e*x^2*(3*e + 2*f*x)) - 3*a*c^2*f*x^2*(18*e^2 - 74*
e*f*x + f*(18*d + 7*f*x^2)) + 3*a^2*c*f*(3*e^2 + 105*e*f*x + f*(3*d + 29*f*
x^2))) + 32*c^3*(4*c^4*d^3*x^3 + 3*a^4*f^2*(16*e + 5*f*x) + 6*a*c^3*d*x*(d^
2 + e^2*x^2 + d*f*x^2) - 2*a^3*c*(2*e^3 + 9*e^2*f*x + f^2*x*(9*d - 10*f*x^2
) + 12*e*f*(d - 3*f*x^2)) - 3*a^2*c^2*(2*d^2*e + 4*d*f*x^2*(3*e + 2*f*x) +
x^2*(2*e^3 + 8*e^2*f*x - 6*e*f^2*x^2 - f^3*x^3))) - 48*b^2*c^2*(a^3*f^2*(25
*e + 63*f*x) - c^3*d*x*(d^2 + e^2*x^2 + d*x*(-6*e + f*x)) + a^2*c*f*x*(-21*
e^2 - 12*e*f*x + 7*f*(-3*d + 7*f*x^2)) + a*c^2*(d^2*(e - 6*f*x) - 2*d*x*(3*
e^2 - 3*e*f*x + 7*f^2*x^2) + x^2*(e^3 - 14*e^2*f*x + 6*e*f^2*x^2 + f^3*x^3)
)))/(12*c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f*(35*b^2*f^2 - 20*
c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/(8*c^(9/2))
```

fricas [B] time = 12.87, size = 3995, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 -
60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*x^4
 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 - 60*
a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^
4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64*
a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*(b^6*c^
2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3*
b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f
^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c
 + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*e)*f^2)
*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f + 5*(7*a*b^7
- 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*(a*b^5*c^2 - 8
*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^
3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2 - 128*a^3*c^5*e^3
 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 - 8*a*b^
2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3)*x
^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*e - (105
*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 - 48*b*c
^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*e^3 - (3
5*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12*(2*(b^4
*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*a^2*b*c^
5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*e - 2*(b
^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^3 - 20*a
^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f^2 + 3*(
64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 - 16*(a*b^
2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448*
a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*a*b^4*c^
3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)*d^2 - 4
*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2 + 24*(8*
a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2)*f + 6*
(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 12*(b^
3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*b^2*c^3
- 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d - (5*a*
b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d^2 - 8*a
^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*sqrt(c*x^
2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*x^3 + (b
^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c^6 + 16*
a^3*b*c^7)*x), -1/24*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5
*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*e)*f^2)*x^4 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7
*a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2
*b^3*c^3 - 64*a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)
*d - 5*(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^
4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a
^4*b*c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^
8 - 46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(
```

```
2*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*
b*c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f
 + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*
(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 +
 16*a^3*b^2*c^3)*e)*f^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(192*a^2*b*c^5*d*e^2 - 128*a^3
*c^5*e^3 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4
- 8*a*b^2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5
)*f^3)*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*
e - (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3
- 48*b*c^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*
e^3 - (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12
*(2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*
a^2*b*c^5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*
e - 2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^
3 - 20*a^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f
^2 + 3*(64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 -
16*(a*b^2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^
3 + 448*a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*
a*b^4*c^3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)
*d^2 - 4*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2
+ 24*(8*a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2
)*f + 6*(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3
- 12*(b^3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*
b^2*c^3 - 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d
 - (5*a*b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d
^2 - 8*a^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*s
qrt(c*x^2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7
- 8*a*b^2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*
x^3 + (b^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c
^6 + 16*a^3*b*c^7)*x)]
```

giac [A] time = 0.45, size = 1401, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((((3*(2*(b^4*c^3*f^3 - 8*a*b^2*c^4*f^3 + 16*a^2*c^5*f^3)*x/(b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6) - (7*b^5*c^2*f^3 - 56*a*b^3*c^3*f^3 + 112*a^2*b*
c^4*f^3 - 12*b^4*c^3*f^2*e + 96*a*b^2*c^4*f^2*e - 192*a^2*c^5*f^2*e)/(b^4*c
^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 4*(32*c^7*d^3 + 12*b^2*c^5*d^2*f + 48*a
*c^6*d^2*f - 24*b^4*c^3*d*f^2 + 168*a*b^2*c^4*d*f^2 - 192*a^2*c^5*d*f^2 - 3
5*b^6*c*f^3 + 279*a*b^4*c^2*f^3 - 588*a^2*b^2*c^3*f^3 + 160*a^3*c^4*f^3 - 4
8*b*c^6*d^2*e + 12*b^3*c^4*d*f*e - 144*a*b*c^5*d*f*e + 60*b^5*c^2*f^2*e - 4
44*a*b^3*c^3*f^2*e + 768*a^2*b*c^4*f^2*e + 12*b^2*c^5*d*e^2 + 48*a*c^6*d*e^
2 - 24*b^4*c^3*f*e^2 + 168*a*b^2*c^4*f*e^2 - 192*a^2*c^5*f*e^2 + 2*b^3*c^4*
e^3 - 24*a*b*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3*(64*b*c^6
*d^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c
^3*d*f^2 - 35*b^7*f^3 + 230*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c
^3*f^3 - 96*b^2*c^5*d^2*e - 96*a*b^2*c^4*d*f*e - 384*a^2*c^5*d*f*e + 60*b^6
*c*f^2*e - 360*a*b^4*c^2*f^2*e + 192*a^2*b^2*c^3*f^2*e + 768*a^3*c^4*f^2*e
+ 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 - 24*b^5*c^2*f*e^2 + 144*a*b^3*c^3*f*
e^2 - 16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^
6))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 + 48*a*b^2*c^4*d^2*f - 24*a*b^4*c^2
*d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^3*c^4*d*f^2 - 35*a*b^6*f^3 + 265*a^2*
b^4*c*f^3 - 504*a^3*b^2*c^2*f^3 + 80*a^4*c^3*f^3 - 12*b^3*c^4*d^2*e - 48*a*
b*c^5*d^2*e - 192*a^2*b*c^4*d*f*e + 60*a*b^5*c*f^2*e - 420*a^2*b^3*c^2*f^2*
e + 624*a^3*b*c^3*f^2*e + 48*a*b^2*c^4*d*e^2 - 24*a*b^4*c^2*f*e^2 + 168*a^2
```

```
*b^2*c^3*f*e^2 - 96*a^3*c^4*f*e^2 - 32*a^2*b*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^
5 + 16*a^2*c^6))*x - (8*b^3*c^4*d^3 - 96*a*b*c^5*d^3 - 192*a^2*b*c^4*d^2*f
+ 72*a^2*b^3*c^2*d*f^2 - 480*a^3*b*c^3*d*f^2 + 105*a^2*b^5*f^3 - 760*a^3*b^
3*c*f^3 + 1296*a^4*b*c^2*f^3 + 48*a*b^2*c^4*d^2*e + 192*a^2*c^5*d^2*e + 768
*a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^
3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2
+ 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a)^
(3/2) - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2
*f*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2
)
```

maple [B] time = 0.03, size = 4635, normalized size = 5.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 5/6*f^3*a/c^2*x^3/(c*x^2+b*x+a)^(3/2)+5/2*f^3*a/c^3*x/(c*x^2+b*x+a)^(1/2)-6
*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e*f-5/2*f^3*a/c^3*b^2/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)*x-33/4*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2
)*x-66*f^3*b^2/c^2*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-1/8*b^4/c^3/(4*a
*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*f+4*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/
2)*x*d*e*f-3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*e*f-48*b*a/(4*a*c-
b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*e*f-24*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(
1/2)*d*e*f+12*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*f^2+12*b^2/c*a/
(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*f+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+
b*x+a)^(3/2)*x*d*f^2+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*f+
12*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+96*e*f^2*a^2/c*b/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-19/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+
a)^(3/2)*x-38*e*f^2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/2*b^3/c
^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e*f+2/3*d^3/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)*b-x^2/c/(c*x^2+b*x+a)^(3/2)*e^3+1/24*b^2/c^3/(c*x^2+b*x+a)^(3/2)*e^
3-2/3*a/c^2/(c*x^2+b*x+a)^(3/2)*e^3+3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*d*f^2+3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e^2*f+35/16*f^3*b^3/c^5/(c*x^2+b*x+a)^(1/2)-5/2*f^3*a/c^(7/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+35/8*f^3*b^2/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))+1/2*f^3*x^5/c/(c*x^2+b*x+a)^(3/2)-35/384*f^3*b^5/c^6/(
c*x^2+b*x+a)^(3/2)-d^2*e/c/(c*x^2+b*x+a)^(3/2)-33/16*f^3*b^2/c^4*a*x/(c*x^2
+b*x+a)^(3/2)-33/8*f^3*b^3/c^4*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+23/16*f^
3*b^5/c^5*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+23/2*f^3*b^5/c^4*a/(4*a*c-b^2)^
2/(c*x^2+b*x+a)^(1/2)-5/4*f^3*a/c^4*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-35/
192*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+35/8*f^3*b^4/c^4/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*f*d^2
+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*d+4*b^2/(4*a*c-b^2)^2/(c*x
^2+b*x+a)^(1/2)*x*f*d^2+4*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*d+2*b
^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*f*d^2+2*b^3/c/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)*e^2*d+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*f*d^2-15/4*e*f^2*b^
4/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3/2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)*d*f
^2+3/2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)*e^2*f+3/8*b^2/c^3*x/(c*x^2+b*x+a)^(3/2
)*d*f^2+3/8*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)*e^2*f-1/16*b^5/c^4/(4*a*c-b^2)/(c
*x^2+b*x+a)^(3/2)*d*f^2-1/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*f-
1/2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f^2-1/2*b^5/c^3/(4*a*c-b^2)
^2/(c*x^2+b*x+a)^(1/2)*e^2*f+b/c^3*a/(c*x^2+b*x+a)^(3/2)*d*f^2+b/c^3*a/(c*x
^2+b*x+a)^(3/2)*e^2*f+3/2/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f^2+3/2
/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2*f+12*e*f^2*a/c^2*x^2/(c*x^2+b*
x+a)^(3/2)+5/2*e*f^2*b/c^2*x^3/(c*x^2+b*x+a)^(3/2)-15/4*e*f^2*b^2/c^3*x^2/(
c*x^2+b*x+a)^(3/2)-15/16*e*f^2*b^3/c^4*x/(c*x^2+b*x+a)^(3/2)+5/32*e*f^2*b^6
/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+5/4*e*f^2*b^6/c^4/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)-3*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^(3/2)+15/2*e*f^2*b/c^3*x/(c*x
```

```
^2+b*x+a)^(1/2)-35/24*f^3*b^6/c^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-33/4*
f^3*b/c^3*a*x^2/(c*x^2+b*x+a)^(3/2)-33*f^3*b^3/c^3*a^2/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*d+8*a/(4*a*c-b^2)^2
/(c*x^2+b*x+a)^(1/2)*b*f*d^2+8*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*e^2*d-
2*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-d^2*e*b^2/c/(4*a*c-b^2)/(c*x^2+
b*x+a)^(3/2)-6*x^2/c/(c*x^2+b*x+a)^(3/2)*d*e*f+1/4*b^2/c^3/(c*x^2+b*x+a)^(3
/2)*d*e*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^3+2/3*b^3/c/(4*a
*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^3-1/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)
^(3/2)*e^3-8*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^3-4*b^2/c*a/(4*a*c-b
^2)^2/(c*x^2+b*x+a)^(1/2)*e^3-4*a/c^2/(c*x^2+b*x+a)^(3/2)*d*e*f-16*d^2*e*b*
c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(
1/2)*x*e^2*f+3/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*f^2+3/4*b^3/c^
3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*f+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+
b*x+a)^(1/2)*d*f^2+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2*f+3/c^
2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f^2-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+
b*x+a)^(1/2)*x*d*f^2-3/2*b/c^2*x/(c*x^2+b*x+a)^(3/2)*d*e*f+1/4*b^4/c^3/(4*a
*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*e*f+2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/
2)*d*e*f-b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^3+5/2*e*f^2*b^5/c^3/(4*a
*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+48*e*f^2*a^2/c^2*b^2/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)+3*e*f^2*a/c^3*b*x/(c*x^2+b*x+a)^(3/2)+6*e*f^2*a^2/c^3*b^2/(4*a*
c-b^2)/(c*x^2+b*x+a)^(3/2)-19/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(
3/2)-19*e*f^2*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-15/2*e*f^2*b^3/c^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+5/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)*x+23/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+23*f^3*b
^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)*x*e^2*d+a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*f*d^2+a/c/(4*a*c-
b^2)/(c*x^2+b*x+a)^(3/2)*b*e^2*d+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x
*f*d^2+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*d+3/c^2*b^2/(4*a*c-b^
2)/(c*x^2+b*x+a)^(1/2)*x*e^2*f-1/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*
x*d*f^2+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*f*d^2-5/4*f^3*a/c^4*b/(
c*x^2+b*x+a)^(1/2)+173/96*f^3*b^3/c^5*a/(c*x^2+b*x+a)^(3/2)-35/8*f^3*b^2/c^
4*x/(c*x^2+b*x+a)^(1/2)+35/16*f^3*b^5/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1
1/2*f^3*b/c^4*a^2/(c*x^2+b*x+a)^(3/2)-7/4*f^3*b/c^2*x^4/(c*x^2+b*x+a)^(3/2)
-35/24*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^(3/2)+35/16*f^3*b^3/c^4*x^2/(c*x^2+b*x
+a)^(3/2)+35/64*f^3*b^4/c^5*x/(c*x^2+b*x+a)^(3/2)-35/384*f^3*b^7/c^6/(4*a*c
-b^2)/(c*x^2+b*x+a)^(3/2)-35/48*f^3*b^7/c^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/
2)+8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^(3/2)-15/2*e*f^2*b/c^(7/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))+3*e*f^2*x^4/c/(c*x^2+b*x+a)^(3/2)+5/32*e*f^2*b
^4/c^5/(c*x^2+b*x+a)^(3/2)-15/4*e*f^2*b^2/c^4/(c*x^2+b*x+a)^(1/2)-x^3/c/(c*
x^2+b*x+a)^(3/2)*d*f^2-x^3/c/(c*x^2+b*x+a)^(3/2)*e^2*f-1/16*b^3/c^4/(c*x^2+
b*x+a)^(3/2)*d*f^2-1/16*b^3/c^4/(c*x^2+b*x+a)^(3/2)*e^2*f-3/c^2*x/(c*x^2+b*
x+a)^(1/2)*d*f^2-3/c^2*x/(c*x^2+b*x+a)^(1/2)*e^2*f+3/2/c^3*b/(c*x^2+b*x+a)^
(1/2)*d*f^2+3/2/c^3*b/(c*x^2+b*x+a)^(1/2)*e^2*f-1/4*b/c^2*x/(c*x^2+b*x+a)^(
3/2)*e^3+1/24*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^3+1/3*b^4/c^2/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^3-3/2*x/c/(c*x^2+b*x+a)^(3/2)*f*d^2-3/2*x/c/
(c*x^2+b*x+a)^(3/2)*e^2*d+1/4*b/c^2/(c*x^2+b*x+a)^(3/2)*f*d^2+1/4*b/c^2/(c*
x^2+b*x+a)^(3/2)*e^2*d-8*d^2*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4/3*d^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*c+32/3*d^3*c^2/(4*a*c-b^2)^2/(c*x^2+b*x
+a)^(1/2)*x+16/3*d^3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^3}{\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.119 $\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=444

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] 2/3*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a*
c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2*d
*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/(c*
x^2+b*x+a)^(3/2)+f^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
5/2)-2/3*(2*b^4*c*e*f+48*a^2*c^3*e*f-b^5*f^2+4*b^2*c^2*e*(-3*a*f+2*c*d)+b^3
*c*(10*a*f^2-c*(2*d*f+e^2))-4*b*c^2*(2*c^2*d^2+8*a^2*f^2+a*c*(2*d*f+e^2))-2
*c*(8*c^4*d^2-2*b^4*f^2+b^2*c*f*(14*a*f+b*e)-c^3*(8*b*d*e-4*a*(2*d*f+e^2))-
c^2*(12*a*b*e*f+16*a^2*f^2-b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)^2/(c*x^2+b
*x+a)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1660, 12, 621, 206}

$$-\frac{2\left(-2cx\left(-c^2\left(16a^2f^2+12abef+b^2\left(-\left(2df+e^2\right)\right)\right)+b^2cf(14af+be)-c^3\left(8bde-4a\left(2df+e^2\right)\right)-2b^4f^2+8\right.\right.}{3c^3\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*
e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*
f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)
) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*
a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x))/(
3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{3c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{3c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{3c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{3c^3\left(b^2-4\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+\right.\right.}{3c^3\left(b^2-4\right.}$$

Mathematica [A] time = 1.26, size = 387, normalized size = 0.87

$$\frac{2\left(b^3\left(-3a^2f^2+18acf^2x^2+c^2\left(-d^2+6dx(fx-e)+ex^2(3e+2fx)\right)\right)+2b^2c\left(21a^2f^2x-2ac\left(d(e-6fx)+x\left(-\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^3
*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x)*
(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6*d*
x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*e +
3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*(3*e
+ 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(-6*e +
 f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2)))))/(3*c^2
*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f^2*Log[b + 2*c*x + 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/c^(5/2)
```

fricas [A] time = 6.52, size = 1581, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)*f^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*
d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 +
 (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 1
2*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^
2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*
c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^
4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2
*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2
+ 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x +
 a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 -
6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2)*f^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(
-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*
d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 +
 (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 1
2*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^
2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*
c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^
4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2
*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2
+ 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x +
 a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 -
6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*x)]
```

giac [A] time = 0.34, size = 587, normalized size = 1.32

$$-\frac{f^2\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}+\frac{2\left(\left(\left(\frac{2\left(8c^5d^2+2b^2c^3df+8ac^4df-2b^4cf^2+14ab^2c^2f^2-16a^2c^3f^2-8bc^4de+b^3c^2fe-\cdots\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -f^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) +
 2/3*(((2*(8*c^5*d^2 + 2*b^2*c^3*d*f + 8*a*c^4*d*f - 2*b^4*c*f^2 + 14*a*b^2
*c^2*f^2 - 16*a^2*c^3*f^2 - 8*b*c^4*d*e + b^3*c^2*f*e - 12*a*b*c^3*f*e + b^
2*c^3*e^2 + 4*a*c^4*e^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^
4*d^2 + 2*b^3*c^2*d*f + 8*a*b*c^3*d*f - b^5*f^2 + 6*a*b^3*c*f^2 - 8*b^2*c^3
*d*e - 4*a*b^2*c^2*f*e - 16*a^2*c^3*f*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2)/(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 + 4*a*b^
2*c^2*d*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2 - b^3*c^2*d*e - 4*a
*b*c^3*d*e - 8*a^2*b*c^2*f*e + 2*a*b^2*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 - 16*a^2*b*c^2*d*f + 3*a^2*b^3
*f^2 - 20*a^3*b*c*f^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e + 32*a^3*c^2*f*e -
```

```
8*a^2*b*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(
3/2)
```

maple [B] time = 0.01, size = 1786, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*d*e/c/(c*x^2+b*x+a)^(3/2)+2/3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-1/
2*x/c/(c*x^2+b*x+a)^(3/2)*e^2+1/12*b/c^2/(c*x^2+b*x+a)^(3/2)*e^2-1/3*f^2*x^
3/c/(c*x^2+b*x+a)^(3/2)-1/48*f^2*b^3/c^4/(c*x^2+b*x+a)^(3/2)-f^2/c^2*x/(c*x
^2+b*x+a)^(1/2)+1/2*f^2/c^3*b/(c*x^2+b*x+a)^(1/2)-2*e*f*b/c*a/(4*a*c-b^2)/(
c*x^2+b*x+a)^(3/2)*x+16/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*d*f-1/24*f^
2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*f^2*b^4/c^2/(4*a*c-b^2)^2/(
c*x^2+b*x+a)^(1/2)*x+1/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*f^
2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+f^2/c^2*b^2/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x-4/3*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-2/3*d*e*b^2/c/
(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/2*e*f*b/c^2*x/(c*x^2+b*x+a)^(3/2)+1/12*e*
f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2/3*e*f*b^4/c^2/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)+1/6*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2+1/6*b^3/c^
2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*f+8/3*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(
1/2)*x*d*f+4/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f+4/3*a/(4*a*c-b^2
)/(c*x^2+b*x+a)^(3/2)*x*d*f+1/3*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*e^2+1
6/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2+f^2/c^(5/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))-2*e*f*x^2/c/(c*x^2+b*x+a)^(3/2)+1/12*e*f*b^2/c
^3/(c*x^2+b*x+a)^(3/2)-4/3*e*f*a/c^2/(c*x^2+b*x+a)^(3/2)+1/2*f^2*b/c^2*x^2/
(c*x^2+b*x+a)^(3/2)+1/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)-1/48*f^2*b^5/c^4/
(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/6*f^2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)
^(1/2)+1/3*f^2*b/c^3*a/(c*x^2+b*x+a)^(3/2)+1/2*f^2/c^3*b^3/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)+2/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2+2/3*a/(4*a*
c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2+8/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*
e^2-16/3*d*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4/3*d^2/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)*x*c+32/3*d^2*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*d
^2*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b-x/c/(c*x^2+b*x+a)^(3/2)*d*f+1/6*b/
c^2/(c*x^2+b*x+a)^(3/2)*d*f+32/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*
f-32/3*d*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-16*e*f*b*a/(4*a*c-b^2)^2
/(c*x^2+b*x+a)^(1/2)*x-8*e*f*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/6*
e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*f^2*b^2/c^2*a/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)*x+4*f^2*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/
3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*f+2/3*a/c/(4*a*c-b^2)/(c*x^2+b*
x+a)^(3/2)*b*d*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2+4/3*b^2/(
4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2+4/3*e*f*b^3/c/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)*x-e*f*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x)
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
[Out] Timed out
```

3.120 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)
/(c*x^2+b*x+a)^(3/2)+2/3*(8*c*d-4*b*e+4*a*f+b^2*f/c)*(2*c*x+b)/(-4*a*c+b^2)
^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1660, 12, 613}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b
^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{8cd-4be+4af+\frac{b^2f}{c}}{2\left(a+bx+cx^2\right)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{\left(8cd-4be+4af+\frac{b^2f}{c}\right)\int \frac{}{(a+}}{3\left(b^2-4ac\right)}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8cd-4be+4af+\frac{b^2f}{c}\right)(b}{3\left(b^2-4ac\right)^2\sqrt{a+bx+c}}$$

Mathematica [A] time = 0.38, size = 147, normalized size = 1.12

$$\frac{8b\left(2a^2f+3ac\left(d-ex+fx^2\right)-2c^2x^2(ex-3d)\right)+16c\left(-a^2e+acx\left(3d+fx^2\right)+2c^2dx^3\right)-4b^2\left(a(e-6fx)-cx\left(\right.\right.}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d + f
*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f -
 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(a +
 x*(b + c*x))^(3/2))
```

fricas [B] time = 4.02, size = 286, normalized size = 2.18

$$\frac{2\left(8\,a^2bf+2\left(8\,c^3d-4\,bc^2e+\left(b^2c+4\,ac^2\right)f\right)x^3+3\left(8\,bc^2d-4\,b^2ce+\left(b^3+4\,abc\right)f\right)x^2-\left(b^3-12\,abc\right)d-2\left(ab^2\right.\right.}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*b
*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b^2
 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e)*x
)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 +
(b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*x)
```

giac [A] time = 0.32, size = 240, normalized size = 1.83

$$\frac{2\left(\left(\left(\frac{2\left(8\,c^3d+b^2cf+4\,ac^2f-4\,bc^2e\right)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\left(8\,bc^2d+b^3f+4\,abcf-4\,b^2ce\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{3\left(2\,b^2cd+8\,ac^2d+4\,ab^2f-b^3e-4\,abce\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x-\frac{b^3d-12\,abcd-8\,a^2bf+2\,ab}{b^4-8\,ab^2c+16\,a^2c}\right.}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(8*c^3*d + b^2*c*f + 4*a*c^2*f - 4*b*c^2*e)*x/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) + 3*(8*b*c^2*d + b^3*f + 4*a*b*c*f - 4*b^2*c*e)/(b^4 - 8*a*b^2*c
```

```
 + 16*a^2*c^2))*x + 3*(2*b^2*c*d + 8*a*c^2*d + 4*a*b^2*f - b^3*e - 4*a*b*c*
e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x - (b^3*d - 12*a*b*c*d - 8*a^2*b*f + 2*
a*b^2*e + 8*a^2*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)^(3/2
)
```

maple [A] time = 0.01, size = 185, normalized size = 1.41

$$\frac{\frac{16}{3}a c^2 f x^3 + \frac{4}{3}b^2 c f x^3 - \frac{16}{3}b c^2 e x^3 + \frac{32}{3}c^3 d x^3 + 8abcf x^2 + 2b^3 f x^2 - 8b^2 ce x^2 + 16b c^2 d x^2 + 8a b^2 f x - 8abcex}{\left(c x^2 + bx + a\right)^{\frac{3}{2}} \left(16a^2c^2 - 8a b^2c + b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d
*x^3+12*a*b*c*f*x^2+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-
12*a*b*c*e*x+24*a*c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2
*e+12*a*b*c*d-b^3*d)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 3.71, size = 175, normalized size = 1.34

$$\frac{2\left(8 f a^2 b - 8 e a^2 c + 12 f a b^2 x - 2 e a b^2 + 12 f a b c x^2 - 12 e a b c x + 12 d a b c + 8 f a c^2 x^3 + 24 d a c^2 x + 3\right.}{3\left(4 a c - b^2\right)^2 \left(c x^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*c^3*d*x^3 - b^3*d + 3*b^3*f*x^2 - 2*a*b^2*e + 8*a^2*b*f - 8*a^2*c*e
- 3*b^3*e*x + 24*a*c^2*d*x + 12*a*b^2*f*x + 6*b^2*c*d*x + 24*b*c^2*d*x^2 -
12*b^2*c*e*x^2 + 8*a*c^2*f*x^3 - 8*b*c^2*e*x^3 + 2*b^2*c*f*x^3 + 12*a*b*c*d
 - 12*a*b*c*e*x + 12*a*b*c*f*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx$

Optimal. Leaf size=51

$$\frac{1}{10}\tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

```
[Out] 1/10*arctan(5/2*(2+x)/(5*x^2+2*x-7)^(1/2))+1/5*arctanh(5*(1+x)/(5*x^2+2*x-7
)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {986, 1029, 203, 207}

$$\frac{1}{10}\tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqr
t[-7 + 2*x + 5*x^2]]/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}dx = -\left(\frac{1}{50}\int \frac{-100-50x}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}dx\right)+\frac{1}{50}\int \frac{}{\sqrt{-7+2x+}}$$

$$= 400\,\text{Subst}\left(\int \frac{1}{160000+100x^2}dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}}\right)+1600\,\text{Sub}$$

$$= \frac{1}{10}\tan^{-1}\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right)$$

Mathematica [C] time = 0.04, size = 81, normalized size = 1.59

$$\left(\frac{1}{10}-\frac{i}{20}\right)\tanh^{-1}\left(\frac{\left(\frac{1}{100}+\frac{i}{50}\right)((100-40i)x+(164-8i))}{\sqrt{5x^2+2x-7}}\right)-\left(\frac{1}{20}-\frac{i}{10}\right)\tan^{-1}\left(\frac{\left(\frac{1}{50}+\frac{i}{100}\right)((-100-40i)x-(16}{\sqrt{5x^2+2x-7}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] (-1/20 + I/10)*ArcTan[((1/50 + I/100)*((-164 - 8*I) - (100 + 40*I)*x))/Sqrt
[-7 + 2*x + 5*x^2]] + (1/10 - I/20)*ArcTanh[((1/100 + I/50)*((164 - 8*I) +
(100 - 40*I)*x))/Sqrt[-7 + 2*x + 5*x^2]]
```

fricas [B] time = 0.76, size = 154, normalized size = 3.02

$$\frac{1}{20}\arctan\left(\frac{27\,x^2+20\,\sqrt{5\,x^2+2\,x-7}\,(x+2)+36\,x}{31\,x^2+16\,x-56}\right)+\frac{1}{20}\arctan\left(-\frac{27\,x^2-20\,\sqrt{5\,x^2+2\,x-7}\,(x+2)+36\,x}{31\,x^2+16\,x-56}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 16
*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)
/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x + 1)
 + 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1) + 26
*x + 9)/x^2)
```

giac [B] time = 0.28, size = 205, normalized size = 4.02

$$-\frac{1}{10}\arctan\left(-\frac{5\,\sqrt{5}\,x+6\,\sqrt{5}\,-5\,\sqrt{5\,x^2+2\,x-7}\,+5}{2\left(\sqrt{5}\,+5\right)}\right)-\frac{1}{10}\arctan\left(\frac{5\,\sqrt{5}\,x+6\,\sqrt{5}\,-5\,\sqrt{5\,x^2+2\,x-7}\,-5}{2\left(\sqrt{5}\,-5\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/(
sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2
*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7)
)^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5) +
65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x - sqr
t(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)
```

maple [B] time = 0.02, size = 144, normalized size = 2.82

$$-\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2}+9}\left(2\operatorname{arctanh}\left(\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2}+9}}{5}\right)+\arctan\left(\frac{5\sqrt{-\frac{4(x+2)^2}{(-x-1)^2}+9}\,(x+2)}{2\left(\frac{4(x+2)^2}{(-x-1)^2}-9\right)(-x-1)}\right)\right)}{10\sqrt{-\frac{\frac{4(x+2)^2}{(-x-1)^2}-9}{\left(1+\frac{x+2}{-x-1}\right)^2}}\left(1+\frac{x+2}{-x-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x)
```

```
[Out] -1/10*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(x+2)^2/(-1-x)^2+9)^
(1/2))+arctan(5/2*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)/(4*(x+2)^2/(-1-x)^2-9)*(x+2
)/(-1-x)))/(-(4*(x+2)^2/(-1-x)^2-9)/(1+(x+2)/(-1-x))^2)^(1/2)/(1+(x+2)/(-1-
x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+12\,x+8\right)\sqrt{5\,x^2+2\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{5\,x^2+2\,x-7}\,\left(5\,x^2+12\,x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)),x)
```

```
[Out] int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(5x+7)}\left(5x^2+12x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)
```

3.122 $\int \frac{1}{\sqrt{a+bx+cx^2}\,\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=1432

$$\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4a\right.}}$$

$$-\overline{}$$

```
[Out] -(cos(2*arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^
(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1
/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2
))^(1/2)))^2)^(1/2)/cos(2*arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-
4*a*c+b^2)^(1/2))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e
+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x
+(-4*a*c+b^2)^(1/2))^(1/2)))*EllipticF(sin(2*arctan((2*c^2*d-b*c*e+b^2*f-2*
a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/
2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2))),1/2*(2+(2*a*f-b*e+2*c*d)*(b
+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*
(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c^2*d+b*f*(b+(-4*a*c+b^2)^(1/2))-c*(b*e+2*a*f
+e*(-4*a*c+b^2)^(1/2)))^(1/2))^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))^(3/2)*(b
^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1
/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((f*x^2+e*x+d)*(4*a*c-(b+(-4*a*c+b
^2)^(1/2))^2)^2/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^
2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)*(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2
)))*(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f
+e*(-4*a*c+b^2)^(1/2)))^(1/2))*((1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*
d-2*c*e*(b+(-4*a*c+b^2)^(1/2))+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c
+b^2)^(1/2))^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2
))^2)-(2*a*f-b*e+2*c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)
))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c
*d-2*a*f+e*(-4*a*c+b^2)^(1/2))))/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d
-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c*x+(-4*a*c+
b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b
^2)^(1/2)))^(1/2))^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c
+b^2)^(1/2))^(1/4)/(4*a*c-(b+(-4*a*c+b^2)^(1/2))^2)/(c*x^2+b*x+a)^(1/2)/(f*
x^2+e*x+d)^(1/2)/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*d-2*c*e*(b+(-4*
a*c+b^2)^(1/2))+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/
(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2)-(2*a*f-b*
e+2*c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(b+2*c*x+(-4
*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*
a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] time = 6.22, antiderivative size = 1432, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size =

$29, \frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {992, 935, 1103}

$$-\frac{\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e
- 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[
b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + e*x + f*x
^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*
f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f -
 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)
)/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c
]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqrt[b^2 -
4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b^2*d +
b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e
+ (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b +
Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - S
qrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*
d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)
]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[((2*c^2*d - b*c*e
 + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[2*a + (b + S
qrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d +
Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])],
(2 + ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f))/(Sqrt[b^2*d + b*(Sqrt[
b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[2*c^2
*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]))
/4])/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - b*c*e + b^2*f - 2*a*c*
f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x
 + f*x^2]*Sqrt[1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b
 + Sqrt[b^2 - 4*a*c])*x))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*
d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^
2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (
b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[
b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

Rule 935

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2
*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sq
rt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 -
b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 992

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r +
2*c*x]*Sqrt[2*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*
c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\,\cdots}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\cdots}{\left(\left(b+\sqrt{b^2-4ac}\right)\cdots\right.}}\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt[4]{b^2d+b\left(\sqrt{b^2-4ac}\,d-ae\right)-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+\sqrt{b^2\cdots}\right.}{\cdots}$$

Mathematica [A] time = 2.34, size = 670, normalized size = 0.47

$$-\frac{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(-\sqrt{e^2-4df}+e+2fx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(\sqrt{e^2-4df}+e+2fx\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)\right)}}\sqrt{-\frac{c\left(\sqrt{b^2-4ac}\sqrt{e\cdots}\right.}{\cdots}}\cdots}{\sqrt{a+x(b+cx)}\sqrt{d+x(e+fx)}\left(f\left(\sqrt{b^2\cdots}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -(((-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[-(
(c*Sqrt[b^2 - 4*a*c]*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(((b + Sqrt[b^2 - 4*a
*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqr
t[-((c*(4*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f] - 2*Sqrt[b^2 - 4*a*c]*f
*x + 2*c*Sqrt[e^2 - 4*d*f]*x - e*(Sqrt[b^2 - 4*a*c] + 2*c*x) + b*(-e + Sqrt
```

```
[e^2 - 4*d*f] + 2*f*x)))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4
*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*EllipticF[ArcSin[Sqrt[(((-b + S
qrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x))]], (2*c*d - b*e + 2*a*f - Sqrt[b^2 - 4*a*c]*Sqrt[e^
2 - 4*d*f])/(2*c*d - b*e + 2*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f])])/(
((-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*Sqrt[(c*Sqrt[b^2 -
 4*a*c]*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(
-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*Sqrt[a + x*(b +
 c*x)]*Sqrt[d + x*(e + f*x)]))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^2+bx+a}\,\sqrt{fx^2+ex+d}}{cfx^4+\left(ce+bf\right)x^3+\left(cd+be+af\right)x^2+ad+(bd+ae)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)/(c*f*x^4 + (c*e + b*f)
*x^3 + (c*d + b*e + a*f)*x^2 + a*d + (b*d + a*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{fx^2+ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

maple [A] time = 0.27, size = 928, normalized size = 0.65

$$8\left(2bf^2x^2-2cefx^2+2befx-8cdfx+2\sqrt{-4df+e^2}\,cfx^2+2\sqrt{-4ac+b^2}\,f^2x^2-2bdf+be^2+2\sqrt{-4df+e^2}\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 8*(2*b*f^2*x^2-2*c*e*f*x^2+2*x^2*c*f*(-4*d*f+e^2)^(1/2)+2*(-4*a*c+b^2)^(1/2
)*f^2*x^2+2*b*e*f*x+2*x*b*f*(-4*d*f+e^2)^(1/2)-8*c*d*f*x+2*x*e*f*(-4*a*c+b^
2)^(1/2)+2*x*f*(-4*d*f+e^2)^(1/2)*(-4*a*c+b^2)^(1/2)-2*b*d*f+b*e^2+b*e*(-4*
d*f+e^2)^(1/2)-2*c*d*e-2*c*d*(-4*d*f+e^2)^(1/2)-2*(-4*a*c+b^2)^(1/2)*d*f+e^
2*(-4*a*c+b^2)^(1/2)+e*(-4*d*f+e^2)^(1/2)*(-4*a*c+b^2)^(1/2))*EllipticF(((
-4*d*f+e^2)^(1/2)*c-(-4*a*c+b^2)^(1/2)*f-b*f+c*e)*(-2*f*x+(-4*d*f+e^2)^(1/2
)-e)/((-4*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2
)^(1/2)+e))^(1/2),(((-4*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f-b*f+c*e)*((-4
*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/((-4*d*f+e^2)^(1/2)*c-(-4*a
*c+b^2)^(1/2)*f+b*f-c*e)/((-4*d*f+e^2)^(1/2)*c-(-4*a*c+b^2)^(1/2)*f-b*f+c*e
))^(1/2))*((-4*d*f+e^2)^(1/2)*(2*c*x+b+(-4*a*c+b^2)^(1/2))*f/((-4*d*f+e^2)^
(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*(
-(-4*d*f+e^2)^(1/2)*(-2*c*x-b+(-4*a*c+b^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c-(
-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*(((-4*d*f+
```

```
e^2)^(1/2)*c-(-4*a*c+b^2)^(1/2)*f-b*f+c*e)*(-2*f*x+(-4*d*f+e^2)^(1/2)-e)/((
-4*d*f+e^2)^(1/2)*c+(-4*a*c+b^2)^(1/2)*f+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)
+e))^(1/2)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^(1/2)/(1/c/f*(-2*f*x+(-4*d*f+e
^2)^(1/2)-e)*(2*f*x+(-4*d*f+e^2)^(1/2)+e)*(-2*c*x-b+(-4*a*c+b^2)^(1/2))*(2*
c*x+b+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*d*f+e^2)^(1/2)/((-4*a*c+b^2)^(1/2)*f-(
-4*d*f+e^2)^(1/2)*c+b*f-c*e)/(c*f*x^4+b*f*x^3+c*e*x^3+a*f*x^2+b*e*x^2+c*d*x
^2+a*e*x+b*d*x+a*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\,\sqrt{fx^2+ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

3.123 $\int \frac{1}{\sqrt{3-x+2x^2}\,\sqrt{2+3x+5x^2}}\,dx$

Optimal. Leaf size=652

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\,\sqrt{6-\left(1-i\sqrt{23}\right)x}\,\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1-\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right)x\right)}{-4x-i\sqrt{23}+1}\right)\cdots}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\,\sqrt{2x^2-x+3}\,\sqrt{5x^2+3x+2}\,\sqrt{-\frac{11\cdots}{(\cdots}}}$$

```
[Out] 1/11*(cos(2*arctan(((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I*23^(1/2
)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2)))^2)^(1/2)/cos(2*arctan(((-3*I+23^(1/2)
)/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I*23^(1/2)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2
)))*EllipticF(sin(2*arctan(((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I
*23^(1/2)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2))),1/22*11^(1/2)*((66*I-22*23^(1
/2)+41*(-23*(3*I-23^(1/2))/(7*I+23^(1/2)))^(1/2)+41*I*((-3*I+23^(1/2))/(7*I
+23^(1/2)))^(1/2))/(3*I-23^(1/2)))^(1/2))*253^(1/2)*(1-4*x-I*23^(1/2))*(6-x
*(1-I*23^(1/2)))^(1/2)*(-1+4*x+I*23^(1/2))^(1/2)*(1-(6-x*(1-I*23^(1/2)))*((
-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/2)/(1-4*x-I*23^(1/2)))*((5*x^2+3*x+2)*(11
*I-23^(1/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2)))^(1/2)*((11-11*(6-x*(1-I*2
3^(1/2)))^2*(3*I-23^(1/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2))-41*(6-x*(1-I
*23^(1/2)))*(I+23^(1/2))/(1-4*x-I*23^(1/2))/(7*I+23^(1/2)))/(1-(6-x*(1-I*23
^(1/2)))*((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/2)/(1-4*x-I*23^(1/2)))^2)^(1/2
)/(23+I*23^(1/2))/((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)/(2*x^2-x+3)^(1/2)/
(5*x^2+3*x+2)^(1/2)/(11-11*(6-x*(1-I*23^(1/2)))^2*(3*I-23^(1/2))/(1-4*x-I*2
3^(1/2))^2/(7*I+23^(1/2))-41*(6-x*(1-I*23^(1/2)))*(I+23^(1/2))/(1-4*x-I*23^
(1/2))/(7*I+23^(1/2)))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {992, 935, 1103}

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\,\sqrt{6-\left(1-i\sqrt{23}\right)x}\,\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1-\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right)x\right)}{-4x-i\sqrt{23}+1}\right)\cdots}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\,\sqrt{2x^2-x+3}\,\sqrt{5x^2+3x+2}\,\sqrt{-\frac{11\cdots}{(\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] (Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (1
 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt[2
3])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23
]))]*(6 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))*Sqrt[(11 - (41*(I +
Sqrt[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x
)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1
- I*Sqrt[23] - 4*x)^2))/(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))]*(6
 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))^2]*EllipticF[2*ArcTan[((-((
3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[6 - (1 - I*Sqrt[23])*x])/Sqrt
```

```
[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 + I*Sqrt[23]])/
88])/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[3
 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 - (41*(I + Sqrt[23])*(6 - (1 -
I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt
[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^
2)])
```

Rule 935

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2
*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sq
rt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 -
b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 992

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r +
2*c*x]*Sqrt[2*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*
c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\,\sqrt{2+3x+5x^2}}\,dx = \frac{\left(\sqrt{-1+i\sqrt{23}+4x}\,\sqrt{6+\left(-1+i\sqrt{23}\right)x}\right)\int \frac{1}{\sqrt{-1+i\sqrt{23}+4x}\,\sqrt{6+\left(-1+i\sqrt{23}\right)x}\cdots}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{\left(2\left(-1+i\sqrt{23}+4x\right)^{3/2}\sqrt{6+\left(-1+i\sqrt{23}\right)x}\,\sqrt{\frac{\left(24-\left(-1+i\sqrt{23}\right)^2\right)\cdots}{\left(180-18\left(-1+i\sqrt{23}\right)+2\left(-1+i\cdots\right.\right.}}\cdots\right.}{\left(24-\cdots\right.}$$

$$= -\frac{\sqrt{\frac{23}{11}}\left(-1+i\sqrt{23}+4x\right)^{3/2}\sqrt{6-\left(1-i\sqrt{23}\right)x}\,\sqrt{\frac{\left(11i-\sqrt{23}\right)\left(2+3x+5x^2\right)}{\left(7i+\sqrt{23}\right)\left(1-i\sqrt{23}-4x\right)^2}}\left(\cdots\right.}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\cdots}$$

Mathematica [A] time = 0.61, size = 390, normalized size = 0.60

$$\frac{\left(-4x+i\sqrt{23}+1\right)\left(10ix+\sqrt{31}+3i\right)\sqrt{\frac{20ix-2\sqrt{31}+6i}{\left(11i+5\sqrt{23}-2\sqrt{31}\right)\left(4ix+\sqrt{23}-i\right)}}\sqrt{\frac{\left(-22-10i\sqrt{23}+4i\sqrt{31}\right)x-\sqrt{713}-i\sqrt{31}-3i\sqrt{23}+63}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(4ix+\sqrt{23}-i\right)}}F\left(\mathrm{si}\right.}{\left(-11i+5\sqrt{23}-2\sqrt{31}\right)\sqrt{\frac{10ix+\sqrt{31}+3i}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(4ix+\sqrt{23}-i\right)}}\sqrt{2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] ((1 + I*Sqrt[23] - 4*x)*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[(6*I - 2*Sqrt[31]
+ (20*I)*x)/((11*I + 5*Sqrt[23] - 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*S
qrt[(63 - (3*I)*Sqrt[23] - I*Sqrt[31] - Sqrt[713] + (-22 - (10*I)*Sqrt[23]
+ (4*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*
I)*x))]*EllipticF[ArcSin[Sqrt[2]*Sqrt[-((-63 + (3*I)*Sqrt[23] + I*Sqrt[31]
+ Sqrt[713] + 2*(11 + (5*I)*Sqrt[23] - (2*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[2
3] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x)))]], (1197 + 41*Sqrt[713])/484])
/((-11*I + 5*Sqrt[23] - 2*Sqrt[31])*Sqrt[(3*I + Sqrt[31] + (10*I)*x)/((11*I
 + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*Sqrt[3 - x + 2*x^2]
*Sqrt[2 + 3*x + 5*x^2])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{5\,x^2+3\,x+2}\,\sqrt{2\,x^2-x+3}}{10\,x^4+x^3+16\,x^2+7\,x+6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)/(10*x^4 + x^3 + 16*x^2 +
 7*x + 6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\,x^2+3\,x+2}\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

maple [A] time = 0.47, size = 418, normalized size = 0.64

$$-\frac{4i\sqrt{5x^2+3x+2}\,\sqrt{2x^2-x+3}\,\left(5i\sqrt{23}+2i\sqrt{31}-11\right)\sqrt{\frac{\left(5i\sqrt{23}-2i\sqrt{31}+11\right)\left(4x-1+i\sqrt{23}\right)}{\left(5i\sqrt{23}+2i\sqrt{31}-11\right)\left(-4x+i\sqrt{23}+1\right)}}\left(-4x+i\sqrt{23}+1\right)^2\sqrt{-}}{23\sqrt{10x^4+x^3+16x^2+7x+6}\,\left(5i\sqrt{23}-2i\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] -4/23*I*(5*x^2+3*x+2)^(1/2)*(2*x^2-x+3)^(1/2)*(5*I*23^(1/2)+2*I*31^(1/2)-11
)*((5*I*23^(1/2)-2*I*31^(1/2)+11)*(-1+4*x+I*23^(1/2))/(5*I*23^(1/2)+2*I*31^
(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*23^(1/2)-4*x+1)^2*(-I*23^(1/2)*(I*31
^(1/2)+10*x+3)/(5*I*23^(1/2)-2*I*31^(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*
```

```
23^(1/2)*(I*31^(1/2)-10*x-3)/(5*I*23^(1/2)+2*I*31^(1/2)-11)/(I*23^(1/2)-4*x
+1))^(1/2)*23^(1/2)*10^(1/2)*EllipticF(((5*I*23^(1/2)-2*I*31^(1/2)+11)*(-1+
4*x+I*23^(1/2))/(5*I*23^(1/2)+2*I*31^(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2),((
5*I*23^(1/2)+2*I*31^(1/2)+11)*(5*I*23^(1/2)+2*I*31^(1/2)-11)/(5*I*23^(1/2)-
2*I*31^(1/2)-11)/(5*I*23^(1/2)-2*I*31^(1/2)+11))^(1/2))/(10*x^4+x^3+16*x^2+
7*x+6)^(1/2)/(5*I*23^(1/2)-2*I*31^(1/2)+11)/((-1+4*x+I*23^(1/2))*(I*23^(1/2
)-4*x+1)*(I*31^(1/2)+10*x+3)*(I*31^(1/2)-10*x-3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\,x^2+3\,x+2}\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\sqrt{5\,x^2+3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)),x)
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\,\sqrt{5x^2+3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**(1/2)/(2*x**2-x+3)**(1/2),x)
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*sqrt(5*x**2 + 3*x + 2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```